

# Towards Abstraction Refinement in TVLA

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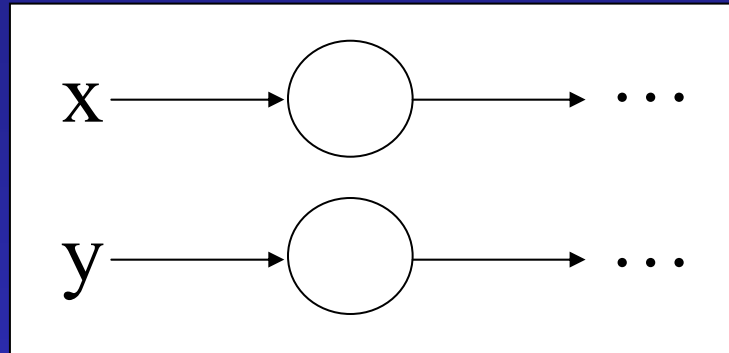
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# Need for Heap Data Analysis

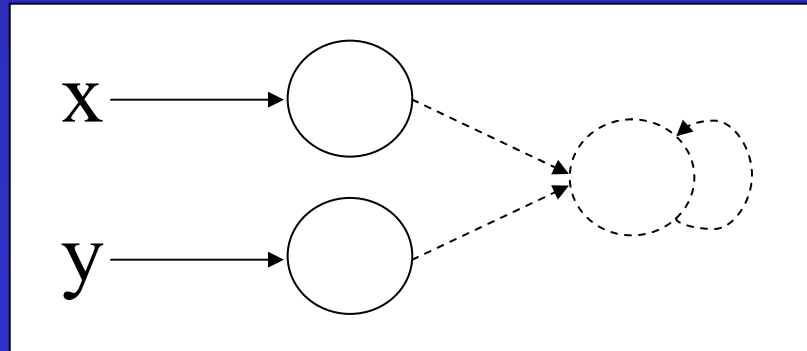
- Talks by WiSA students
  - Static analyses for de-obfuscation of code
  - Limited ability to analyze heap data
  - Need understanding of possible shapes
    - Want to handle unbounded heap object creation
    - Java objects (e.g. threads) are in heap

