COMPUTER STUDIES OF PLANETARY-TYPE EVOLUTION

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Donald Greenspan

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John Collier

Computer Sciences Technical Report #297

November 1977

Computer Sciences Department
University of Wisconsin
1210 West Dayton Street
Madison, Wisconsin 53706

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ABSTRACT

In this paper a new, computer approach to the study of the interactions of particles with differing masses is applied to the study of planetary type evolution. The formulation contains an inherent self-reorganization property in which particles self-stratify in accordance with their masses. Computer examples are described and discussed.

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1. Introduction

Recently [4], a new computer approach was developed for modeling the interactions of particles with differing masses. In this approach, matter is approximated by finite sets of particles, each of which is treated as an aggregate of molecules, and the active forces are appropriately rescaled. Inherent in the formulation is a natural, self-reorganization property, in which the particles self-stratify in accordance with their masses. After a precise mathematical and physical extension of the method, so as to include both long-range and short-range forces, we will apply it to the study of planetary-type evolution.

2. Basic Mathematical Definitions and Formulas

For positive time step Δt, let $t_k = k\Delta t$, $k = 0,1,2 \ldots$. For $i = 1,2,3,\ldots,N$, let particle P_i have mass m_i and at time t_k let P_i be located at $\vec{r}_{i,k} = (x_{i,k},y_{i,k})$, have velocity $\vec{v}_{i,k} = (v_{i,k,x},v_{i,k,y})$, and have acceleration $\vec{a}_{i,k} = (a_{i,k,x},a_{i,k,y})$. Let position, velocity and acceleration be related by the "leap-frog" formulas ([3], p. 107):

$$\vec{v}_{i,\frac{1}{2}} = \vec{v}_{i,0} + \frac{\Delta t}{2}\vec{a}_{i,0} \tag{2.1}$$

$$(2.2)\qquad \vec{v}_{i,k+\frac{1}{2}} = \vec{v}_{i,k-\frac{1}{2}} + (\Delta t)\,\vec{a}_{i,k} \qquad , k = 1,2,\ldots .$$

$$(2.3)\qquad \vec{r}_{i,k+1} = \vec{r}_{i,k} + (\Delta t)\,\vec{v}_{i,k+\frac{1}{2}} \qquad , k = 0,1,2,\ldots .$$

If $\vec{F}_{i,k}$ is the force acting on P_i at time t_k, where $\vec{F}_{i,k} = (F_{i,k,x}, F_{i,k,y})$, then we assume that force and acceleration are related by

$$(2.4)\qquad \vec{F}_{i,k} = m_i\,\vec{a}_{i,k} .$$

Once an exact structure is given to $\vec{F}_{i,k}$, the motion of each particle will be determined recursively and explicitly by (2.1)-(2.4) from prescribed initial data.

In the present paper, we will want $\vec{F}_{i,k}$ to incorporate both short-range and long-range components, and this will be implemented as follows. At time t_k, let $r_{ij,k}$ be the distance between P_i and P_j. Let G_{ij} (coefficient of molecular-type attraction), H_{ij} (coefficient of molecular-type repulsion), β_{ij} (exponent of molecular-type attraction), α_{ij} (exponent of molecular-type repulsion) and G^*_{ij} (gravitational constant) be constants determined by P_i and P_j, subject to the constraints (see [6]): $G_{ij} \geq 0$, $H_{ij} \geq 0$, $\alpha_{ij} \geq \beta_{ij} \geq 2$, $G^*_{ij} \geq 0$. Then the force $(\bar{F}_{i,k,x}, \bar{F}_{i,k,y})$ exerted on P_i by P_j is defined by

$$(2.5)\qquad \bar{F}_{i,k,x} = \left[\frac{-G_{ij}}{r_{ij,k}^{\beta_{ij}}} + \frac{H_{ij}}{r_{ij,k}^{\alpha_{ij}}} - \frac{G^*_{ij}}{r_{ij,k}^{2}} \right] \frac{(m_i m_j)(x_{i,k} - x_{j,k})}{r_{ij,k}}$$

$$(2.6) \qquad \bar{F}_{i,k,y} = \left[\frac{-G_{ij}}{r_{ij,k}^{\beta_{ij}}} + \frac{H_{ij}}{r_{ij,k}^{\alpha_{ij}}} - \frac{G^*_{ij}}{r_{ij,k}^{2}} \right] \frac{(m_i m_j)(y_{i,k}-y_{j,k})}{r_{ij,k}} .$$

The total force $(F_{i,k,x}, F_{i,k,y})$ on P_i due to all the other (N-1) particles is given by

$$(2.7) \qquad F_{i,k,x} = \sum_{\substack{j=1 \\ j\neq 1}}^{N} \bar{F}_{i,k,x} \; ; \quad F_{i,k,y} = \sum_{\substack{j=1 \\ j\neq 1}}^{N} \bar{F}_{i,k,y} .$$

The formulation (2.1)-(2.7) is explicit and economical though non-conservative. Conservation of energy and momenta can be achieved [3], but only through an implicit, less economical approach. Throughout, the time step to be used in (2.1)-(2.3) will always be $\Delta t = 10^{-4}$ and a comprehensive FORTRAN program for implementation of (2.1)-(2.7) is given in the Appendix of [5]. It should be noted, in addition, that although all the present computations were executed on an IBM 7094, the way in which (2.1)-(2.7) will be applied lends itself directly to parallel computation also [2].

3. Basic Physical Assumptions and Definitions

We will consider a system of 137 particles, so that N = 137. This parameter was determined solely by economic constraints. Next, we fix the parameters $\alpha_{ij} \equiv 6, \beta_{ij} \equiv 4$, which were shown to be viable in previous computations [4] with relatively smaller particle sets. Unless indicated otherwise, initial particle positions will always be those shown in Figure 1 and listed precisely in the $x_{i,o}$, $y_{i,o}$ columns

of Table I. These positions were generated in such a fashion that the configuration is approximately circular and is, for zero initial velocities and constant masses, relatively stabel [3].

The entire configuration will be set into counterclockwise rotation as follows. In terms of the angular velocity parameter $\dot{\theta}$ and a perturbation parameter ε, let

$$(3.1) \qquad v_{i,0,x} = \pm |\dot{\theta} y_{i,0}| \pm \frac{m_i}{2000}\varepsilon, \quad v_{i,0,y} = \pm |\dot{\theta} x_{i,0}| \pm \frac{m_i}{2000}\varepsilon ,$$

where the choices of the signs will be made as follows. Choose the signs before the absolute value terms in (3.1) by setting $\varepsilon = 0$ and using the rule

$$\begin{aligned}
(x_{i,0},y_{i,0}) &\in \text{Quadrant I} \Rightarrow v_{i,0,x} \le 0,\ v_{i,0,y} \ge 0 \\
(x_{i,0},y_{i,0}) &\in \text{Quadrant II} \Rightarrow v_{i,0,x} \le 0,\ v_{i,0,y} \le 0 \\
(x_{i,0},y_{i,0}) &\in \text{Quadrant III} \Rightarrow v_{i,0,x} \ge 0,\ v_{i,0,y} \le 0 \\
(x_{i,0},y_{i,0}) &\in \text{Quadrant IV} \Rightarrow v_{i,0,x} \ge 0,\ v_{i,0,y} \ge 0 .
\end{aligned}$$

As will be discussed later, the signs before the ε terms will be determined at random.

Still another parameter which will be important will be a distance parameter D which will determine whether the long range forces or the short range forces predominate. Wi will choose $D>1$ and will use the "switching" rule

$$(3.2) \qquad r_{ij,k} < D \Rightarrow G^*_{ij} \equiv 0 .$$

FIGURE 1

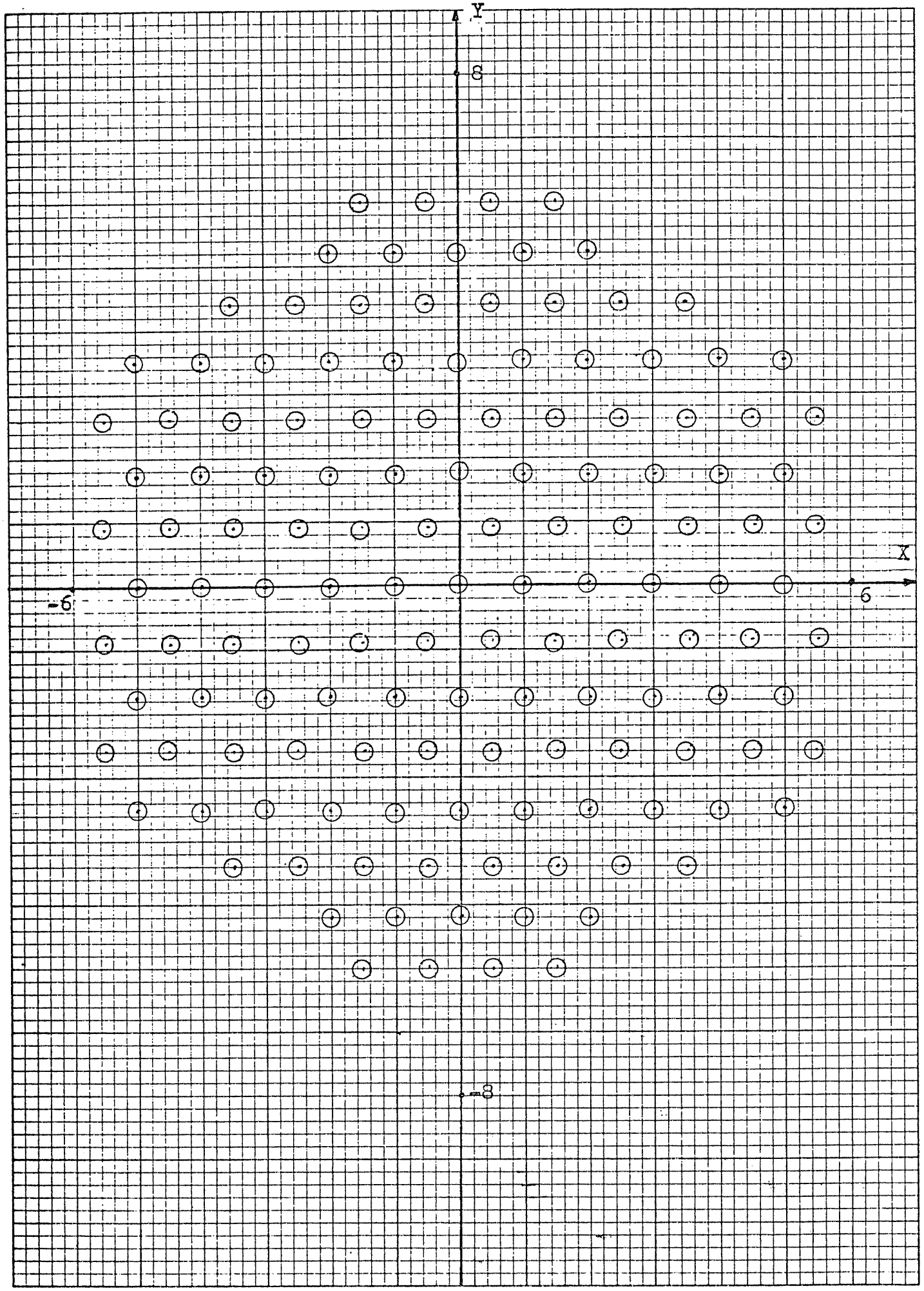


TABLE 1

i	m_i	$x_{i,o}$	$y_{i,o}$	$v_{i,o,x}$	$v_{i,o,y}$
1	10000.000	-2.013	3.447	-63.700	-58.093
2	10000.000	.487	2.590	-60.548	-48.656
3	8000.000	-4.489	-2.614	50.506	-57.819
4	8000.000	1.496	.874	-36.596	34.207
5	8000.000	-2.503	2.595	-29.911	-50.305
6	6000.000	2.005	-5.198	49.930	22.043
7	6000.000	-.991	-1.736	36.643	-34.130
8	6000.000	5.497	-.869	-33.072	8.022
9	6000.000	-5.494	.858	26.435	-52.036
10	6000.000	-1.002	1.727	-23.062	-32.590
11	6000.000	.989	5.197	-8.967	-26.022
12	6000.000	-1.991	-3.469	44.047	-38.052
13	6000.000	4.504	-2.599	39.992	11.172
14	6000.000	3.499	4.302	-11.928	16.192
15	6000.000	2.002	-1.726	23.000	36.942
16	6000.000	-3.004	3.448	-42.176	-17.779
17	6000.000	4.991	1.742	-36.756	50.254
18	4000.000	-3.499	2.601	10.161	6.379
19	4000.000	-.498	-4.334	-2.860	-21.963
20	4000.000	4.996	.002	-19.979	.281
21	4000.000	-1.498	-4.337	-2.833	-26.232
22	4000.000	-5.507	2.590	-30.227	-42.401
23	4000.000	-3.001	-1.737	-13.200	-32.448
24	4000.000	.510	-6.064	44.571	-17.775
25	4000.000	-1.990	-5.207	40.949	-27.638
26	4000.000	5.007	-3.458	33.898	-.198
27	4000.000	1.007	-3.464	33.939	-16.214
28	4000.000	3.007	-1.731	27.145	-8.222
29	4000.000	-2.004	.001	-19.751	11.913
30	4000.000	4.496	.880	-23.345	37.608
31	4000.000	1.493	2.607	-30.421	25.221
32	4000.000	4.993	3.470	-34.039	39.965
33	2000.000	1.508	-6.055	34.193	15.821
34	2000.000	.008	-5.198	30.864	10.301
35	2000.000	-.992	-5.198	30.465	5.822
36	2000.000	.508	-4.328	27.603	11.559
37	2000.000	1.508	-4.325	27.164	15.831
38	2000.000	3.505	-2.595	20.310	24.138
39	2000.000	-3.495	-2.601	20.675	-4.072
40	2000.000	.005	-1.728	17.189	9.776
41	2000.000	3.502	-.865	13.802	23.993
42	2000.000	-2.498	-.871	13.871	-.112
43	2000.000	3.002	.005	9.612	22.053
44	2000.000	-1.498	.869	6.335	4.096
45	2000.000	-.498	.872	6.963	8.070
46	2000.000	-2.498	.869	6.860	.649
47	2000.000	.999	1.732	2.741	14.224
48	2000.000	-5.001	1.726	3.227	-9.785
49	2000.000	-4.001	1.726	3.055	-5.651
50	2000.000	-.001	3.462	-4.250	10.021
51	2000.000	.496	4.332	-7.248	11.301
52	2000.000	-1.504	4.329	-7.790	3.488
53	2000.000	1.992	5.201	-30.436	-2.226
54	2000.000	-.508	6.058	-34.086	-11.939

TABLE 1 (continued)

i	m_i	$x_{i,o}$	$y_{i,o}$	$v_{i,o,x}$	$v_{i,o,y}$
55	2000.000	1.004	-5.202	10.675	-5.577
56	2000.000	2.504	-4.329	7.105	.065
57	2000.000	.001	-3.462	3.390	-9.927
58	2000.000	-2.499	-2.605	.397	-19.737
59	2000.000	-4.999	-1.738	-3.122	-30.063
60	2000.000	4.498	-.866	-6.699	7.907
61	2000.000	-1.502	-.875	-6.675	-15.910
62	2000.000	3.998	.004	-9.577	5.837
63	2000.000	.498	.868	-13.788	-7.132
64	2000.000	-4.502	.862	-13.455	-27.916
65	2000.000	-3.502	.865	-13.429	-23.805
66	2000.000	-.005	1.728	-16.122	-10.123
67	2000.000	-4.505	2.592	-19.992	-27.953
68	2000.000	.995	3.458	-23.847	-5.399
69	2000.000	-1.005	3.458	-23.940	-14.888
70	2000.000	1.492	4.331	-27.334	-4.517
71	2000.000	-.508	4.328	-27.502	-12.539
72	2000.000	-1.508	6.055	-34.261	-15.724
73	2000.000	3.504	-4.329	7.267	3.980
74	2000.000	-2.496	-4.335	7.436	-20.447
75	2000.000	4.001	-3.456	3.750	5.980
76	2000.000	-.999	-3.462	3.939	-13.799
77	2000.000	-4.999	-3.468	3.800	-30.306
78	2000.000	5.501	-2.596	.415	12.575
79	2000.000	-1.499	-2.605	.369	-15.970
80	2000.000	1.005	-1.732	16.755	-6.301
81	2000.000	-3.995	-1.738	16.866	-26.138
82	2000.000	-.498	-.872	13.320	-11.742
83	2000.000	.502	-.872	13.561	-7.918
84	2000.000	-5.498	-.878	13.698	-31.834
85	2000.000	2.002	.001	10.032	-2.358
86	2000.000	-3.998	-.008	9.796	-25.846
87	2000.000	3.502	.871	6.472	4.005
88	2000.000	5.502	.874	6.538	11.540
89	2000.000	1.999	1.731	2.607	-1.494
90	2000.000	5.499	2.604	-.471	11.759
91	2000.000	-1.501	2.595	-.292	-16.074
92	2000.000	1.999	3.461	-4.224	-1.783
93	2000.000	2.496	4.331	-7.565	.214
94	2000.000	-2.504	4.325	-7.488	-20.654
95	2000.000	.496	6.058	-14.298	-7.829
96	2000.000	-1.492	-6.065	34.425	-15.952
97	2000.000	-3.492	-4.335	27.649	-23.920
98	2000.000	2.005	-3.459	23.911	-2.091
99	2000.000	-3.995	-3.468	24.091	-26.184
100	2000.000	-.495	-2.602	20.272	-12.374
101	2000.000	.505	-2.602	20.206	-7.595
102	2000.000	4.005	-1.726	17.254	6.058
103	2000.000	1.502	-.869	13.649	-3.339
104	2000.000	-4.498	-.878	13.329	-28.419
105	2000.000	-.998	-.002	9.984	-14.004
106	2000.000	-4.998	-.008	9.795	-30.092
107	2000.000	2.502	.871	6.578	.041
108	2000.000	2.999	1.731	2.686	1.761

