1: FA

~~Given a word ω, we will prove:~~

~~$\forall n.\ |\omega| = n \Rightarrow (\omega \in L(A) \Leftrightarrow \omega \in$~~

Proof sketch that our Reverse (NWA) alg works

Given a word ω, let $\overleftarrow{\omega}_k$ and $\overrightarrow{\omega}_k$ be rightmost & leftmost k chars in ω.

Want: for $n \in 0 \dots |w|$

This is the complex rule

$$\frac{(c,\sigma,e) \in \delta_c \quad (x,c,\tau,r) \in \delta_r}{\begin{array}{l}(r,\tau,x) \in \delta_c^{rev} \\ (r',\tau,x) \\ (e,r,\sigma,c) \\ (e,r',\sigma,c) \quad \text{in } \delta_r^{rev} \\ (e',q_0,\sigma,c') \\ \text{(for each } q_0 \in Q_0^{rev}\text{)}\end{array}}$$

<u>Backwards</u> induction

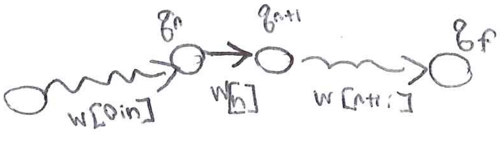
M no ε but perhaps nondet

Given a word w:

If $w \in L(M)$, then for each $n \in 0 \dots |w|$ after reading $w[0:n]$, it must be able to reach a state q_n w/ $\delta(q_n, w[n:]) \ni q_f$ (q_f final)

If $n = |w|$, then q_n must be q_f, and we are done. I guess. Somehow.

If $n < |w|$, then



2-FA

By induction,

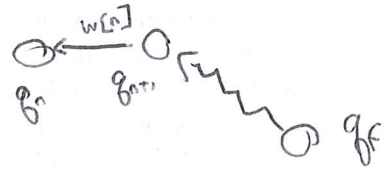
M^R running $w^R[n+1:]$ $(w[n+1:])^R$

~~the first |w| - (n+1) chars in w, i.e. those correspond~~

takes us from q_f to q_{n+1};

Because M contains $(q_n, w[n], q_{n+1})$,
M^R contains $(q_{n+1}, w[n], q_n)$.

Thus M^R (can) goes from q_f to q_n on $(w[n:])^R$



So if $w \in L(M)$,
$\forall n \in 0 \ldots |w|$

$\exists q_n$ s.t. $\delta_M(q_0, w[0:n]) \ni q_n$
$\exists q_f$ and $\delta_{M^R}(q_f, (w[n:])^R) \ni q_n$.

Base case is now $n = |w|$

Let q_n be in $Q_f \cap \delta(q_0, w)$. First bit trivial.

2nd bit trivial as $q_f = q_n$ gives $\delta_{M^R}(q_f, (w[|w|:])^R)$

$= \delta_{M^R}(q_f, \varepsilon) = q_f$

NWA start

If $w \in L(M)$

then for n in $0 \dots |w|$

$\exists\, q_0, q_n, q_F$ st.

$\{ \delta_M(q_0, w[0:n]) \ni q_n$

$\{ \delta_{M^R}(q_F, (w[n:])^R) \ni q_n$

→ if $(w[n:])^R$ has no pending returns (in rev string - "()" ok, "())" ok, "(()" bad);

ow $\delta_{M^R}(\dots) \ni q_n'$

3 → NWA

$w \in L(M)$

Base case same.

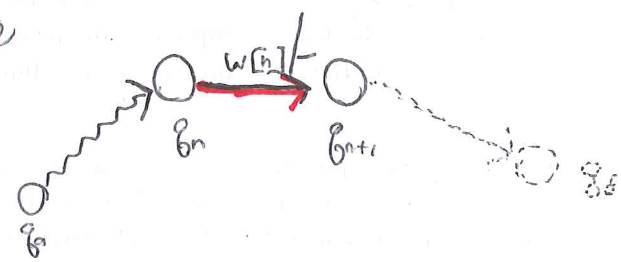
Let $q_n \in \delta(q_0, w) \cap Q_f$, and $q_f = q_n$.