# Linked List Abstractions

- Informally

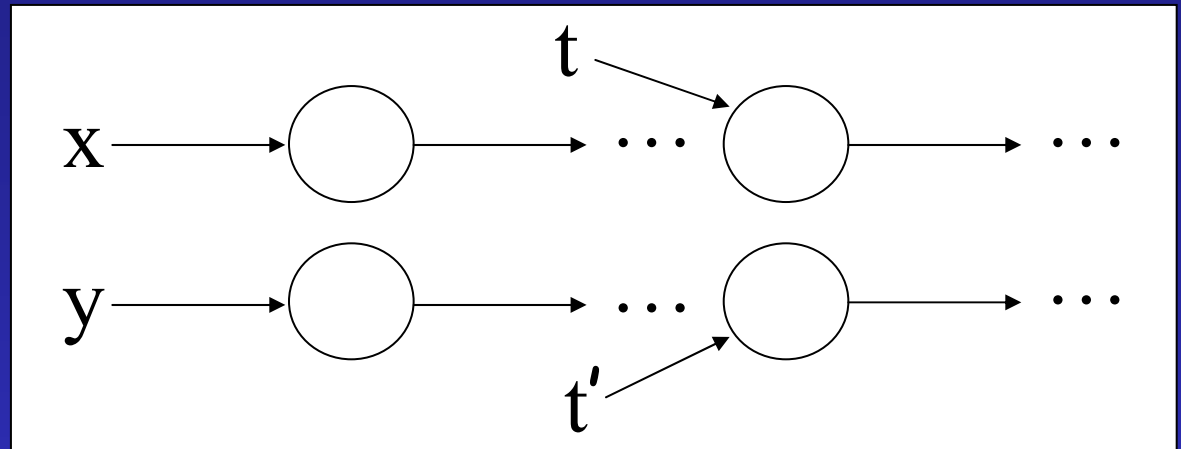


- Formally

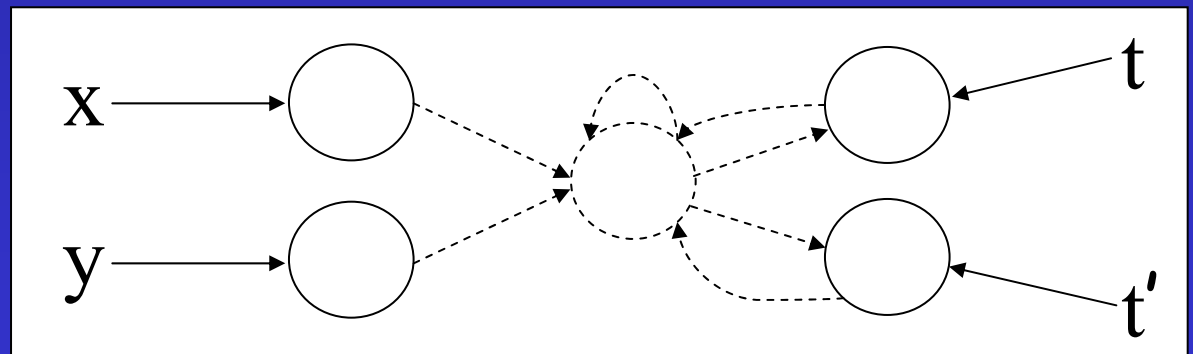


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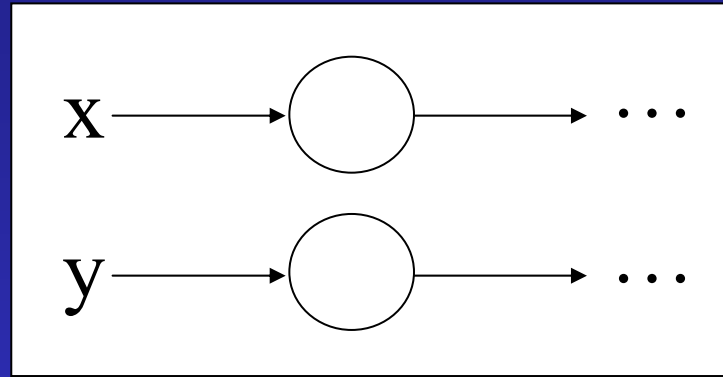


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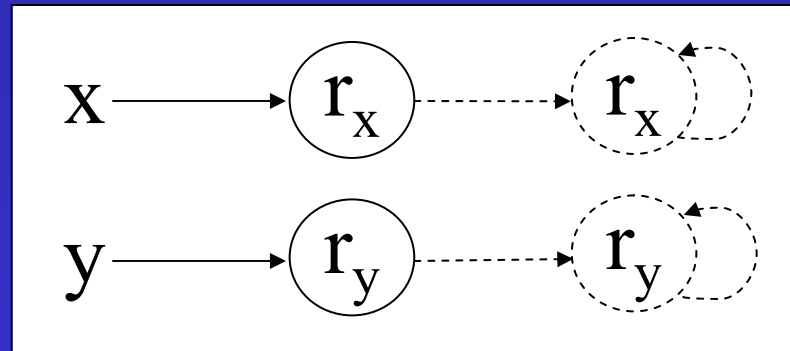


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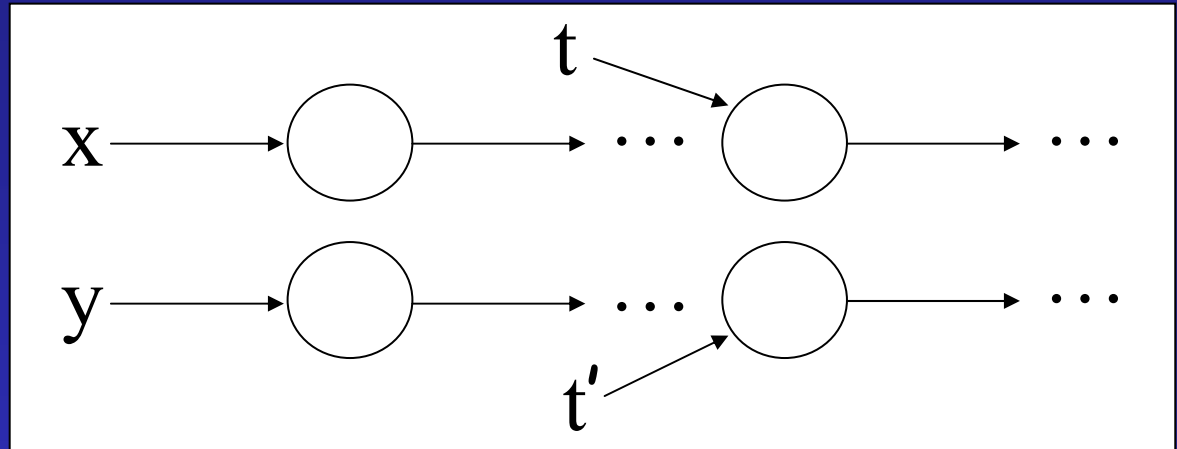