TABLE 1 (concluded)

i	m_i	$x_{i,o}$	$y_{i,o}$	$v_{i,o,x}$	$v_{i,o,y}$
109	2000.000	-3.001	1.725	3.732	-21.165
110	2000.000	3.499	2.601	-.030	4.079
111	2000.000	2.495	2.605	-20.882	19.700
112	2000.000	-.505	2.602	-21.368	8.316
113	2000.000	2.995	3.465	-23.897	21.952
114	2000.000	-4.005	3.456	-23.030	-6.161
115	2000.000	-3.508	4.329	-26.368	-4.922
116	2000.000	-2.008	5.199	-30.894	1.100
117	2000.000	1.492	6.065	-34.222	16.098
118	2000.000	-.496	-6.058	13.606	8.178
119	2000.000	3.001	-3.455	3.874	22.206
120	2000.000	-2.999	-3.461	3.481	-1.801
121	2000.000	1.501	-2.595	.616	16.211
122	2000.000	2.501	-2.595	.397	20.005
123	2000.000	-5.499	-2.604	.373	-11.792
124	2000.000	5.001	-1.722	-3.288	29.701
125	2000.000	-1.999	-1.731	-3.065	1.824
126	2000.000	2.498	-.865	-6.705	20.480
127	2000.000	-3.502	-.871	-6.532	-4.177
128	2000.000	-.002	.002	-9.843	10.323
129	2000.000	.998	.002	-9.866	13.895
130	2000.000	-3.002	-.001	-9.997	-2.150
131	2000.000	3.995	1.738	-14.825	26.147
132	2000.000	-2.005	1.729	-17.136	3.032
133	2000.000	4.495	2.608	-20.408	27.820
134	2000.000	3.995	3.468	-23.657	25.962
135	2000.000	-5.005	3.456	-23.512	-9.969
136	2000.000	-1.008	5.202	-30.861	5.542
137	2000.000	-.008	5.202	-30.831	9.907

(3.3) $\quad r_{ij,k} \geq D \Rightarrow G_{ij} \equiv H_{ij} \equiv 0$.

Further, as in [4], we will fix $G_{ij} \equiv H_{ij} \equiv 5$ whenever (3.2) is valid. Thus, once D is assigned, G_{ij} and H_{ij} are completely determined, but G^*_{ij} is, as yet, still arbitrary whenever (3.3) is valid.

In the assignment of particle masses, let us first consider the chemical composition of Earth, as shown in Table II. The proportionate <u>total</u> mass for each element is shown in the right-most column. If one regroups all the elements in Table II as shown in Table III, then one observes that, in order, the total masses of groups I-V decrease, while the individual particle masses increase. It is this observation which is incorporated approximately into our model as follows. We will consider five groups, consisting of 2, 3, 12, 15, and 105 particles, respectively, with individual particle masses of 10000, 8000, 6000, 4000, and 2000 units, respectively. The actual assignment of a mass to each particle will be implemented later by a random process.

Next, while in rotation we will require a rule for determining the physical state of each particle. Such matters can be exceedingly complex [1] so that, for the present, we will use, as in [5], the following intuitive notions. Consider four particles P_1, P_2, P_3, P_4 , each of the same mass, located at (0,1), (-0.87, 0.5), (0.87,0.5), (0,0) , respectively. If each particle is assigned $\vec{0}$ velocity, then the three particles P_1, P_2, P_4 form a three particle bond [3], since, by (2.5)-(2.7), the local force on any one due to the other two is zero. Similarly, P_1, P_3, P_4 form a three particle bond. We will then call P_1, P_2, P_3 and P_4 <u>solid</u> particles. To explore the change of state to

TABLE II - CHEMICAL COMPOSITION OF EARTH

Element	Relative number of atoms	Element's atomic weight	Product
Hydrogen	40000.0	1.008	40320
Helium	3100.0	4.003	12409
Carbon	3.5	12.01	42
Nitrogen	6.6	14.008	92
Oxygen	21.5	16.000	344
Neon	8.6	20.183	174
Sodium	0.04	22.997	1
Magnesium	0.91	24.31	22
Aluminum	0.09	26.98	2
Silicon	1.0	28.09	28
Phosphorous	0.01	30.07	0
Sulfur	0.37	32.06	12
Argon	0.15	39.948	6
Calcium	0.05	40.08	2
Iron	0.6	55.85	34
Nickel	0.03	58.71	2

TABLE III - REGROUPING OF EARTH'S ELEMENTS

Group	Elements	Total Products
I	Hydrogen, Helium	52729
II	Carbon through Oxygen	478
III	Neon through Aluminum	199
IV	Silicon through Calcium	48
V	Iron, Nickel	36

fluid particles, we now change only the initial velocity of P_1 to $(0,v)$, where $v>0$, and seek the smallest value of v for which P_1 relocates monotonically upward, in accordance with (2.1)-(2.7), so that the given bonds are broken and new ones formed by the triplet P_1, P_2, P_3 and by the triplet P_2, P_3, P_4. In this fashion, the speed v assigned to P_1 has enabled the particles to change their bonds easily, which we call a fluid state, and, in particular, a liquid state. For $D = 2.1$, so that (3.2) is valid, these values of v are given in Table IV in the "v-liquid" column for the different masses to be considered. (Of course, these results are also valid for any $D>2.1$). However, it will be more convenient to identify a particle as being a liquid particle by its temperature, which is defined as follows [3]. The instantaneous temperature $T'_{i,k}$ of P_i at t_k is defined by its kinetic energy, that is

$$(3.4) \qquad T'_{i,k} = \frac{1}{2} m_i \, v^2_{i,k} .$$

Since these numbers can be relatively large due to the magnitudes m_i, the normalized instantaneous temperature $T^*_{i,k}$ of P_i at t_k is defined by

$$T^*_{i,k} = T'_{i,k}/10^4 .$$

The temperature $T_{i,k}$ of P_i at t_k is defined by

$$(3.5) \qquad T_{i,k} = \frac{1}{M} \sum_{j=k-M}^{k} T^*_{i,j} ,$$

TABLE IV - Change of State Velocities and Temperatures

m_i	v-liquid	v-gas	temp-liquid	temp-gas
10000	100	170	11370	30200
8000	90	160	8190	22600
6000	78	140	4640	12500
4000	65	110	1160	5300
2000	50	80	710	2130

where M is a positive integer, and where (3.5) is an average over M time steps, thus corresponding to the fact that temperature is a quantity which is measured over a finite, positive time period [3]. Computations have shown that it is reasonable in the examples which follow to choose $M = 500$, so that we will use

$$(3.6) \qquad T_{i,k} = \frac{1}{500} \sum_{j=k-500}^{k} T^*_{i,j} \ .$$

The temperatures at which each particle changes state from solid to liquid are listed in the "temp-liquid" column of Table IV.

The critical velocities and temperatures of gas particles, as shown in Table IV, were determined more simply by considering only three particles P_1, P_2, P_3, located at (0,0.87), (-0.5,0), (0.5,0), respectively, with initial velocities (0,v), (0,0), (0,0), respectively, and by determining the positive parameter v for which $|P_1P_2| = |P_1P_3| > D$. The results shown are for $D = 2.3$. Intuitively, when $|P_1P_2| > D$, all molecular-type forces are zero, by (2.3), so that P_1 moves "freely", which is characteristic of gas particles.

Note that "temperature", as defined above, is a phenomenon of a particles "local" velocity, that is, its velocity relative to neighboring particles. Thus, when a particle is rotating within a large system, the gross system velocities should have no effect on the particle's temperature and must be subtracted out before the temperature calculation is performed. The velocity of the centroid of the system and of the average angular velocity of the system are utilized for this purpose in the following way to determine the temperature of P_i at t_k as P_i rotates

within the system. First, at time t_k, let the mass center of the system be $(\bar{x}_k, \bar{y}_k)$ and let the average linear velocity, $(\bar{v}_{x,k}, \bar{v}_{y,k})$, of the system be defined by

$$(3.6) \qquad \bar{v}_{x,k} = \frac{\sum(m_i v_{x,i,k})}{\sum m_i} \ , \quad \bar{v}_{y,k} = \frac{\sum(m_i v_{y,i,k})}{\sum m_i} \ ,$$

where the summations of (3.6) are taken over all particles of the system. Then P_i's position $(x^*_{i,k}, y^*_{i,k})$ and velocity $(v^*_{i,k,x}, v^*_{i,k,y})$ relative to the mass center are defined by

$$(3.7) \qquad x^*_{i,k} = x_{i,k} - \bar{x}_k \ , \quad y^*_{i,k} = y_{i,k} - \bar{y}_k$$

$$(3.8) \qquad v^*_{i,k,x} = v_{i,k,y} - \bar{v}_{x,k} \ , \quad v^*_{i,k,y} = v_{i,k,y} - \bar{v}_{y,k} \ .$$

Next, out of $v^*_{i,k,x}$ and $v^*_{i,k,y}$ we wish to take the angular rotation of the system, which is done as follows. Introduce the normal and tangent velocity components, $v^*_{i,k,n}$ and $v^*_{i,k,t}$, respectively, of P_i at t_k, by the usual formulas

$$(3.9) \qquad v^*_{i,k,n} = [v^*_{i,k,y}\, y^*_{i,k} + v^*_{i,k,x}\, x^*_{i,k}]/R_i$$

$$(3.10) \qquad v^*_{i,k,t} = [-v^*_{i,k,x}\, y^*_{i,k} + v^*_{i,k,y}\, x^*_{i,k}]/R_i \ ,$$

where

$$(3.11) \qquad R_i = [(x^*_{i,k})^2 + (y^*_{i,k})^2]^{\frac{1}{2}}$$

Since, in general, $\dot{\theta} = v_t/R$, we define the average angular velocity, $\bar{\dot{\theta}}$, of the system by

$$(3.12) \qquad \bar{\dot{\theta}} = \frac{\sum(\dot{\theta}_i m_i)}{\sum m_i} = \frac{\sum^{(m_i \frac{v^*_{i,k,t}}{R_i})}}{\sum m_i} \quad ,$$

where the summations are taken over all the particles of the system. Finally, the speeds $v^2_{i,k}$, used in (3.4) to calculate the temperature by (3.6), are given by

$$(3.13) \qquad v^2_{i,k} = (v^*_{i,k,t} - \bar{\dot{\theta}}R_i)^2 + (v^*_{i,k,n})^2 \quad .$$

As a last consideration, we will allow for radiation into and out of the system. This will be accomplished as follows. Consider the vertical strip regions

$$k\gamma \le x \le (k+1)\gamma \quad ; \quad \gamma > 0, \; k = 0, \pm 1, \pm 2, \pm 3, \ldots \quad .$$

In each strip, the particles with maximum and minimum y coordinates are called outer particles. An outer particle will be called a light-side particle, that is, it faces a sun, if $y_{i,k} \ge \bar{y}_k$. Otherwise, it is called a dark-side particle. Light-side particles will receive radiant heat while dark-side particles will emanate radiant heat. This is implemented as follows. Every k^* steps, outer particle velocities are reset by the rule

$$(3.14) \qquad \begin{aligned} &\text{light-side particle:} \quad \vec{v}_{i,k} \to 1.001\, \vec{v}_{i,k} \\ &\text{dark-side particle:} \quad \vec{v}_{i,k} \to 0.9955\, \vec{v}_{i,k} \quad . \end{aligned}$$

4. Examples

A variety of examples were run with various combinations of parameter choices selected from $\dot{\theta} = 4,5$; $\varepsilon = 5,10$; $k^* = 50,100$; $D = 2.1,2.3$; $\gamma = 0.1, 1.0,2.0$; and $G^*_{ij} = 0.01,0.001$. From these we will first describe two, in both of which the parameter choices are $\dot{\theta} = 4$, $\varepsilon = 10$, $D = 2.3$, $k^* = 100$, $\gamma = 1$, and $G^*_{ij} = 0.001$. Later, some general remarks will be made about other computations. Initial positions and velocities were generated in the manner described in Section 3 and, for completeness, are given in Table I.

Example 1. In this example, we consider the mass distribution given in Table I. This distribution was generated at random, but under the constraint that the relatively heavy particles be more centrally located than the relatively light ones. Such a constraint is reasonable because heavy particles on the periphery of a rotating system often have sufficient momentum to escape the system, which was verified computationally for the present system. Thus, in effect, we are starting with a system from which some of the outer particles which will escape are considered to have already escaped.

The rotational motion of the system, relative to the centroid, is shown in Figures 2(a)-(j) at the respective times t_{10000}, t_{15000}, t_{20000}, t_{25000}, t_{30000}, t_{35000}, t_{40000}, t_{45000}, t_{50000}, and t_{60000}. The five different size circles, from largest to smallest, represent particles from the five sets of masses, from largest to smallest, respectively. For emphasis, the interior of the circle representing any particle of mass greater than 2000 has been darkened and this darkened set will be referred to as the "heavier particles". When any particle's distance to the centroid exceeded

15 units, that particle was considered to have escaped the system and it was dropped from all further considerations.

The system's self-reorganization with the heavier particles located centrally is clear from the figures. Figures 2(a)-(c) indicate that the process occurs in two steps: first the heavier particles form into several clusters, and then these clusters relocate centrally. By the time t_{30000}, the system has been reduced to sixty four particles and has achieved a state of relative stability. It is of interest to note that all the particles which have escaped from the system have mass 2000. At time t_{60000}, the geometric center of the system is (-0.06,-0.09), which is almost, but not quite, identical with the centroid. It is interesting to note that at this time the two heaviest particles are not located at the centroid. Such anomalies with regard to the centroid are often present in lunar type bodies. To explain such phenomena, let us examine Figures 3(a)-(j), which show the state of solidification at the very same times as those in Figures 2(a)-(j). Using Table IV, we have shown the solid particles by means of hexagonal enclosures and the gas particles by means of triangular enclosures. All other particles are liquid. If one studies the two heaviest particles in Figure 3(c) and observes the relative positions of the neighboring solid particles, then it is apparent that these particles have already formed into a rigid, noncircular configuration which does not change during the remainder of the motion. It is this very early solidification which prevents those inner motions which are necessary for the heaviest particles to relocate to the centroid of the entire system.

Figure 4 shows the oscillatory dissipation of system kinetic energy until time t_{60000}, at which time the system is relatively stable.