Inductive case

$w[n]$ internal plays out as before (maybe w/ staying same level)

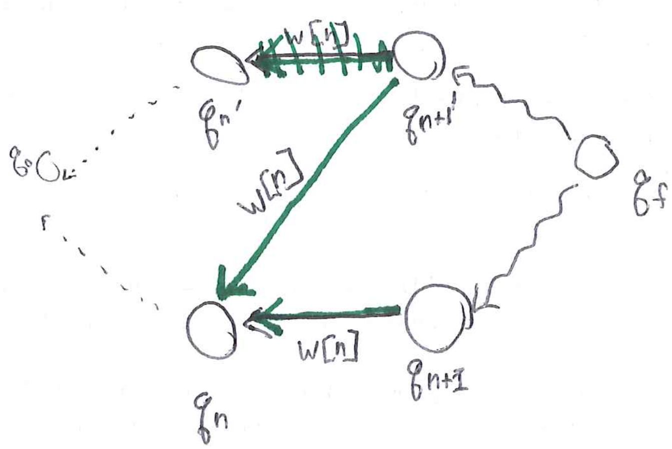
$w[n]$ any return

M:



▬ = return trans

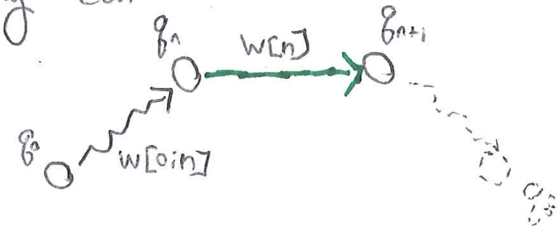
turns into a call:



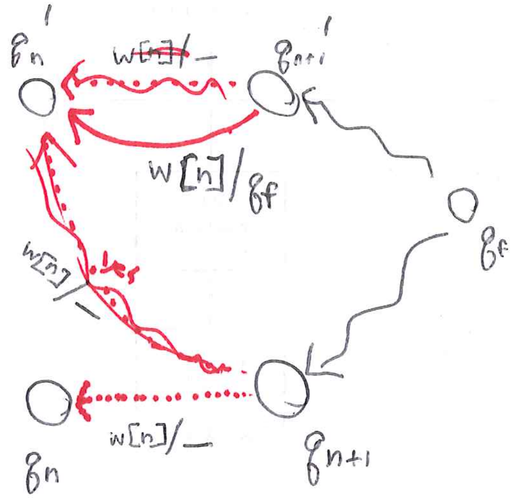
— = call trans

This is legal regardless of ~~properness~~ $(w[n:])^R$, so if q_{n+1} was reachable, so is q_n (& likewise for $q_{n+1}' \to q_n'$)

In both cases, a following return will be matched w[n] pending call



then "$(w[n])^R$" is pending return, so we are in q_{n+1}'



and can take $(q_{n+1}', q_f, w[n], q_n')$

And still in primed state

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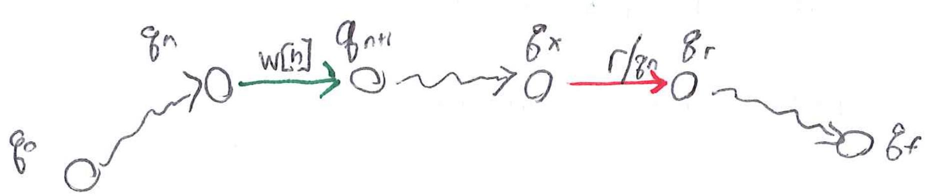
w[n] matched call



so $(q_n, w[n], q_{n+1}) \in \delta_c$

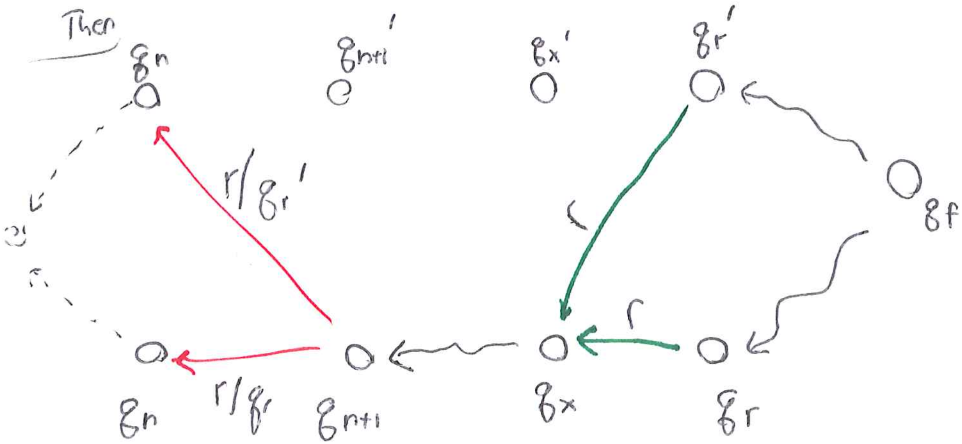
$(q_x, q_n, \cancel{q_x}, q_r) \in \delta_r$

so (q_{n+1}, q_r, r, q_n)

(q_{n+1}, q_r', r, q_n')

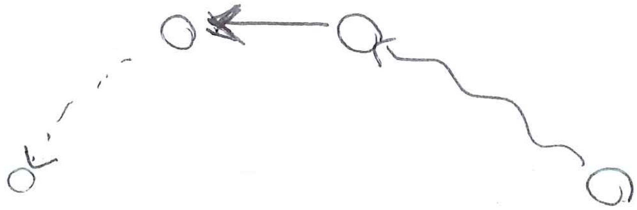
in δ_r^{rev}

Then



(if $w^R \in L(M^R)$ then $w \in L(M)$)

Basically same



~~w[n] internal~~

Stmt:

if $w^R \in L(M^R)$, then for $n \in 0 \ldots |w|$

$\varphi(n)$ $\exists q_0, q_n, q_f$ st.