- Formally

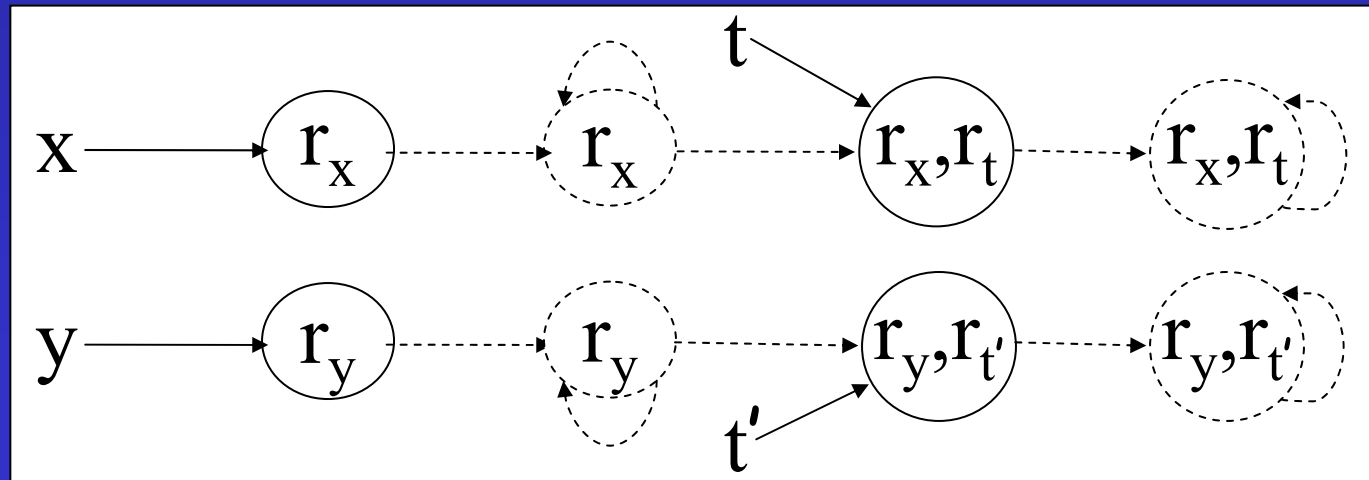


# Linked List Abstractions

- Informally



- Formally



# Abstraction Refinement

- Devising good analysis abstractions is hard
  - Precision/cost tradeoff
    - Too coarse: “Unknown” answer
    - Too refined: high space/time cost
- Start with simple (and cheap) abstraction
- Successively refine abstraction
  - Adaptive algorithm

# Abstraction Refinement

- Iterative process
  - Create an abstraction (e.g. set of predicates)
  - Run analysis
  - Detect indefinite answers (stop if none)
  - Refine abstraction (e.g. add predicates)
  - Repeat above steps



# Previous Work on Abstraction Refinement

- Counterexample guided [Clarke et al]
  - Finds shortest invalid prefix of spurious counterexample
  - Splits last state on prefix into less abstract ones
- SLAM toolkit [Ball, Rajamani]
  - Temporal safety properties of software
  - Identifies correlated branches