FIGURE 2 (a) t_{10000}

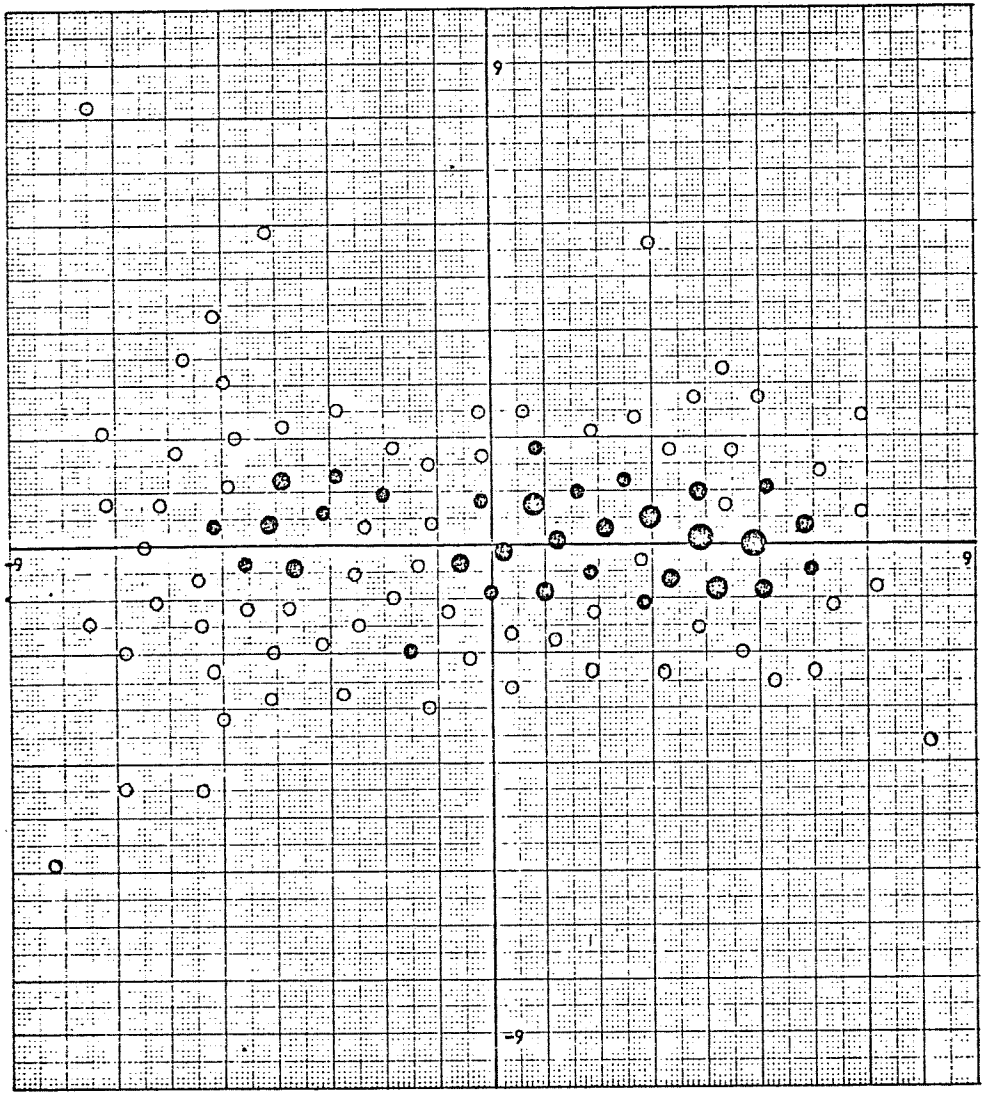


FIGURE 2 (b) t_{15000}

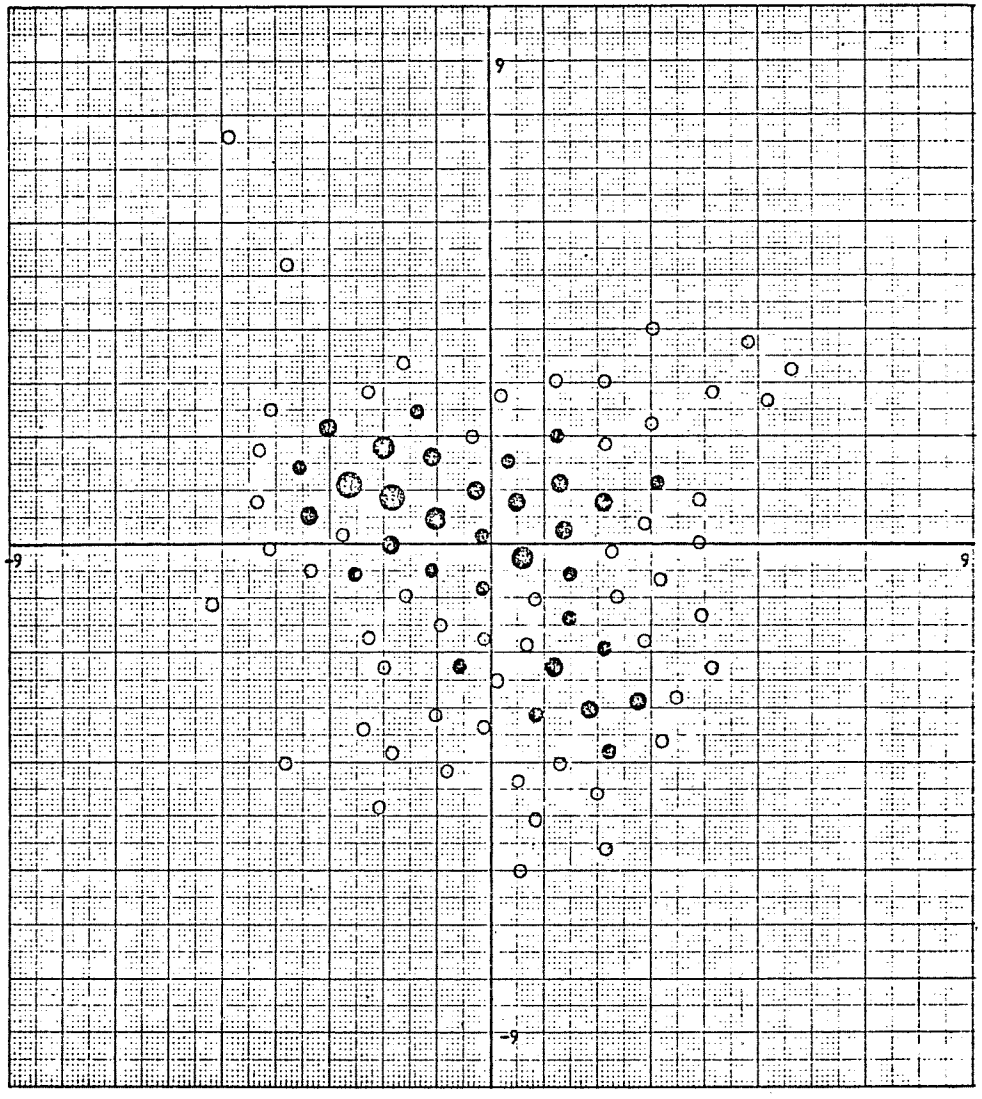


FIGURE 2 (c) t_{20000}

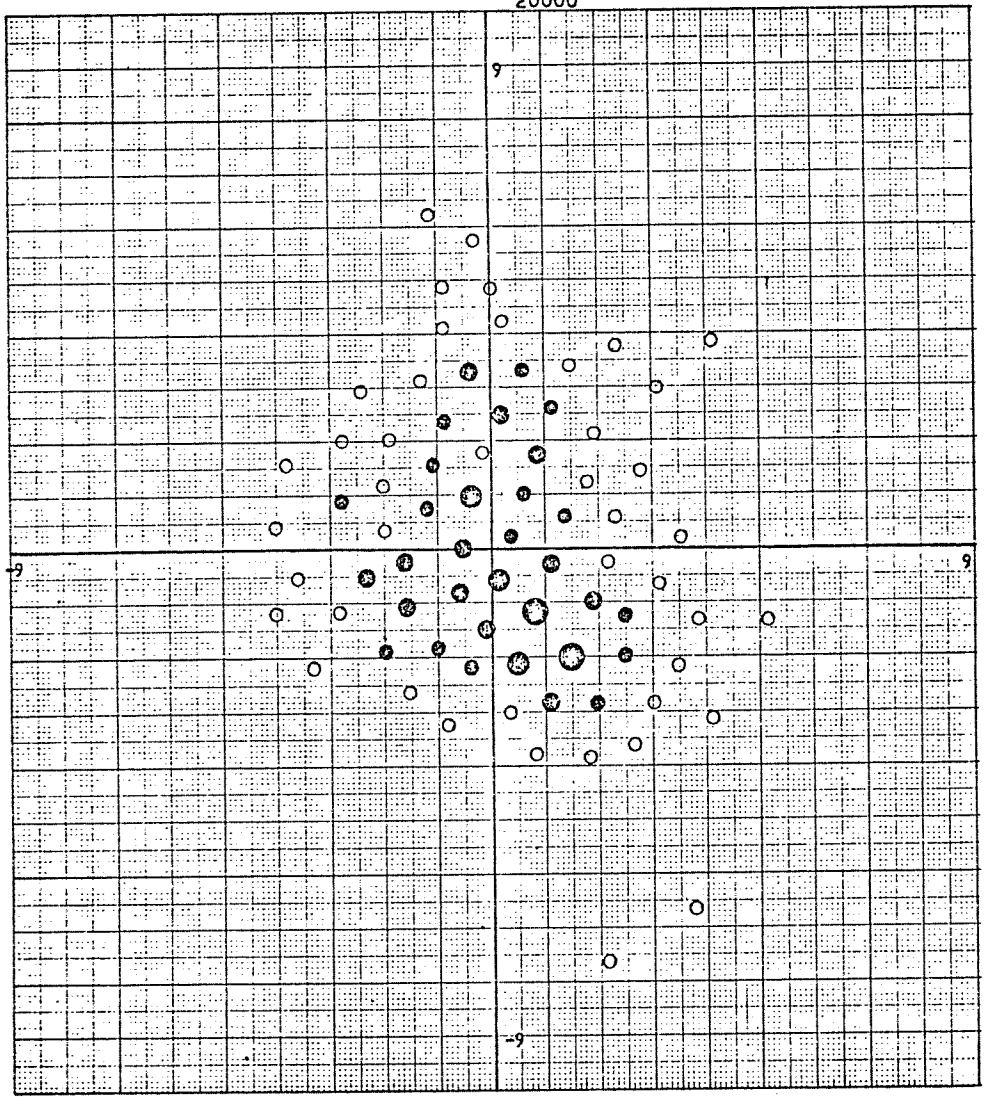


FIGURE 2 (d) t_{25000}

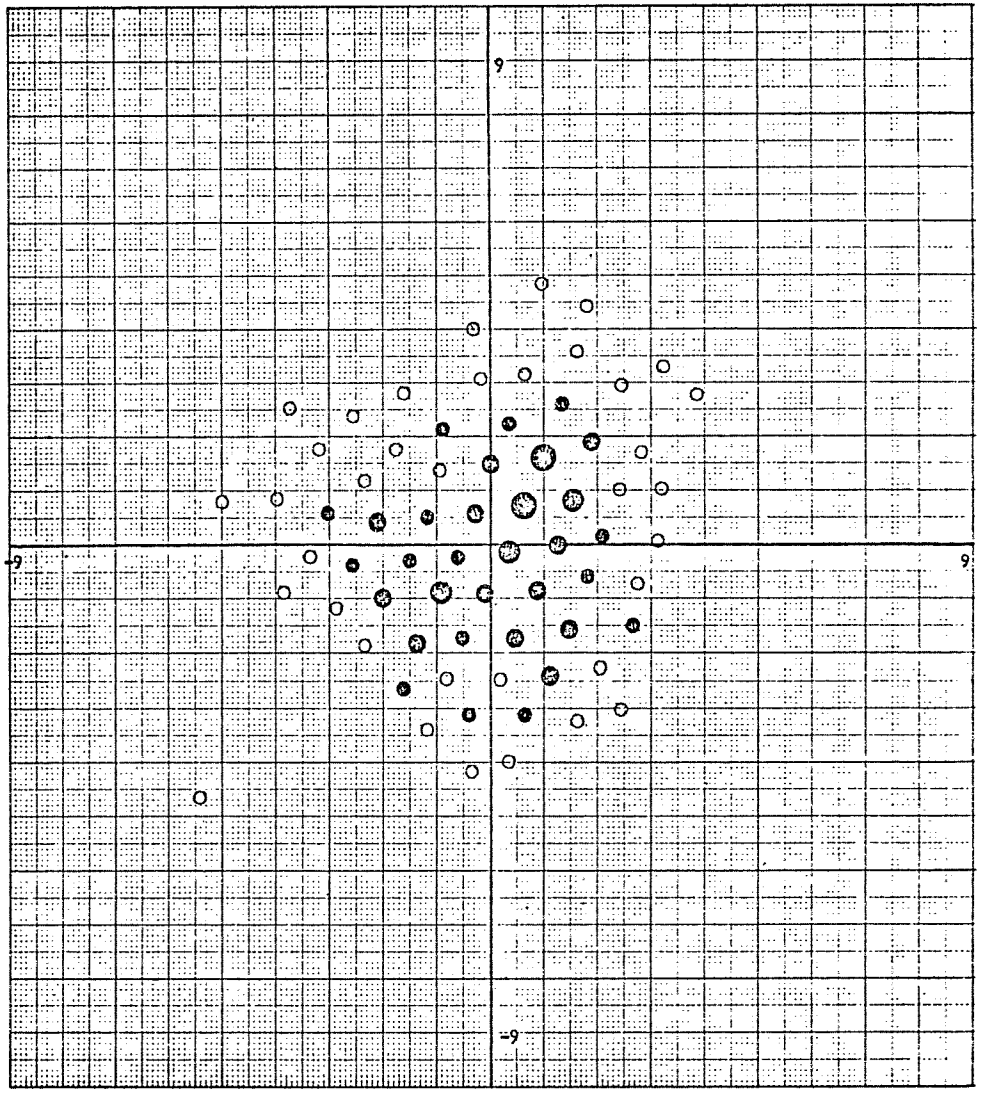


FIGURE 2 (e) t_{30000}

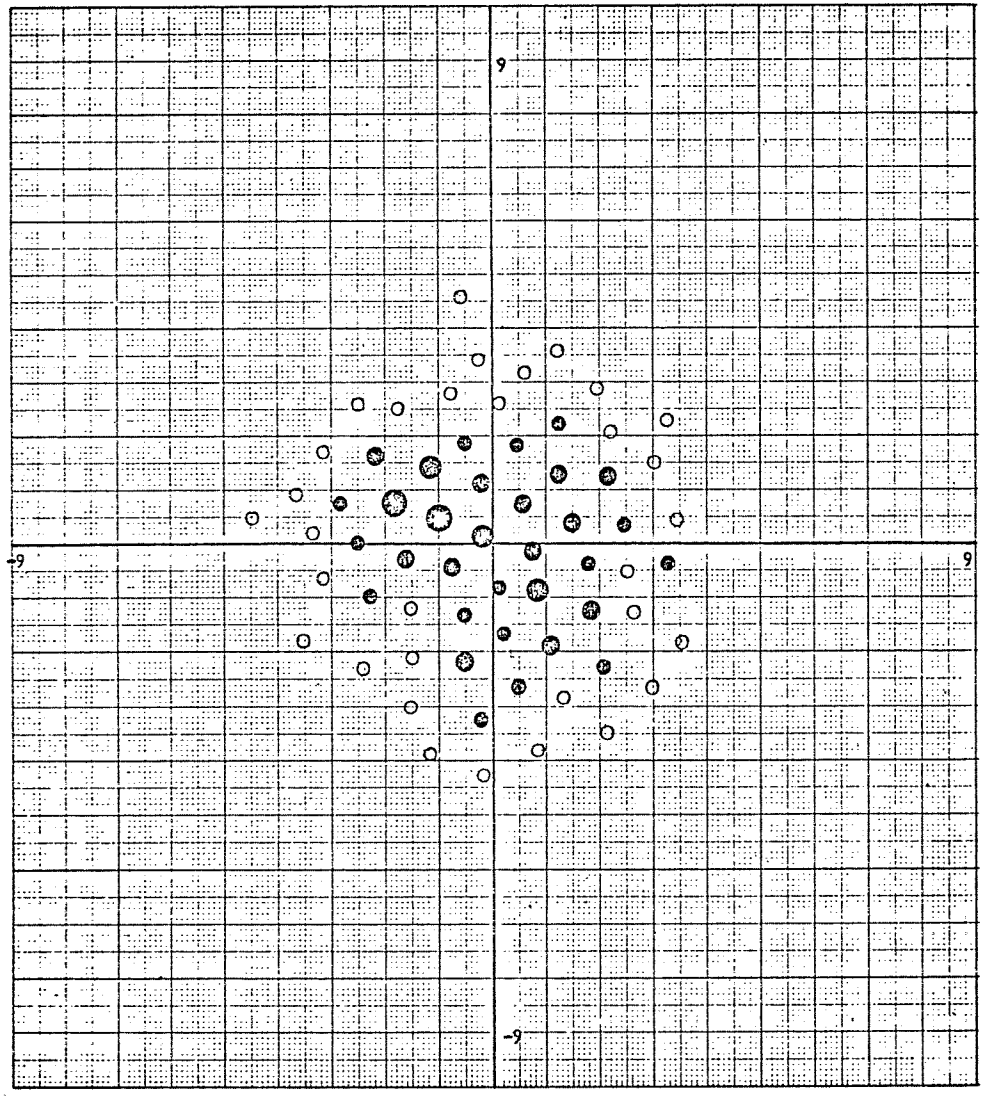


FIGURE 2 (f) t_{35000}

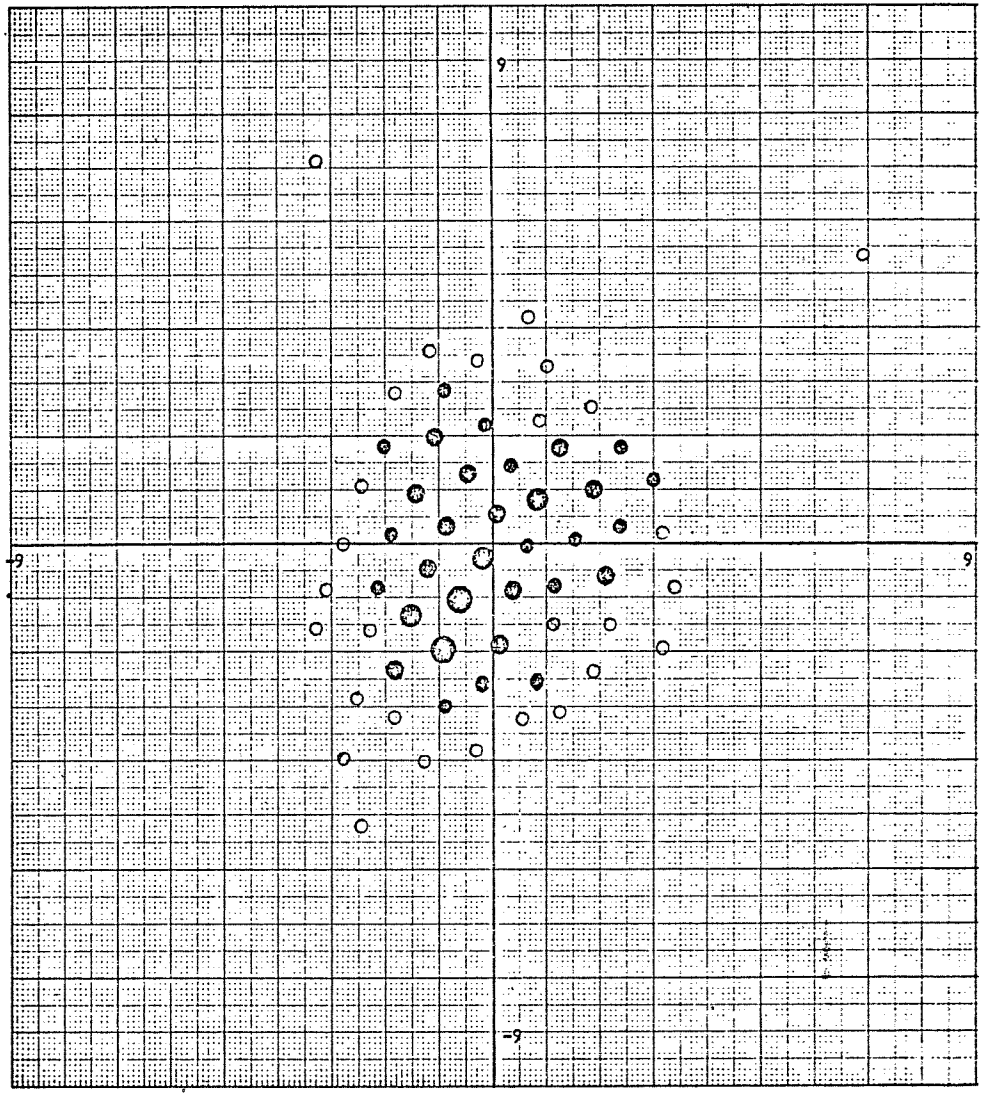


FIGURE 2 (g) t_{40000}

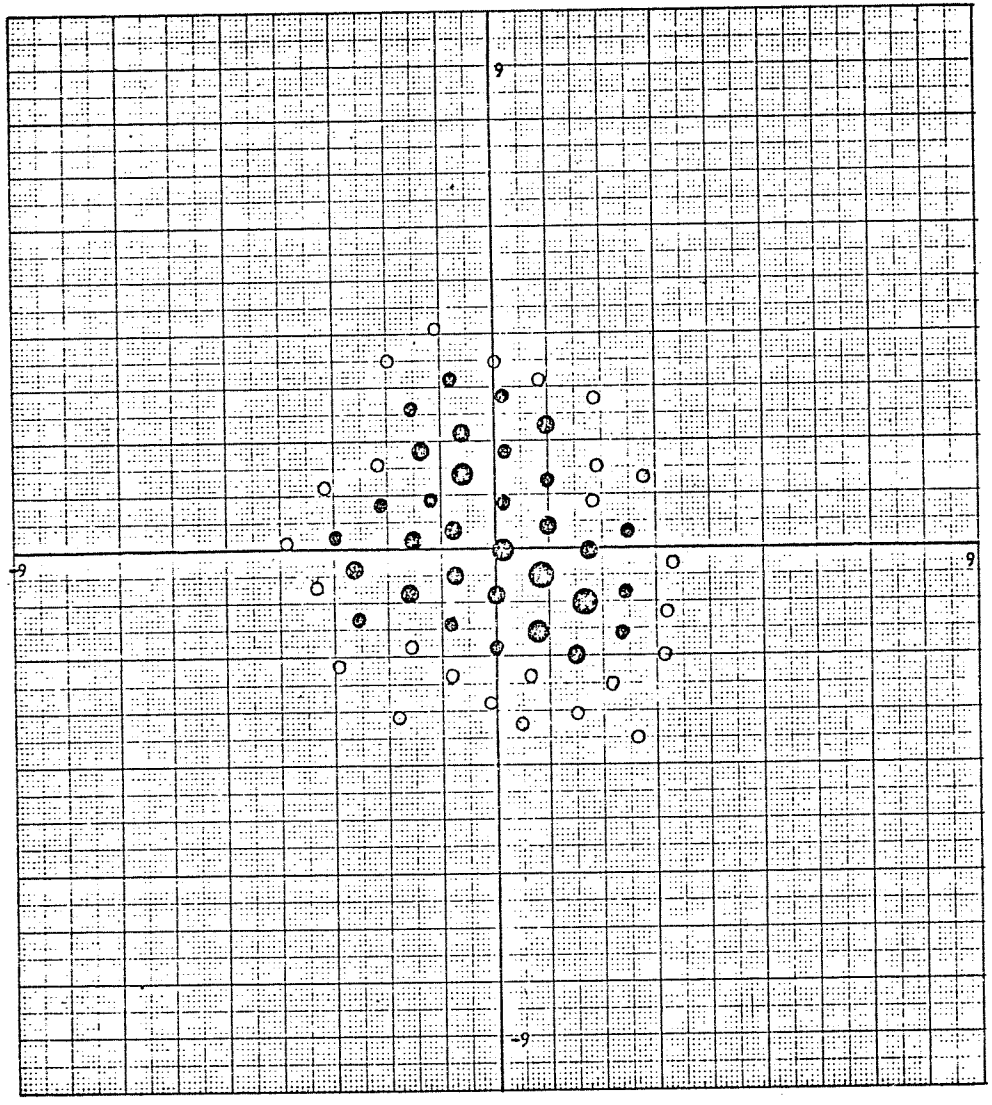


FIGURE 2 (h) t_{45000}

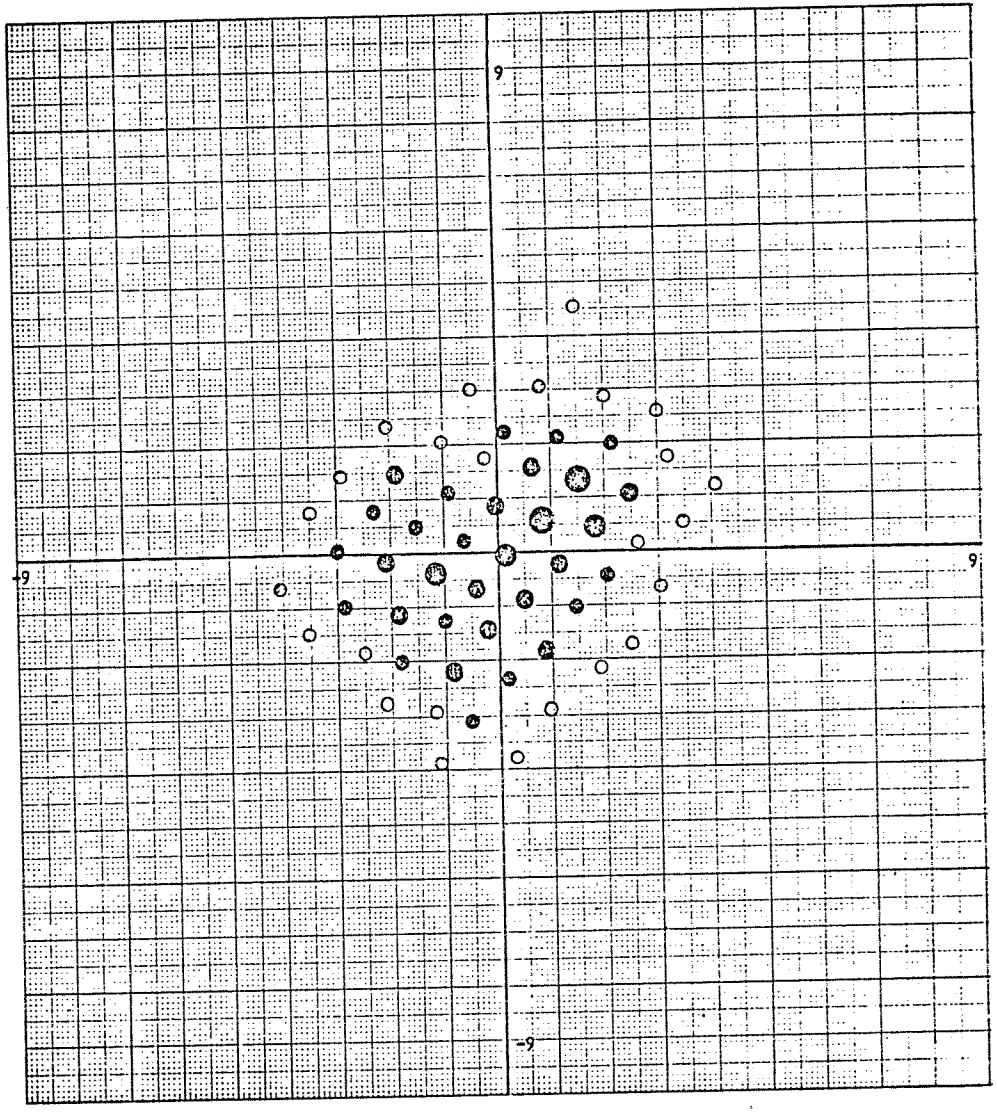


FIGURE 2 (i) t_{50000}

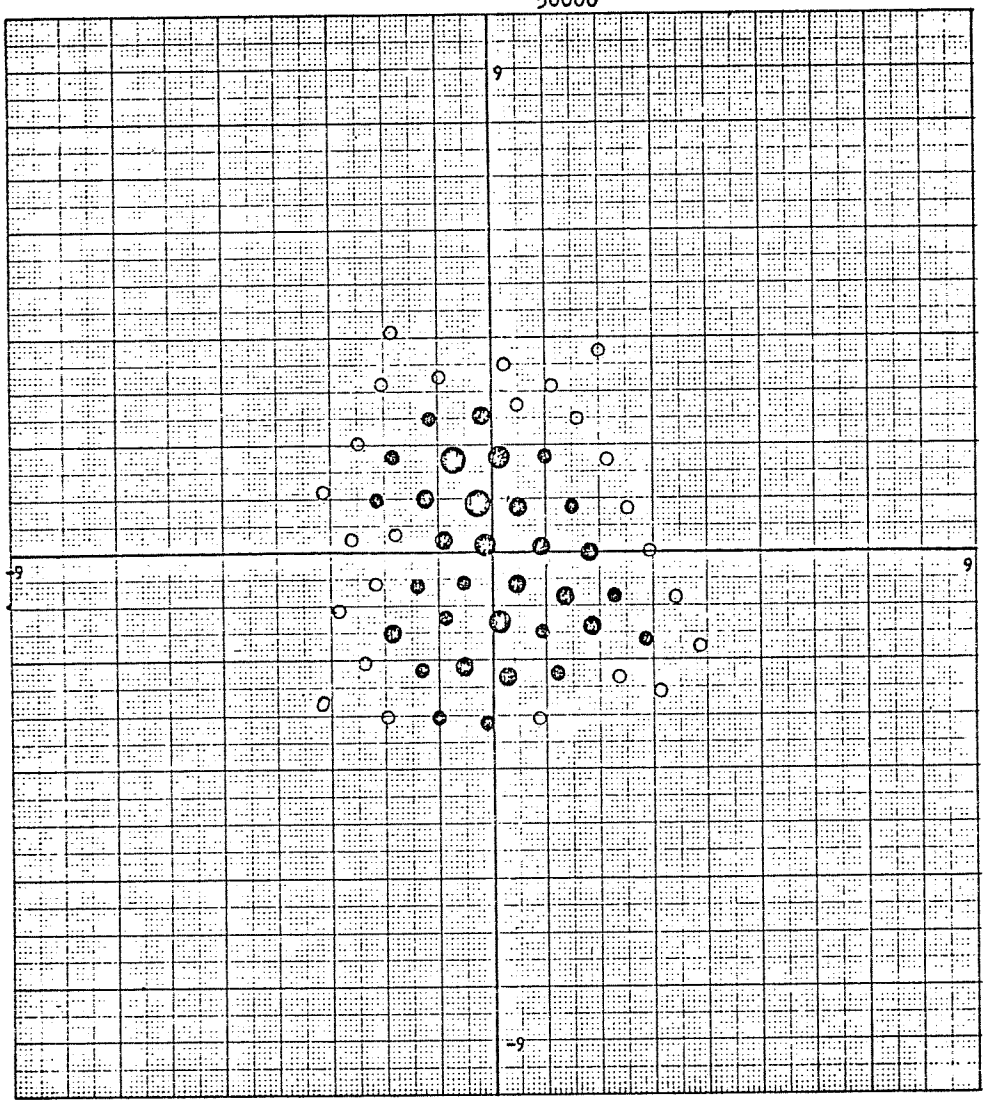


FIGURE 2 (j) t_{60000}

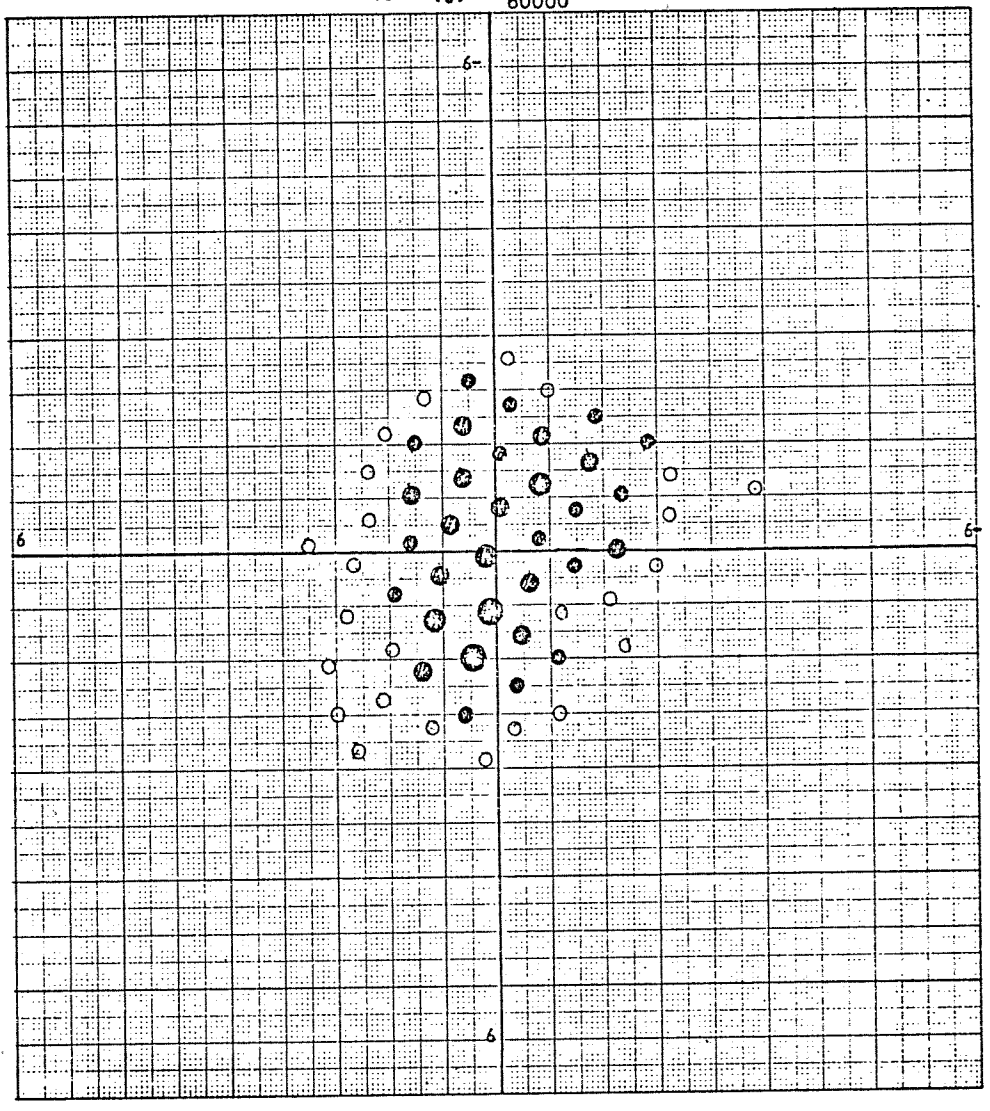


FIGURE 3 (a) t_{10000}

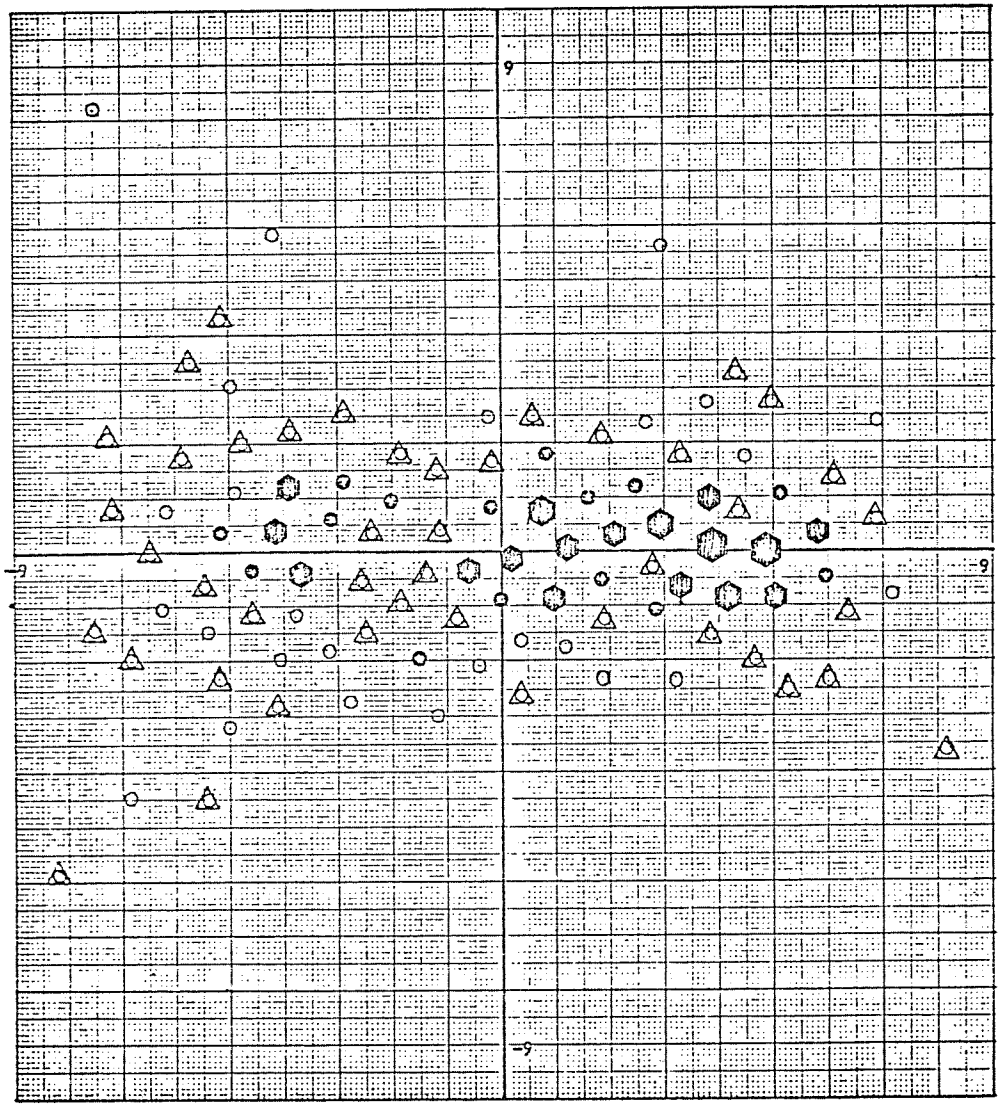


FIGURE 3 (b) t_{15000}

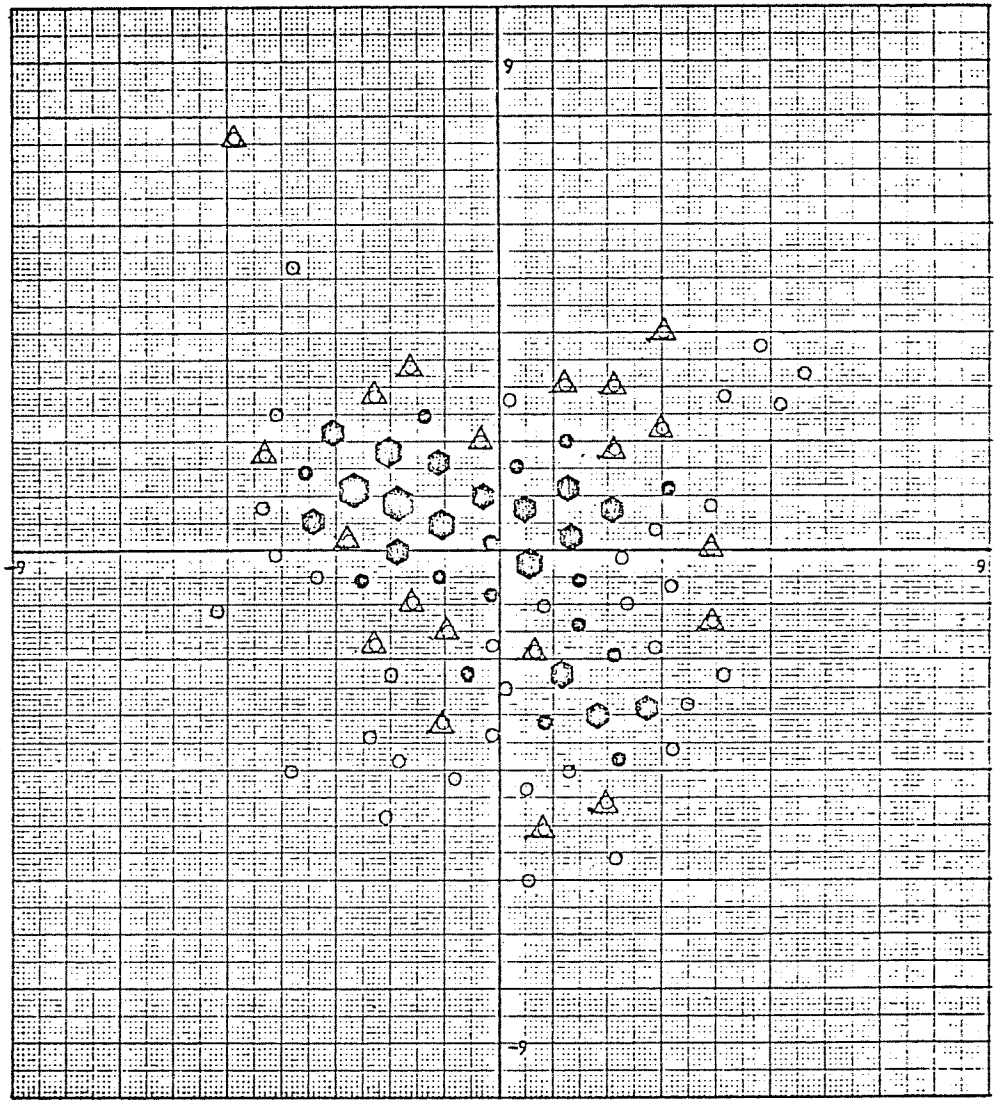