~~[illegible]~~

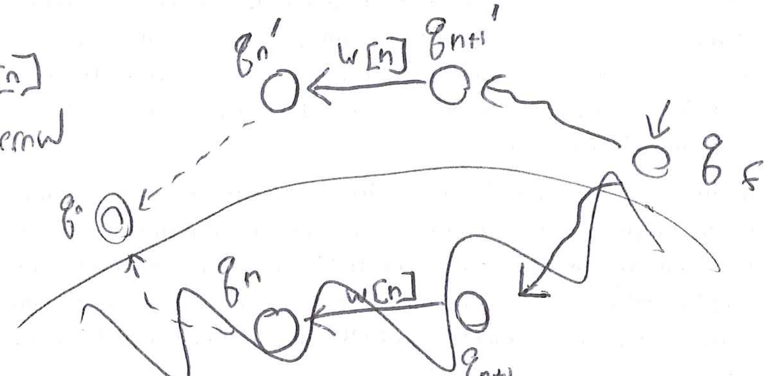
And • Or • $\delta_{M^R}(q_f, (w[n:])^R) \ni q_n$ and $w[n:]$ has no pending calls

• $\delta_{M^R}(q_f, (w[n:])^R) \ni q_n'$ $w[n:]$ has pending calls

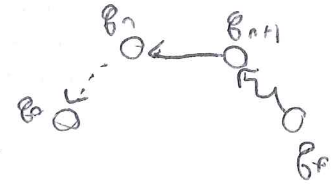
• $\delta_M(q_0, w[0:n]) \ni q_n$

Base ~~Base:~~ ~~$\varphi(|w|)$ ho~~ $\varphi(0)$ holds: take path from $q_0 \to q_f$ that witnesses acceptance, and $q_n = q_f$

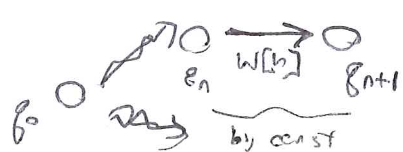
Ind: w[n] internal



or



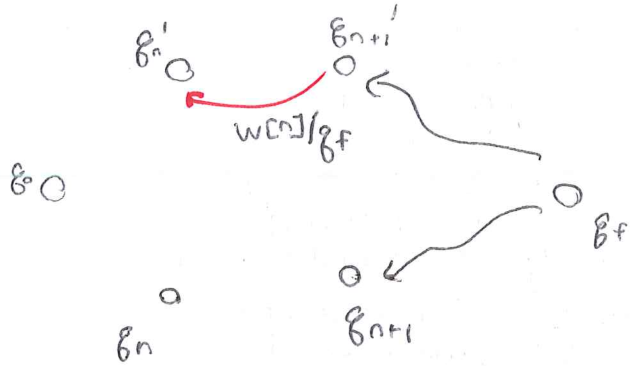
Know ind that



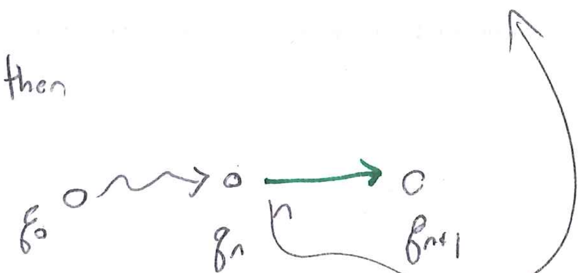
$w[n]$ pending call (in forward; return in back)

in q'_{n+1}

(not q_n)



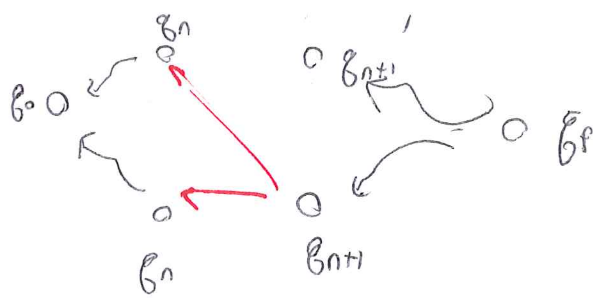
well then



as that is the only way to get this trans in M^R

$w[n]$ matched call in q'_{n+1}

one of these paths must go



so