# Abstraction Refinement for TVLA: New Challenges

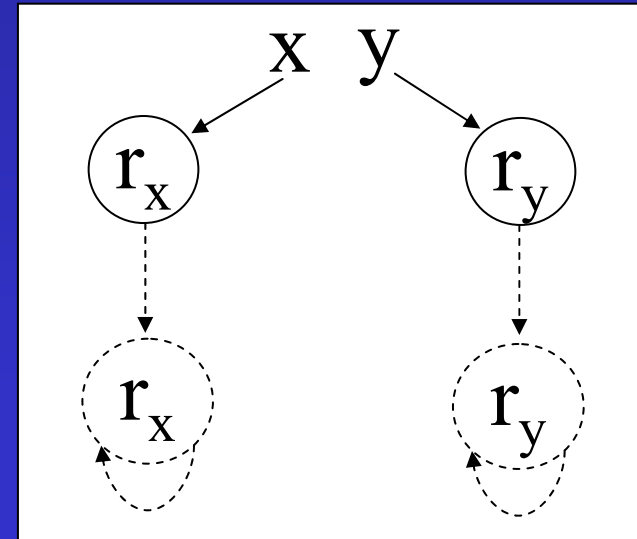
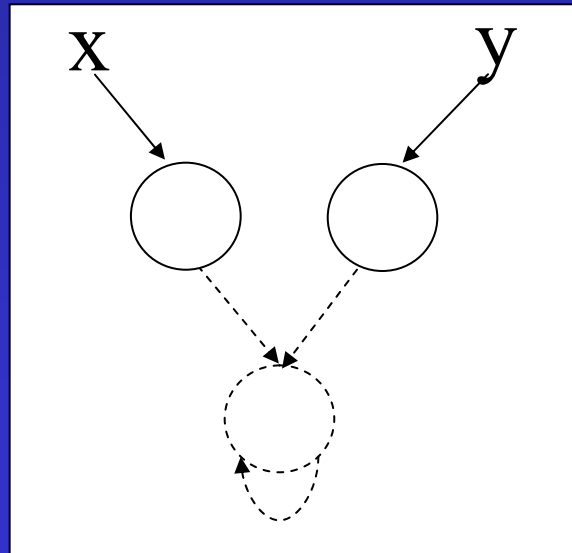
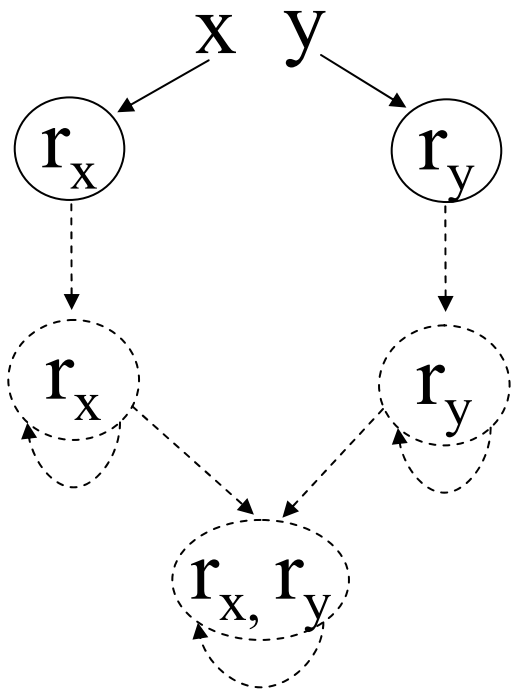
- Need to refine abstractions of linked data structures
  - Identify appropriate new distinctions between
    - Nodes
    - Structures
- Need to derive the associated abstract interpretation
  - What are the update formulas?

# Control Over the Merging of Nodes

- Unary abstraction predicates

$$r_x(v) = \exists v' : (x(v') \xrightarrow{n^*} v)$$

- Distinguish nodes reachable from x (and y)
- Can now tell if lists are disjoint

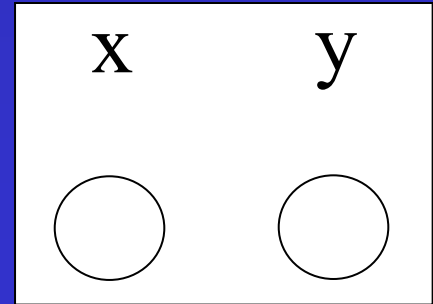
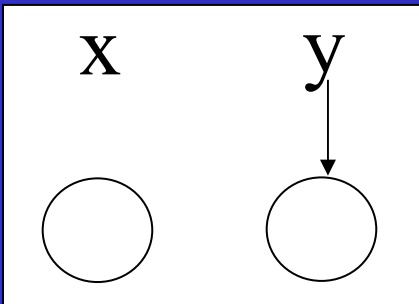
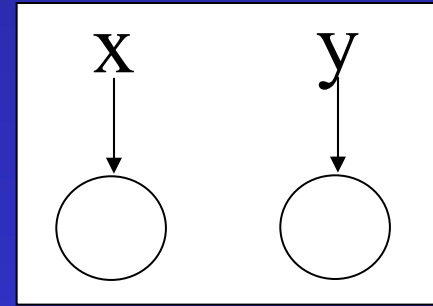
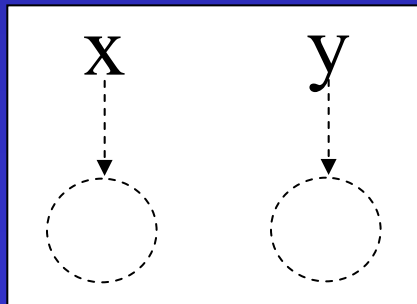
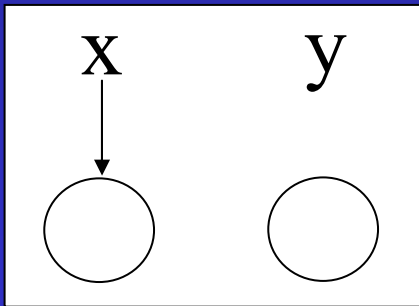