FIGURE 3 (c) t_{20000}

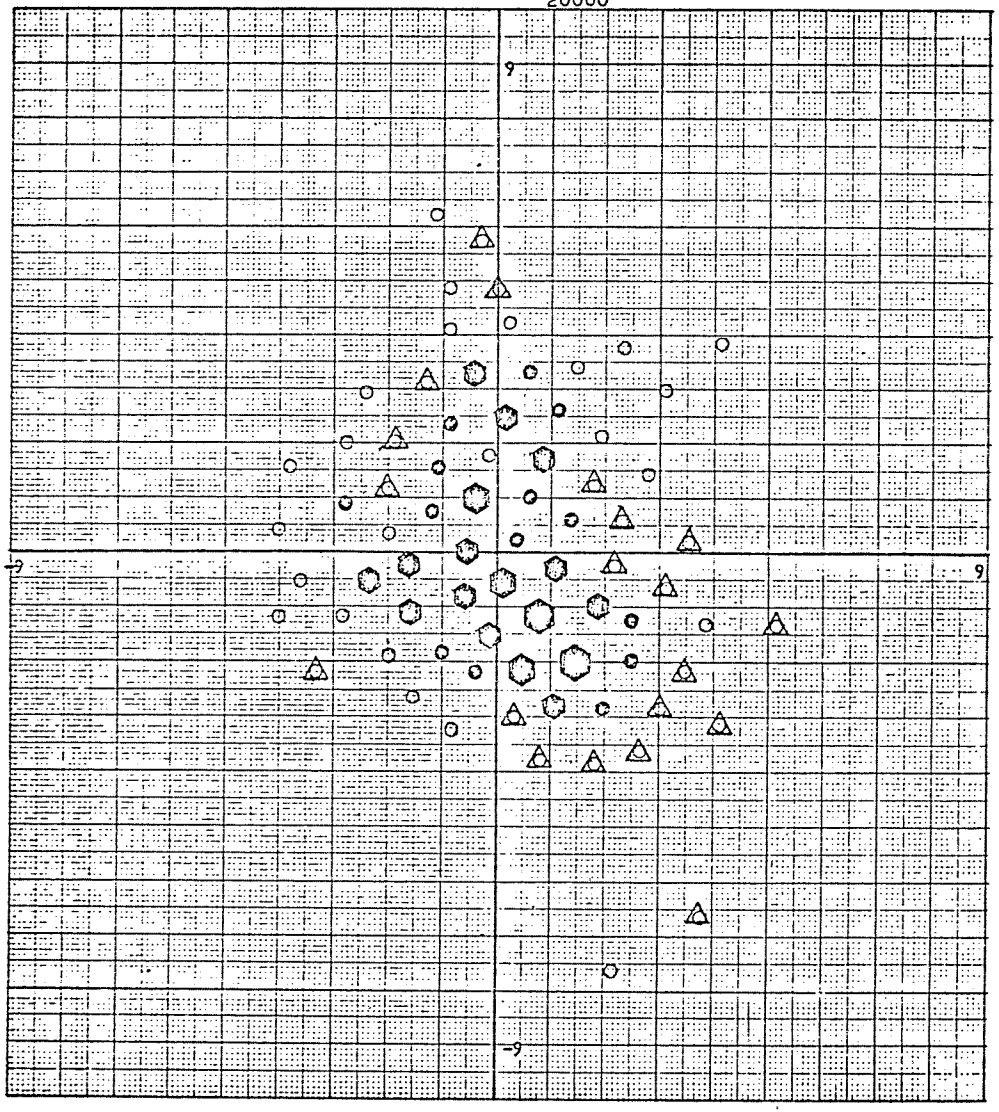


FIGURE 3 (d) t_{25000}

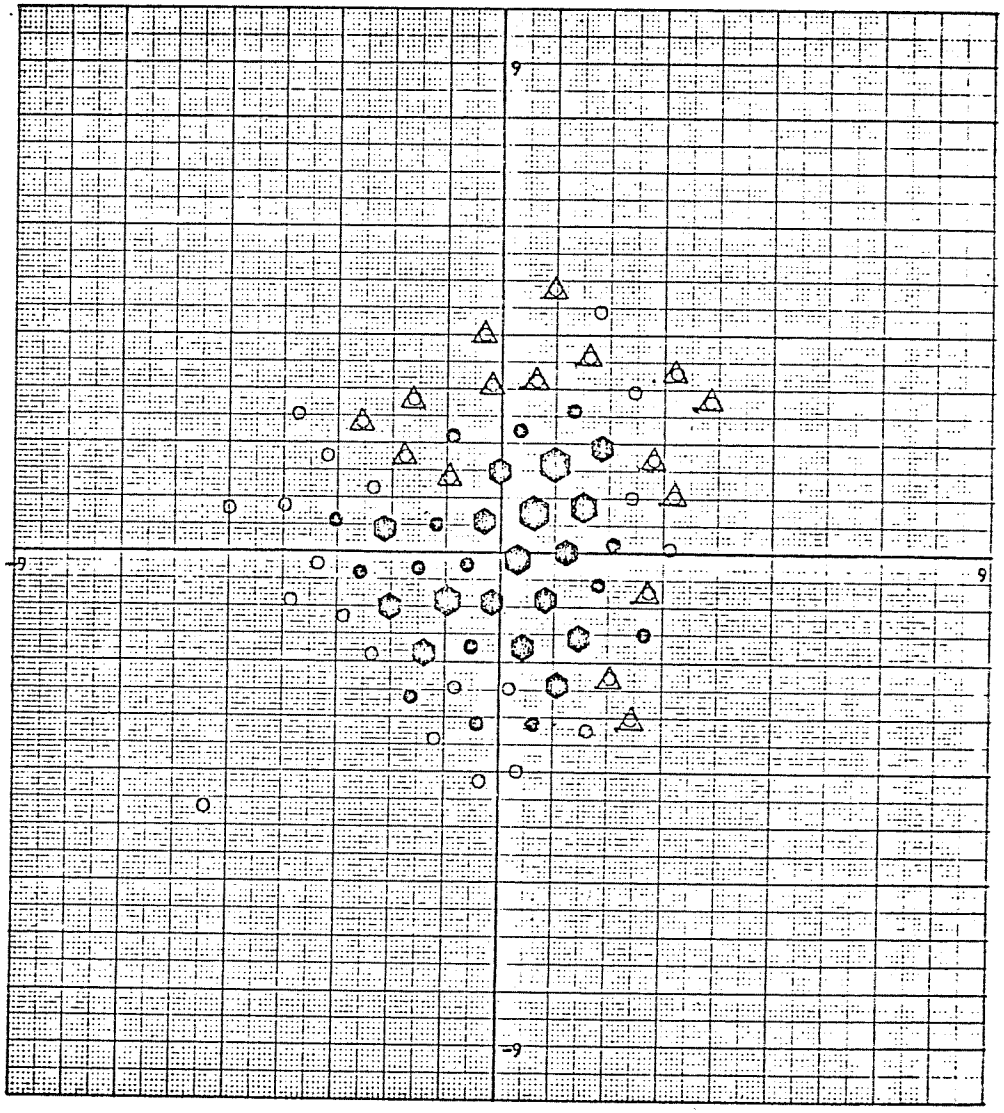


FIGURE 3 (e) t_{30000}

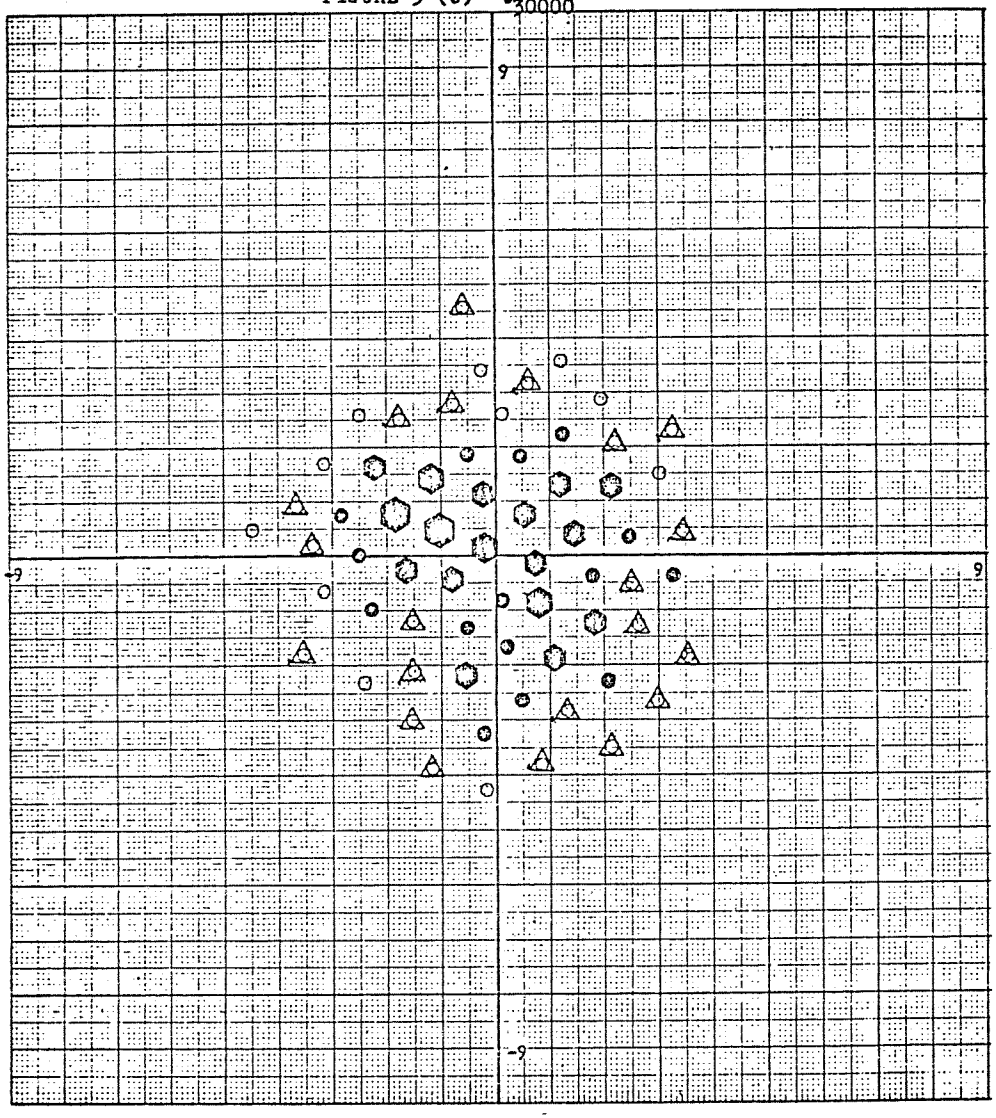


FIGURE 3 (f) t_{35000}

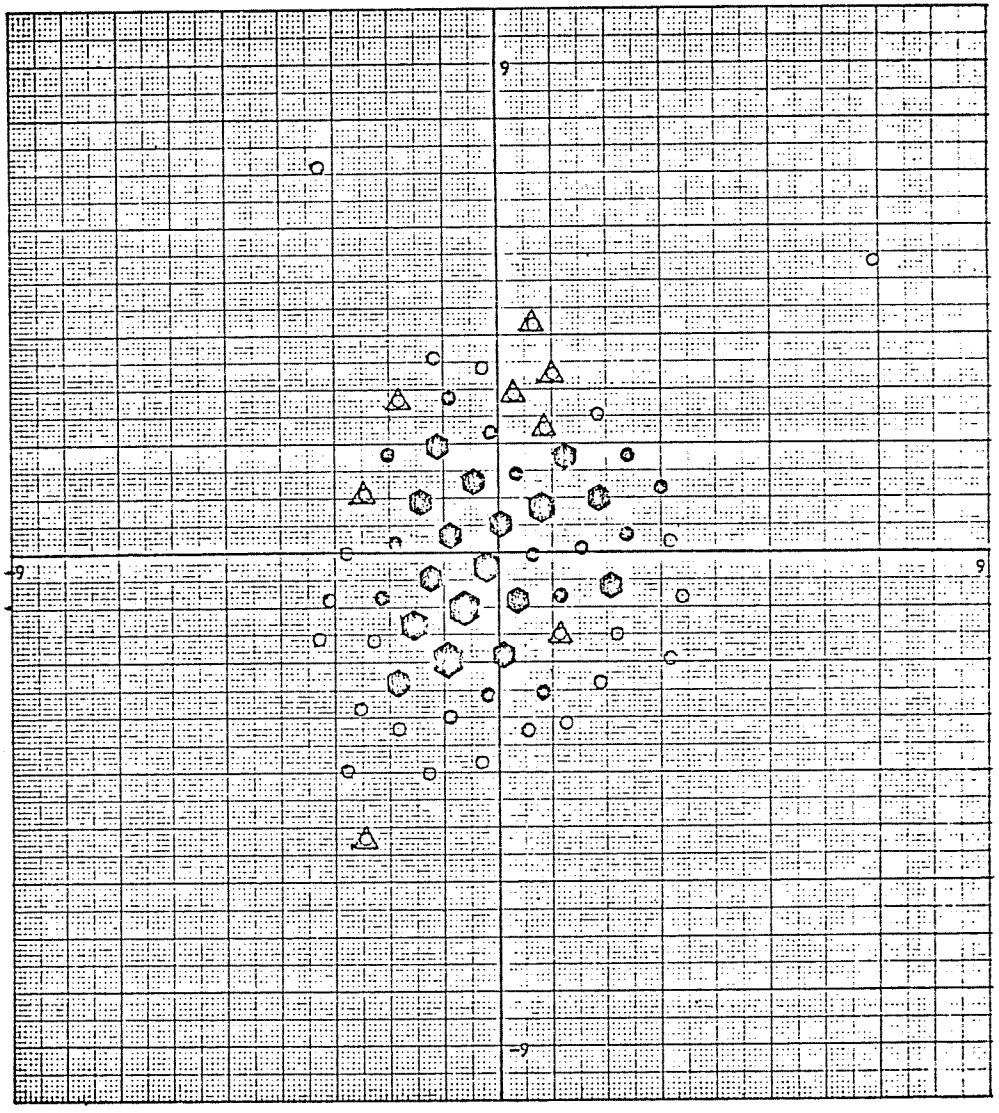


FIGURE 3 (g) t_{40000}

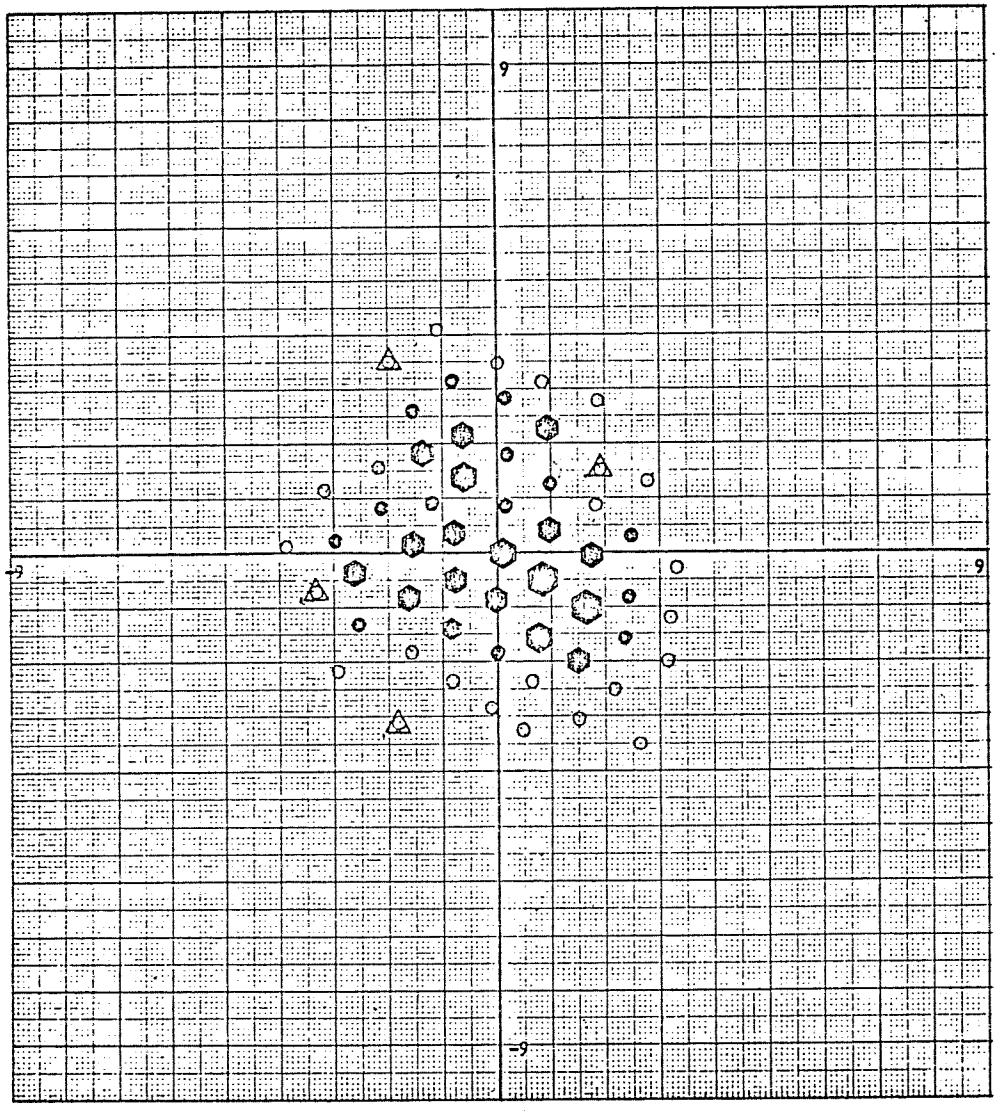


FIGURE 3 (h) t_{45000}

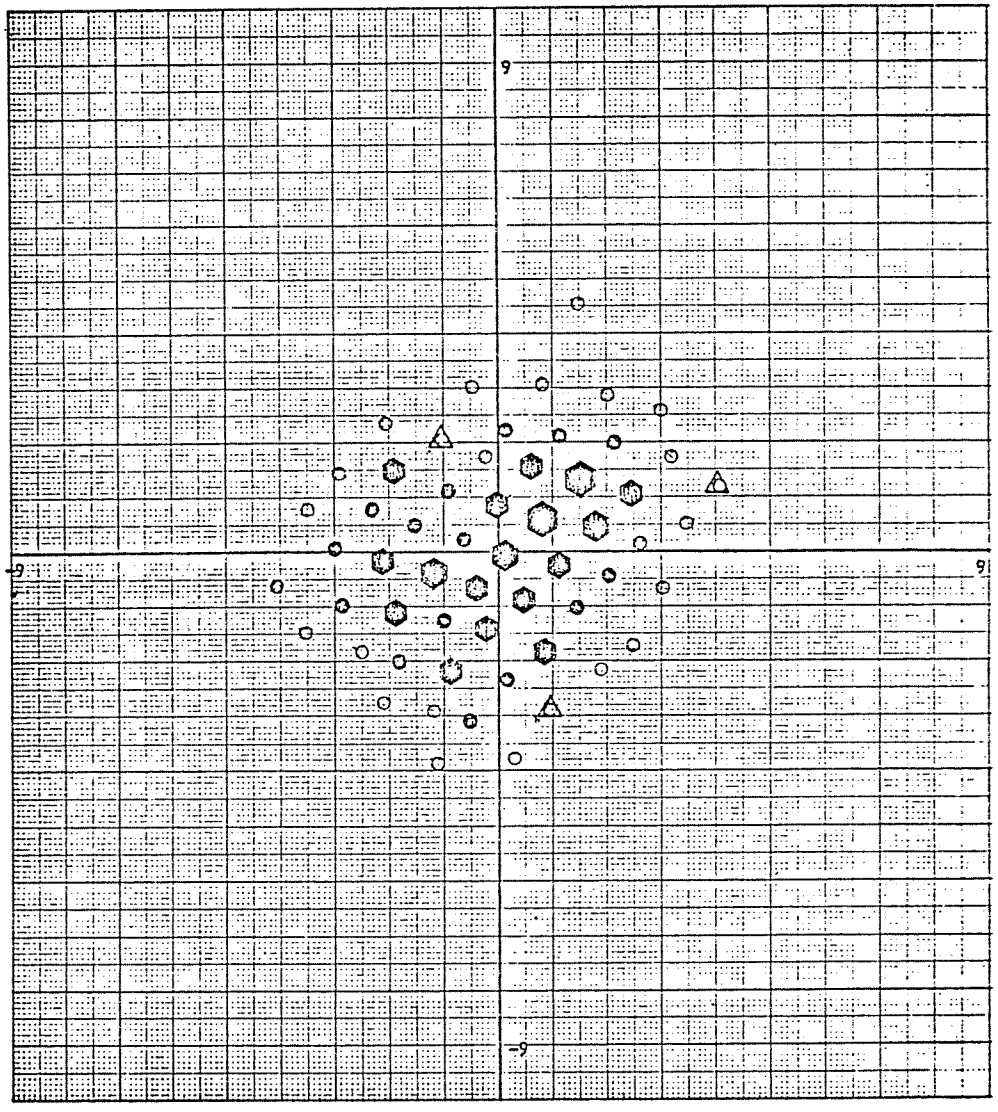


FIGURE 3 (i) t_{50000}

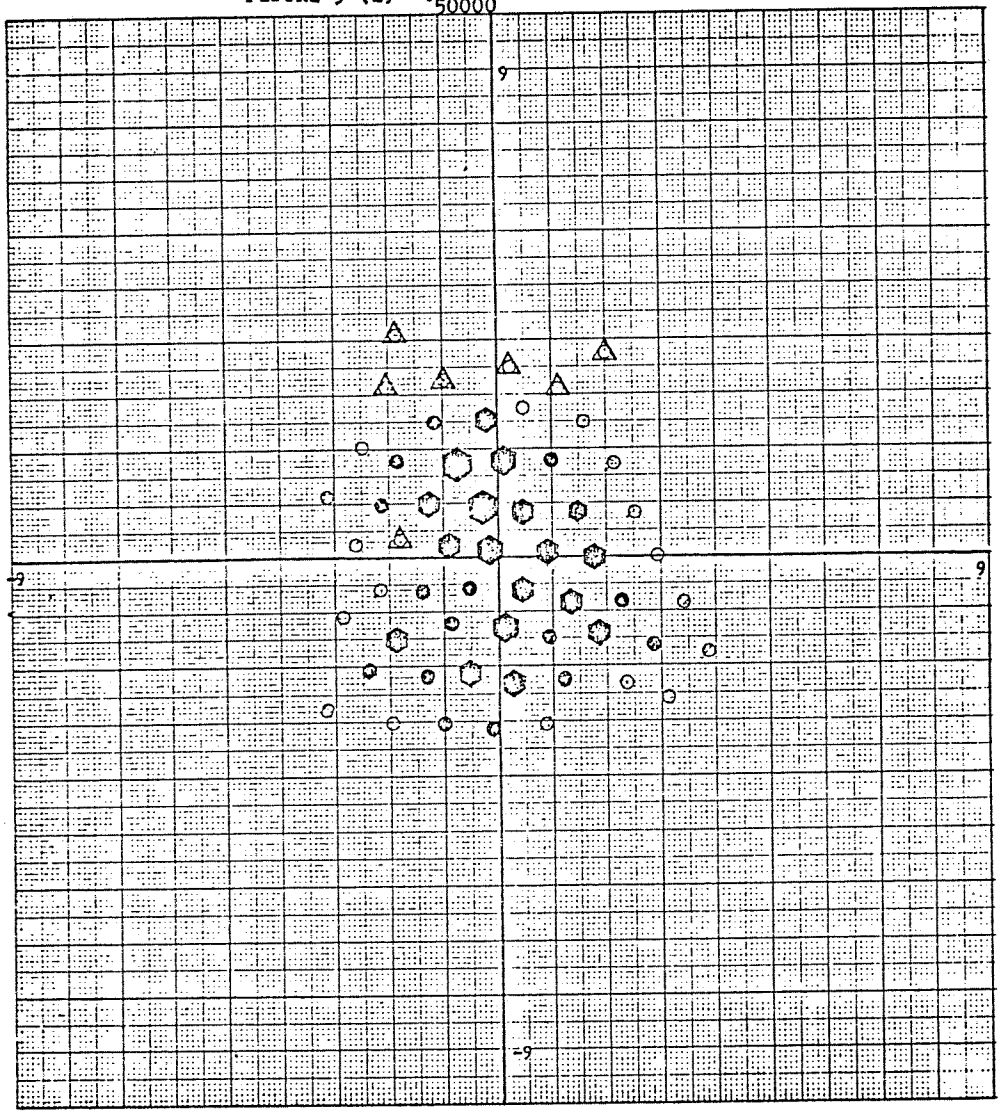


FIGURE 3 (j) t_{60000}

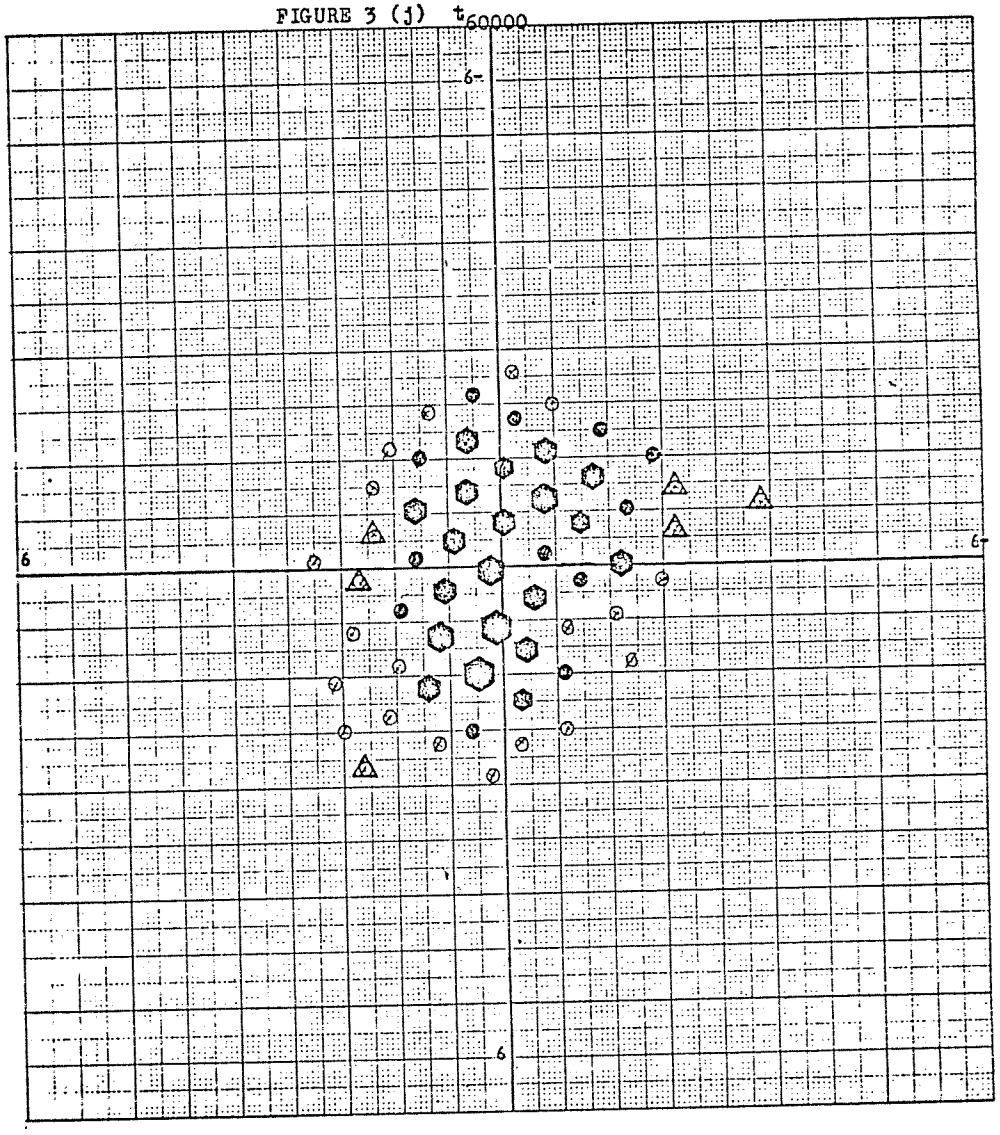
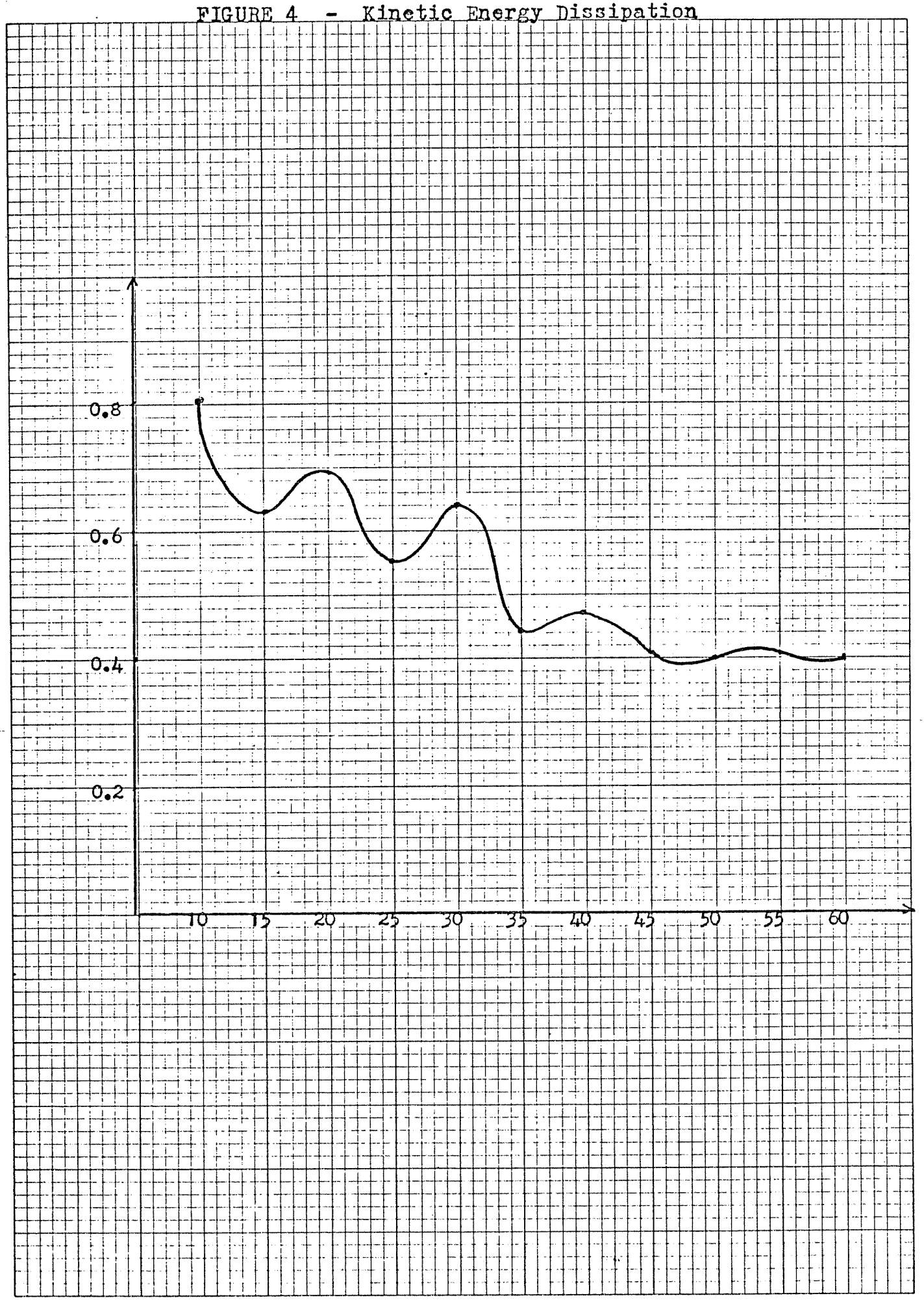


FIGURE 4 - Kinetic Energy Dissipation



Example 2. In order to avoid the early solidification process discussed in Example 1, the following nine changes in mass were made: $m_1 = m_3 = m_5 = m_6 = m_8 = m_9 = m_{11} = m_{17} = 