# Control Over the Merging of Structures

- Nullary abstraction predicates

$$nn_x() = \exists v : x(v)$$

- Distinguish structures based on whether x is NULL
- Can now tell that x is NULL whenever y is (and vice versa)

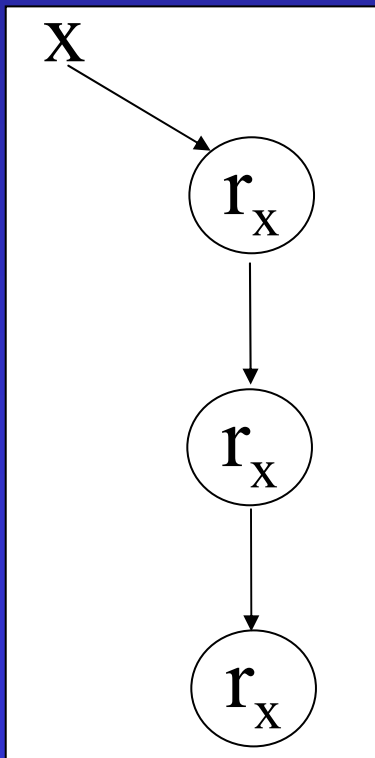


# Need for Update Formulas

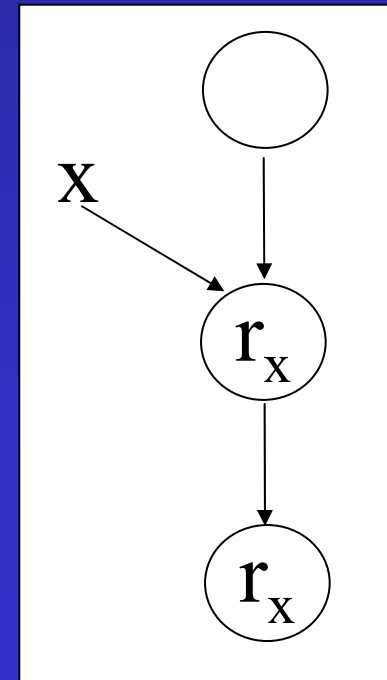
- Re-evaluating formulas is imprecise

$$r_x(v) = \exists v' : (x(v') \wedge n^*(v',v))$$

Action “ $x = x \rightarrow n$ ”