4000$, $m_7 = 8000$. All other considerations were identical and the system was again studied through t_{60000}. The resulting motion is shown in Figures 5(a)-(i) at the respective times t_{10000}, t_{15000}, t_{20000}, t_{30000}, t_{35000}, t_{40000}, t_{45000}, t_{50000}, t_{60000}. The increase in fluidity has resulted by t_{20000} in a separation into two distinct subsystems. It is the larger subsystem, shown to the left in Figure 5(c), which is then studied through t_{60000}. At t_{30000} this system has 52 particles, as shown in Figure 5(d). By the time t_{60000}, the largest particle is at the center of the system, the geometric center is (-0.004, -0.02), and the system is almost circular. The density clearly decreases monotonically from the center outward. The solidification process is shown in Figures 6(a)-(c), and Figure 6(c), at time t_{60000}, is certainly consistent with current theories on the structure of the interiors of planets.

5. Remarks

The results of the variety of other examples run can only be summarized easily as follows. As in Examples 1 and 2 of Section 4, only minor changes in input parameters often resulted in dramatic changes in dynamical behavior [4]. This is, of course, consistent with the diversity one observes among the planets and moons of the solar system.

Other computations, aimed at duplicating the development of galaxy arms, all ended in failure. The systems invariably self-reorganized into one or more relatively elliptic or circular subsystems. This suggests to us the strong possibility that more than gravitation is involved