$x = x \rightarrow n$

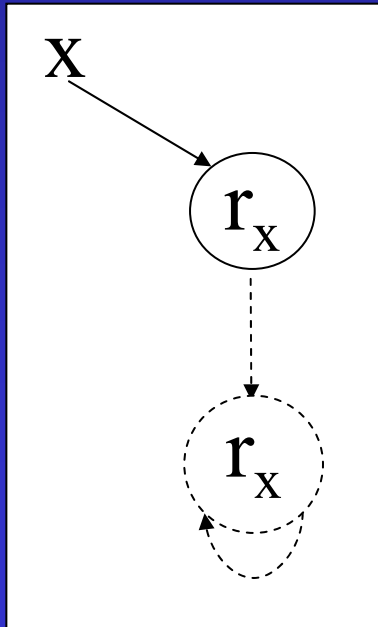


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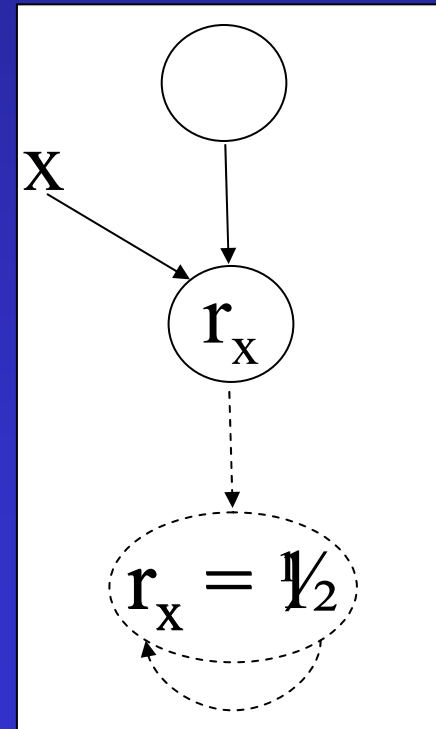
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$$r_x(v) = \exists v' : (x(v') \wedge n^*(v',v))$$

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$x = x \rightarrow n$



# Goal: Create Update Formulas Automatically

- Currently: user provides all update formulas
  - A lot of work
  - Error prone
  - Precludes iterative refinement
- Idea: Finite differencing of formulas

$$F(\varphi) = p_{\varphi} \rightarrow \varphi^{\Delta}$$

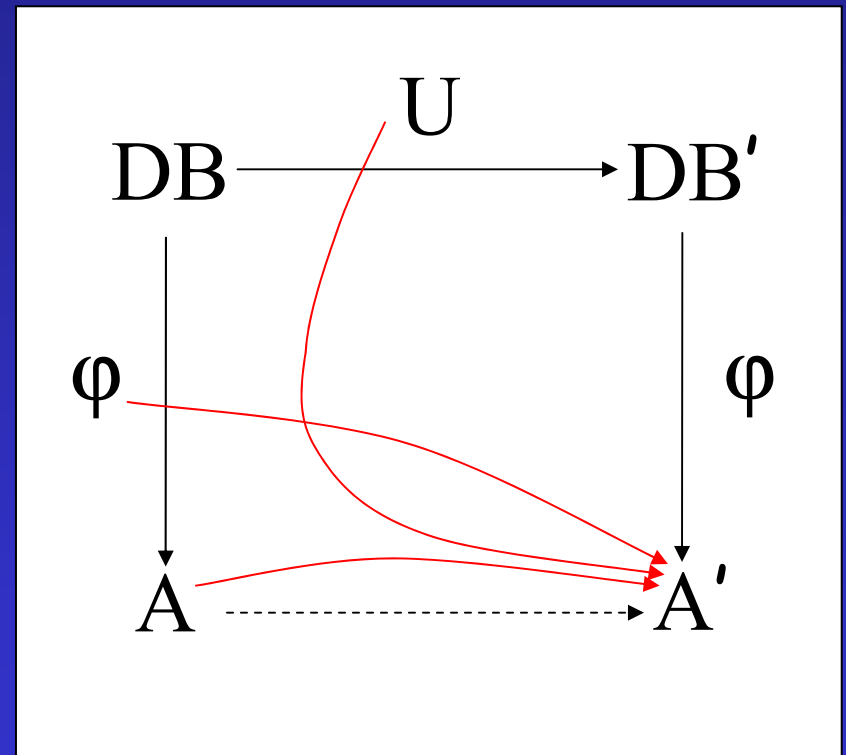
# Past Work on Finite Differencing

- Paige: SETL
- Horwitz & Teitelbaum: Relational Algebra
- Liu & Teitelbaum: Functional Programs