FIGURE 5 (a) t_{10000}

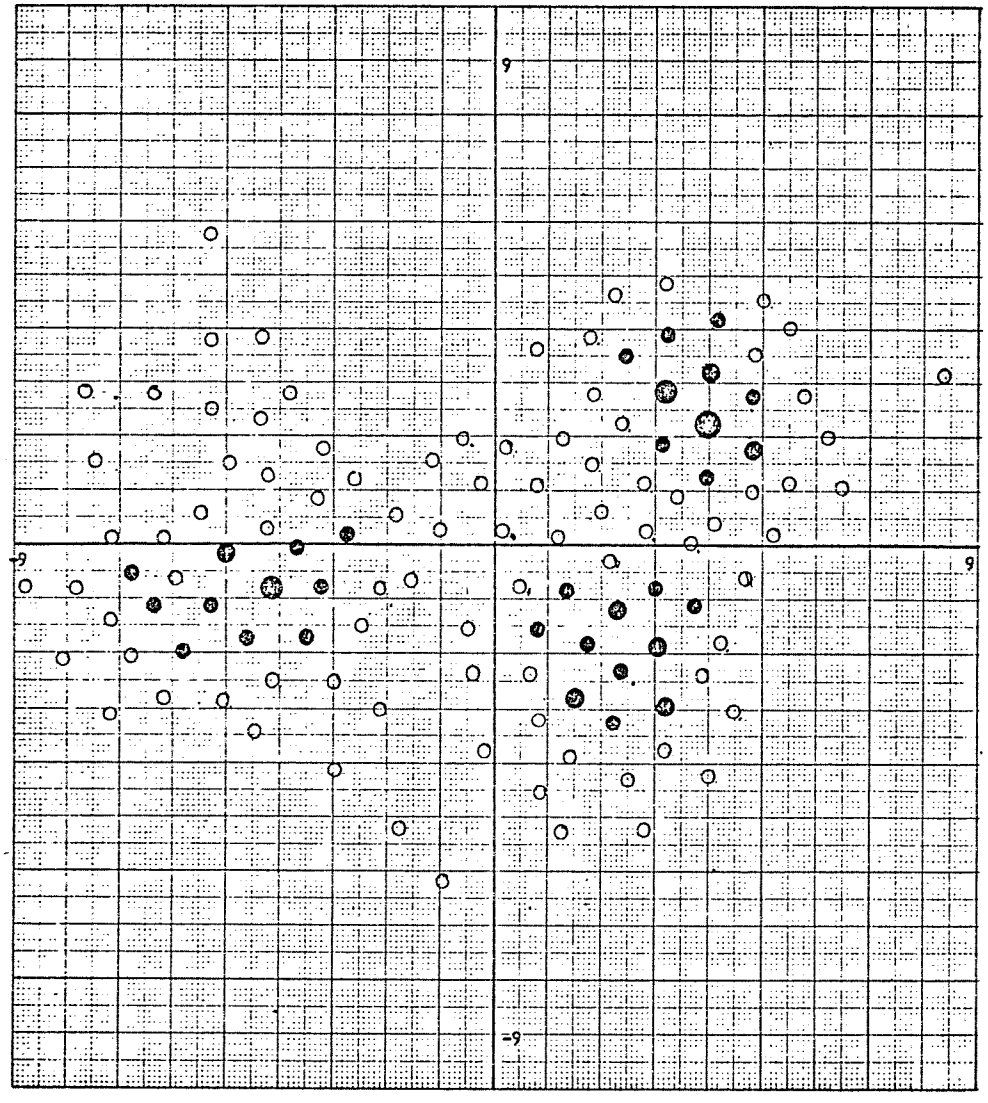


FIGURE 5 (b) t_{15000}

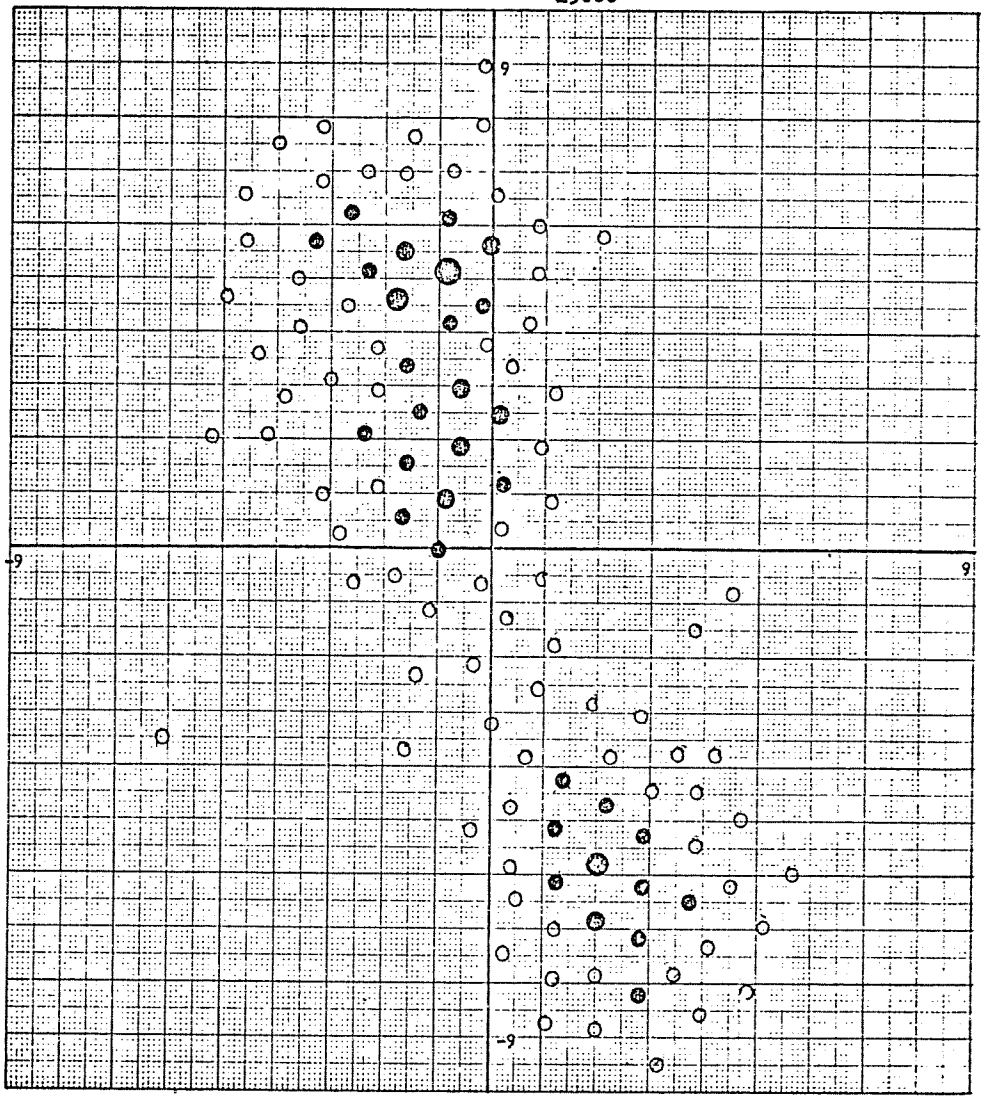


FIGURE 5 (c) t_{20000}

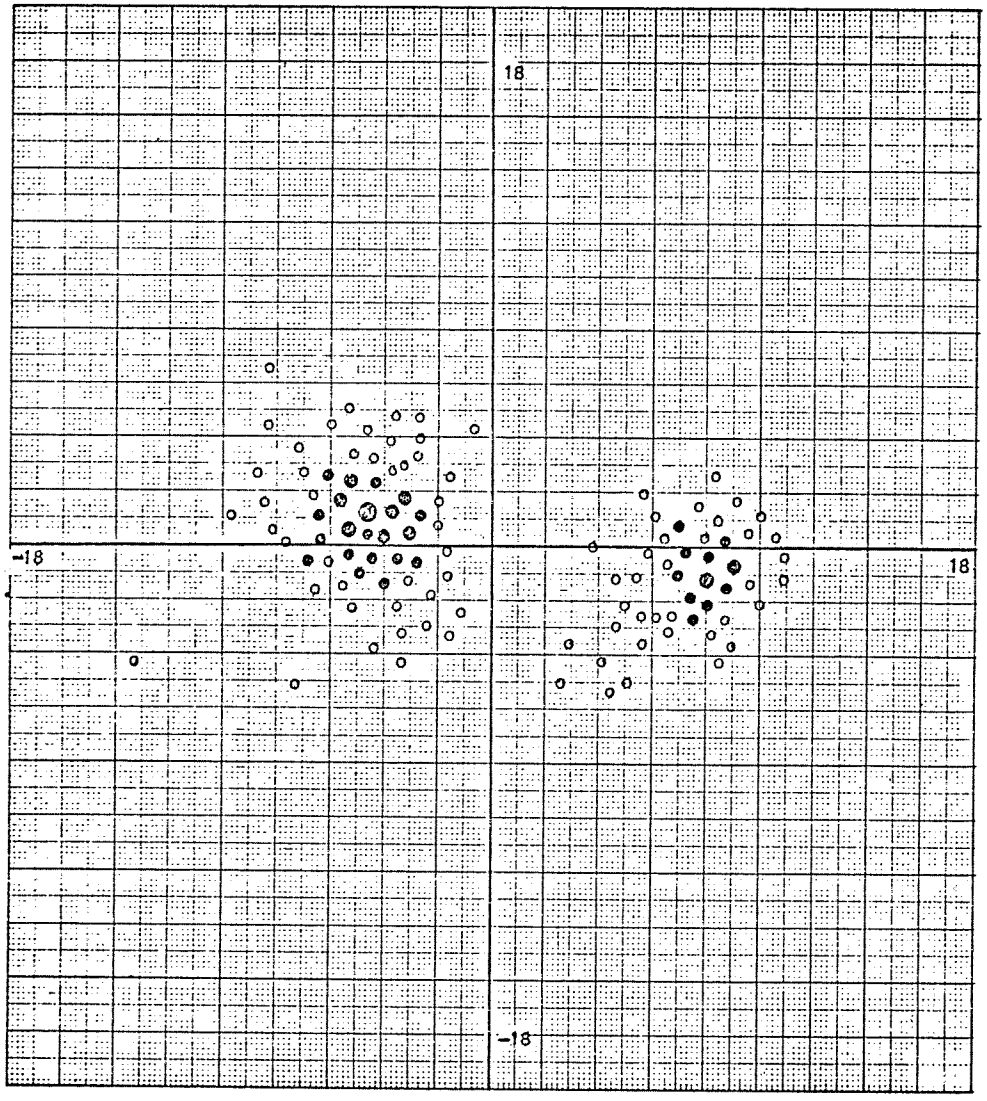


FIGURE 5 (d) t_{30000}

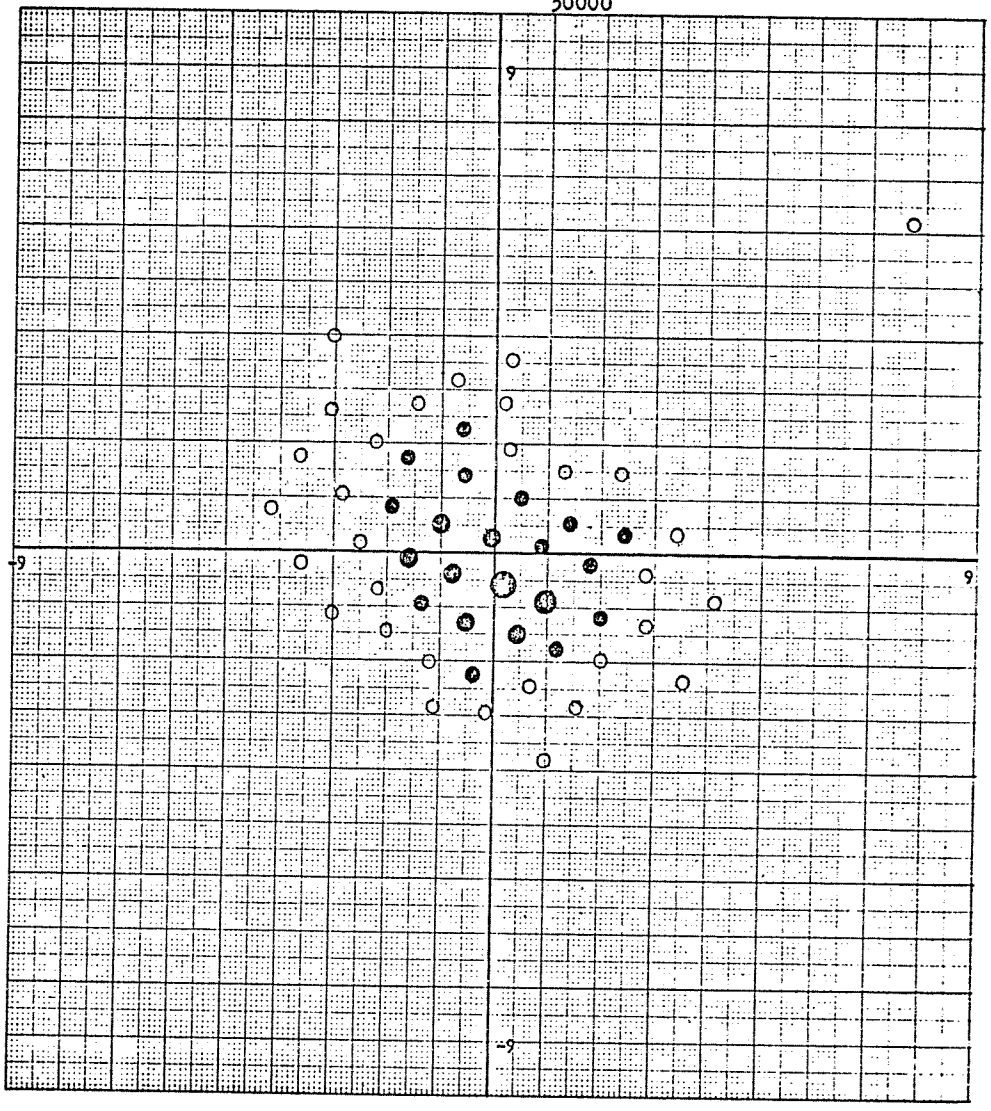