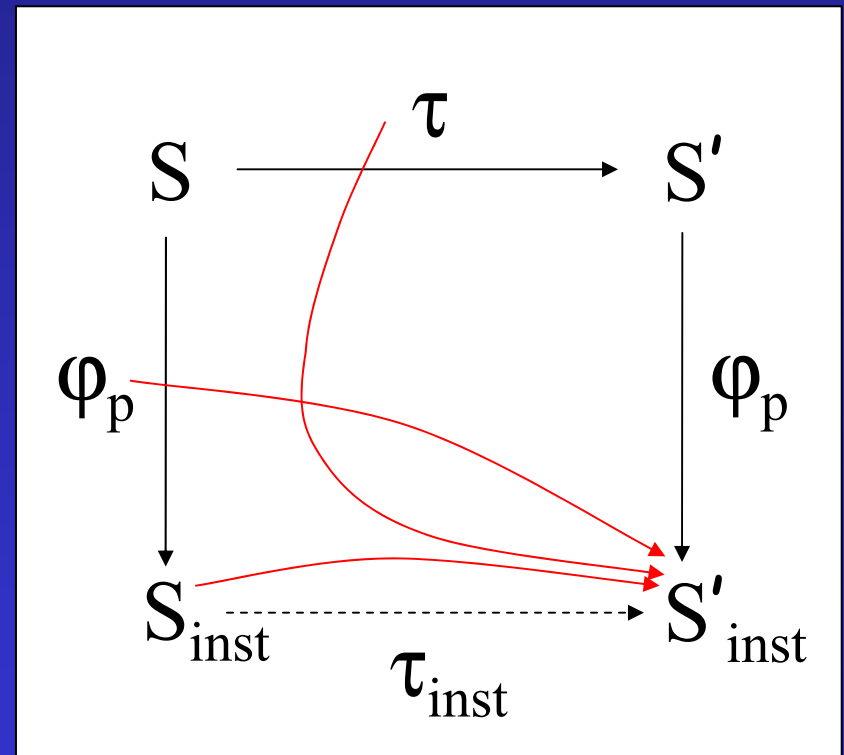
# Database View Maintenance

- $DB'$  - updated database
- $U$  – database update
- $\varphi$  - query
- $A$  – answer



# Finite Differencing of first-order formulas

- $S'$  - updated core structure
- $\tau$  - core update
- $\varphi_p$  - instrumentation predicate formula
- $S_{inst}$  - instrumentation structure



# Differencing $\approx$ Differentiation

$$0^\Delta = 0$$

$$1^\Delta = 0$$

$$(\varphi \oplus \psi)^\Delta = \varphi^\Delta \oplus \psi^\Delta$$

$$(\varphi \otimes \psi)^\Delta = \varphi \otimes \psi^\Delta + \varphi^\Delta \otimes \psi$$

$$c' = 0$$

$$(f+g)'(x) = f'(x)+g'(x)$$

$$(f * g)'(x) = f'(x) * g(x) + f(x) * g'(x)$$

# Finite Differencing of Formulas

- $F(\varphi) = p_{\varphi} \oplus \varphi^{\Delta}$  - too naïve
  - $\oplus$  yields  $\frac{1}{2}$  when either argument is  $\frac{1}{2}$
  - No ability to “generate” or “kill” facts
- **Idea: split into negative and positive change**
  - $F(\varphi) = p_{\varphi} ? \neg\Delta^{-}(\varphi) : \Delta^{+}(\varphi)$
  - More precise
  - Natural for static analysis problems

# Laws for $\Delta^+$

$$\Delta^+(0) = 0 \quad \Delta^+(1) = 0$$

$$\Delta^+(\neg\varphi) = \Delta^-(\varphi)$$

$$\Delta^+(\varphi \rightarrow \psi) = (\Delta^+(\varphi) \rightarrow \Delta^+(\psi)) \rightarrow (\Delta^+(\varphi) \rightarrow \Delta^+(\psi))$$

$$\Delta^+(\psi)$$

$$\Delta^+(\exists v : \varphi) = (\exists v : \Delta^+(\varphi)) \rightarrow (\exists v : \varphi)$$

# Example: reachability

$$\begin{aligned}\Delta^+(\mathbf{r}_x(\mathbf{v})) &= \Delta^+(\exists v' : (\mathbf{x}(v') \text{ } \rightarrow \text{ } \mathbf{n}^*(v', \mathbf{v}))) \\ &= (\exists v' : \Delta^+(\mathbf{x}(v') \text{ } \rightarrow \text{ } \mathbf{n}^*(v', \mathbf{v}))) \\ &\quad \rightarrow \neg(\exists v : (\mathbf{x}(v') \text{ } \rightarrow \text{ } \mathbf{n}^*(v', \mathbf{v}))) \\ &= (\exists v' : \Delta^+(\mathbf{x}(v')) \text{ } \rightarrow \text{ } \mathbf{F}(\mathbf{n}^*(v', \mathbf{v})) \text{ } \rightarrow) \\ &\quad \mathbf{F}(\mathbf{x}(v')) \text{ } \rightarrow \text{ } \Delta^+(\mathbf{n}^*(v', \mathbf{v})) \\ &\quad \rightarrow \neg(\exists v' : (\mathbf{x}(v') \text{ } \rightarrow \text{ } \mathbf{n}^*(v', \mathbf{v})))\end{aligned}$$

# Evaluation






- Programs
  - Singly linked list manipulation
  - Doubly linked list manipulation
  - SLL sorting routines
  - Information flow tests
- Measure of success
  - Fraction of automatically generated formulas without loss of precision relative to best hand-crafted formulas
  - Greedy exploration

# Results

- More than half of formulas auto generated
  - Worst case: 0 of 2, best case 11 of 15
  - All problems due to transitive closure
- Analysis time increase
  - 3% (decrease) to 9% (avg. <2%)



# Example revisited: reachability

$$\begin{aligned} \Delta^+(\mathbf{r}_x(\mathbf{v})) = & \\ & (\exists \mathbf{v}' : \Delta^+(\mathbf{x}(\mathbf{v}')) \text{  } \mathbf{F}(\mathbf{n}^*(\mathbf{v}', \mathbf{v})) \text{ } \\ & \quad \mathbf{F}(\mathbf{x}(\mathbf{v}')) \text{  } \Delta^+(\mathbf{n}^*(\mathbf{v}', \mathbf{v}))) \\ & \text{  } \neg(\exists \mathbf{v}' : (\mathbf{x}(\mathbf{v}') \text{  } \mathbf{n}^*(\mathbf{v}', \mathbf{v}))) \end{aligned}$$

# Solution: factor out TC

- Use power of instrumentation
  - Save TC info in instrumentation predicate
  - $t_n(v_1, v_2) = n^*(v_1, v_2)$
- Use in other instrumentation predicates
  - $r_x(v) = \exists v' : x(v') \wedge t_n(v', v)$
  - $c_n(v) = \exists v' : n(v', v) \wedge t_n(v, v')$

# Example revisited: reachability

$$\begin{aligned} \Delta^+(\mathbf{r}_x(\mathbf{v})) = & \\ & (\exists v' : \Delta^+(\mathbf{x}(v')) \rightarrow F(\mathbf{t}_n(v', \mathbf{v})) \rightarrow \\ & F(\mathbf{x}(v')) \rightarrow \Delta^+(\mathbf{t}_n(v', \mathbf{v}))) \\ & \rightarrow \neg(\exists v' : (\mathbf{x}(v') \rightarrow \mathbf{t}_n(v', \mathbf{v}))) \end{aligned}$$

# Results (improved)

- Almost 90% of formulas auto generated
  - Worst case: 9 of 11, best cases 3 of 3, 11 of 12
  - Only two kinds of hand-crafted updates needed
    - predicate  $t_n(v1,v2)$  for  $x \rightarrow n = \text{null}$
    - predicate  $t_n(v1,v2)$  for  $x \rightarrow n = y$
- Analysis time increase
  - 5% (decrease) to 11% (avg. <4%)

# Conclusions

- FD – fully handles first-order formulas
- Performance effect – minimal
- Challenges – Transitive Closure
  - Updating TC information
  - Preserving unchanged TC information
    - Key: avoid recomputation

# Future Work

- Generation of new instrumentation predicates
  - Formula generation
  - Generate update formulas via finite differencing
- Abstraction refinement loop

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# Laws for $\Delta^+$

$$\Delta^+(0) = 0 \quad \Delta^+(1) = 0$$

$$\Delta^+(v = w) = 0$$

$$\Delta^+(\neg\varphi) = \Delta^-(\varphi)$$

$$\Delta^+(\varphi \multimap \psi) = (\Delta^+(\varphi) \multimap \neg\psi) \multimap (\neg\varphi \multimap \Delta^+(\psi))$$

$$\Delta^+(\varphi \multimap \psi) = (\Delta^+(\varphi) \multimap F(\psi)) \multimap (F(\varphi) \multimap \Delta^+(\psi))$$

$$\Delta^+(\exists v : \varphi) = (\exists v : \Delta^+(\varphi)) \multimap \neg(\exists v : \varphi)$$

$$\Delta^+(\forall v : \varphi) = (\exists v : \Delta^+(\varphi)) \multimap (\forall v : F(\varphi))$$