FIGURE 5 (e) t_{35000}

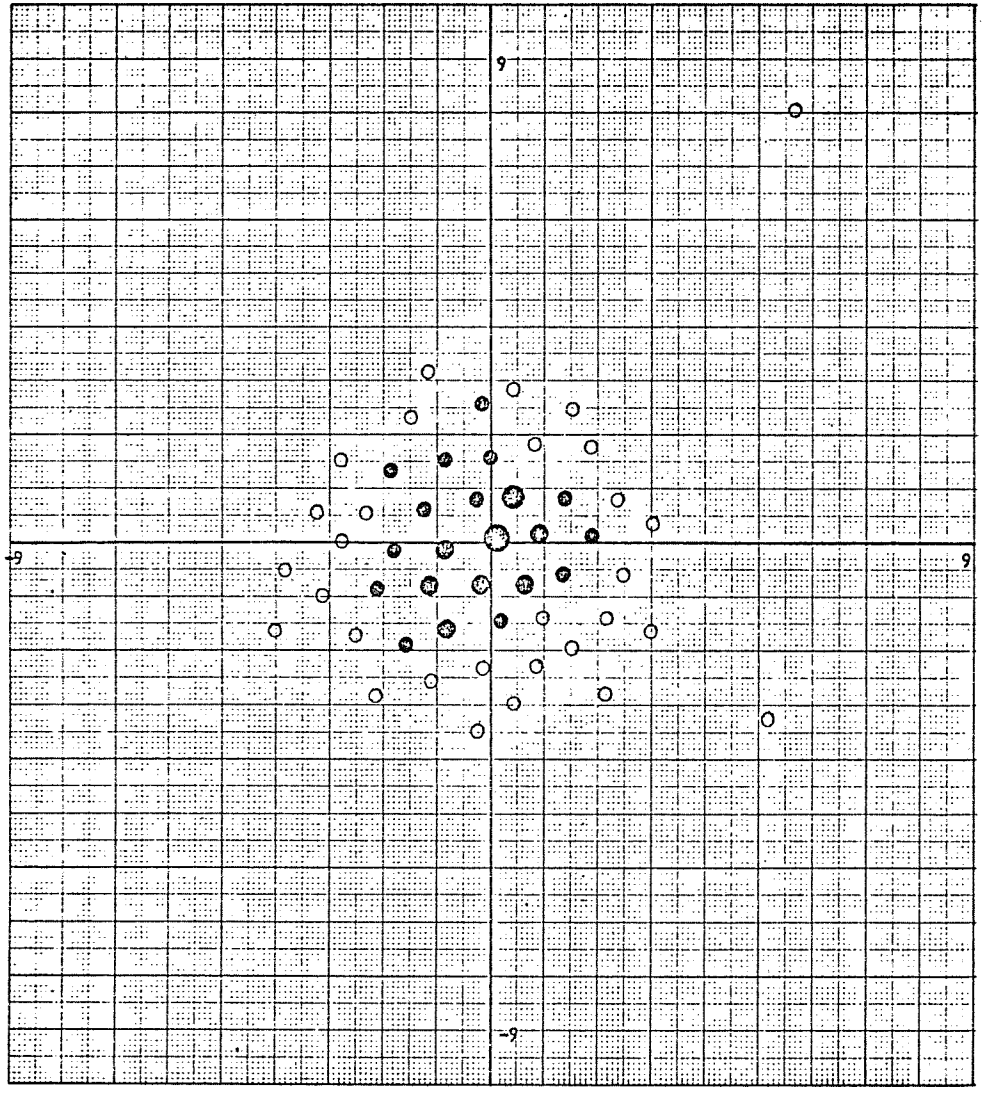


FIGURE 5 (f) t_{40000}

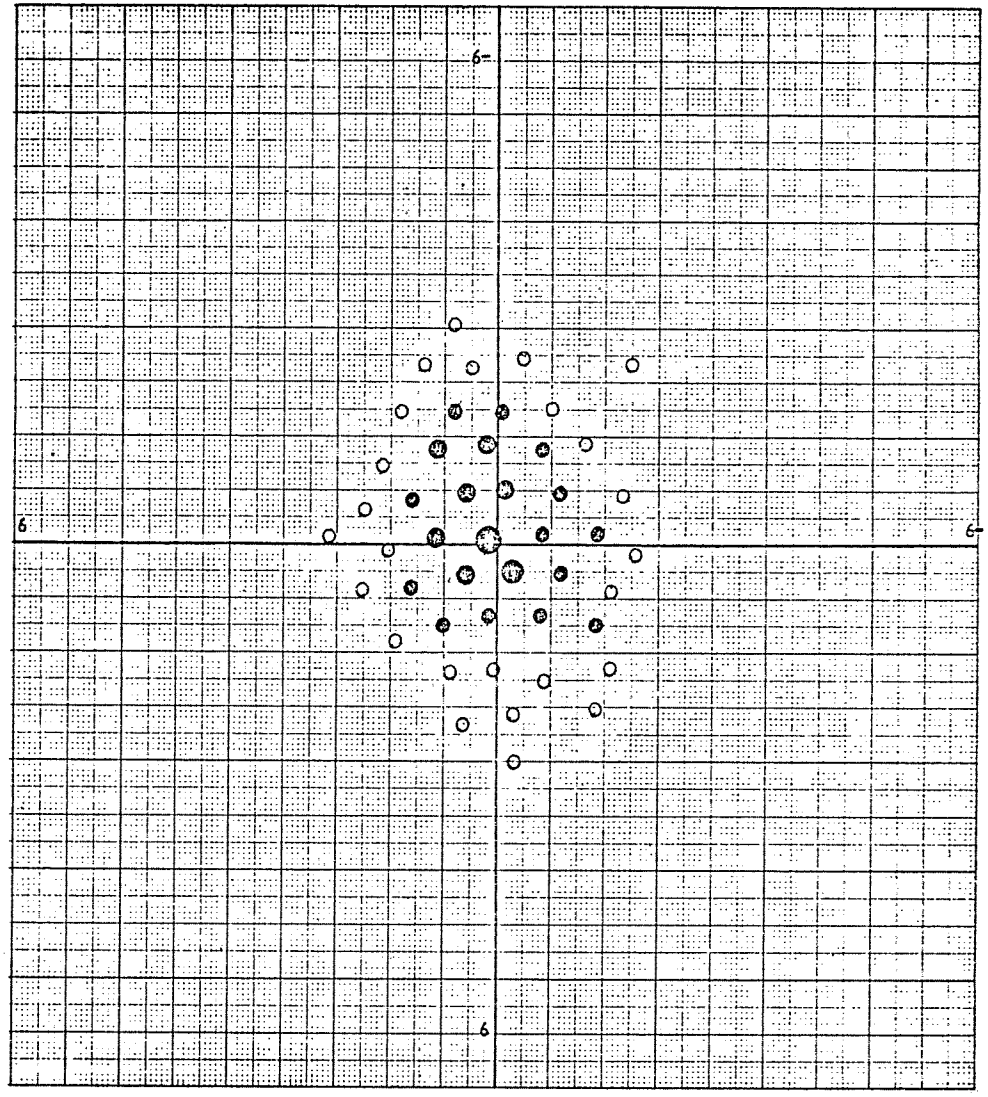


FIGURE 5 (g) t_{45000}

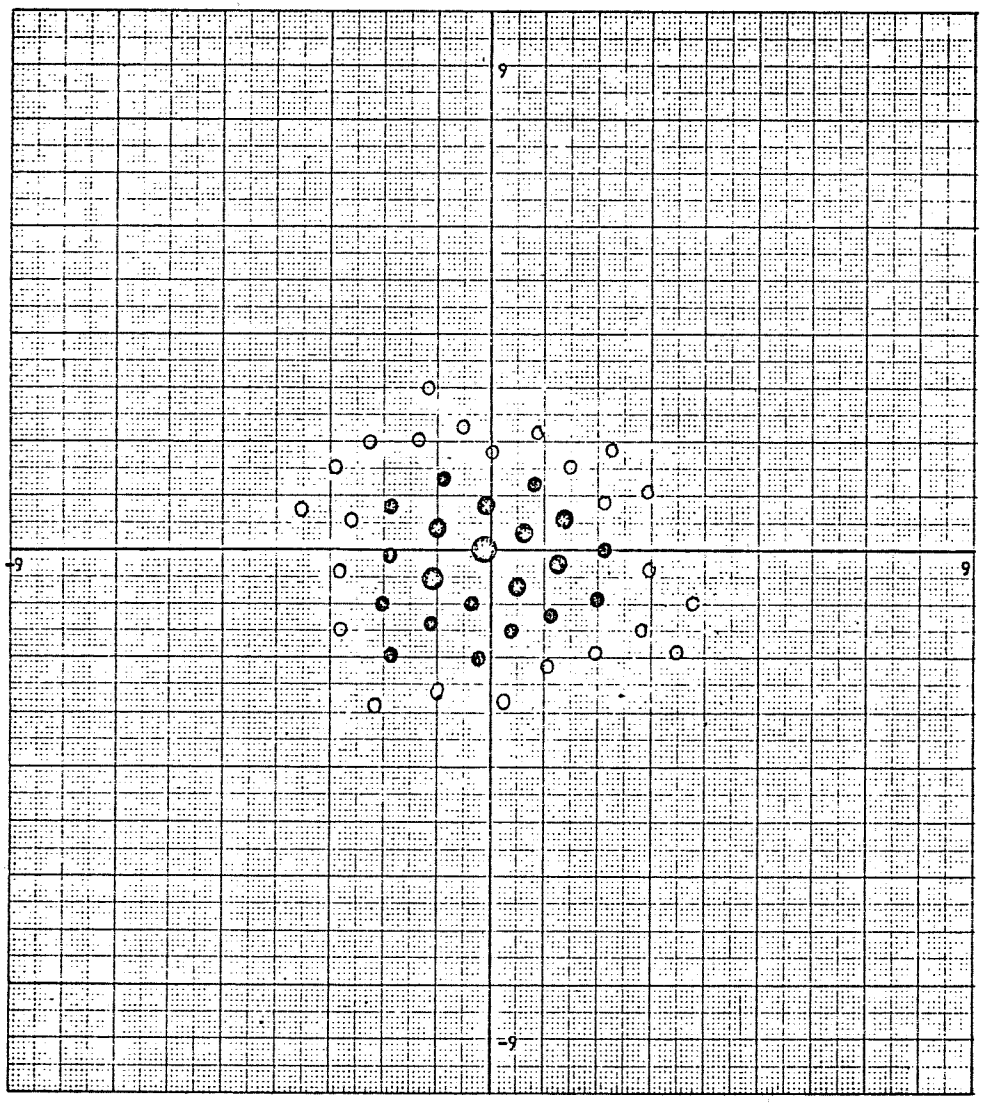


FIGURE 5 (h) t_{50000}

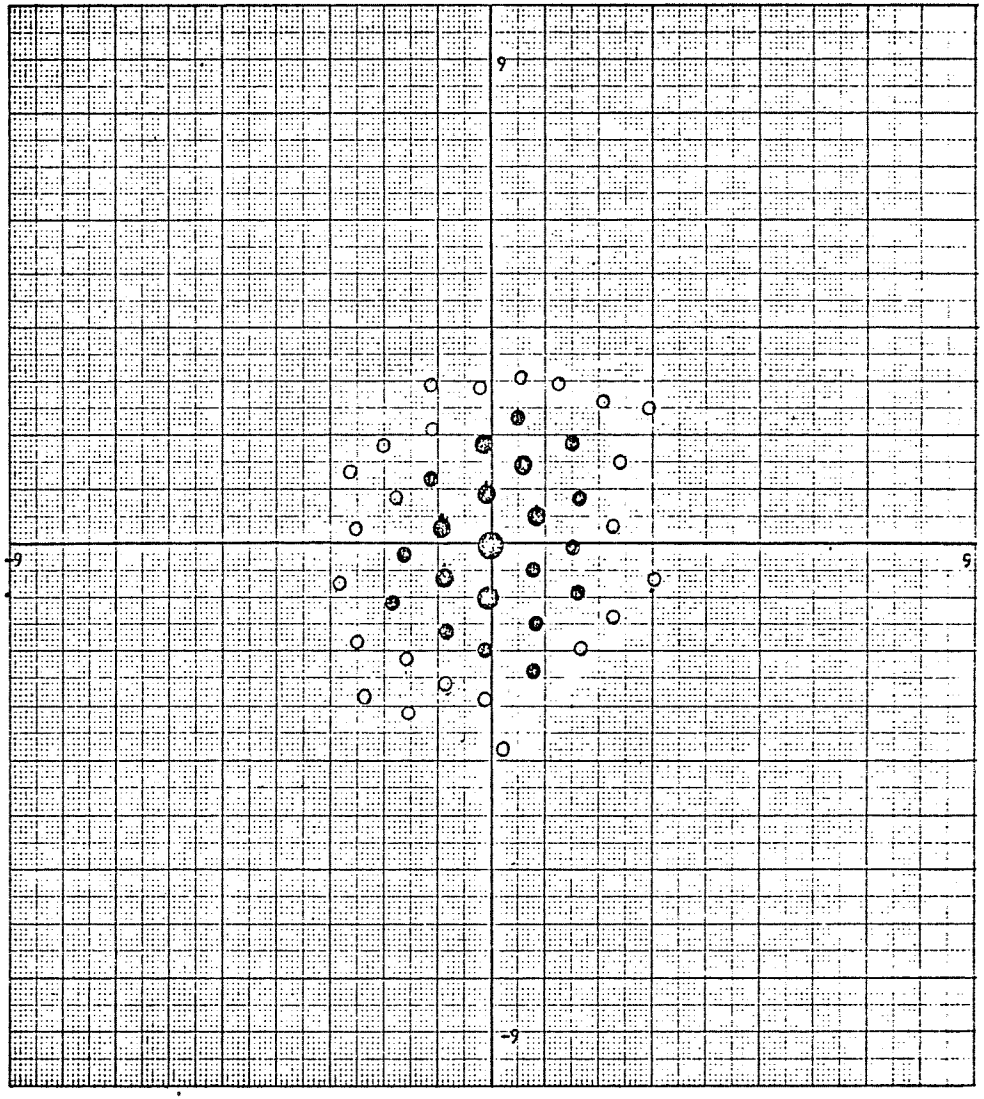


FIGURE 5 (i) t_{60000}

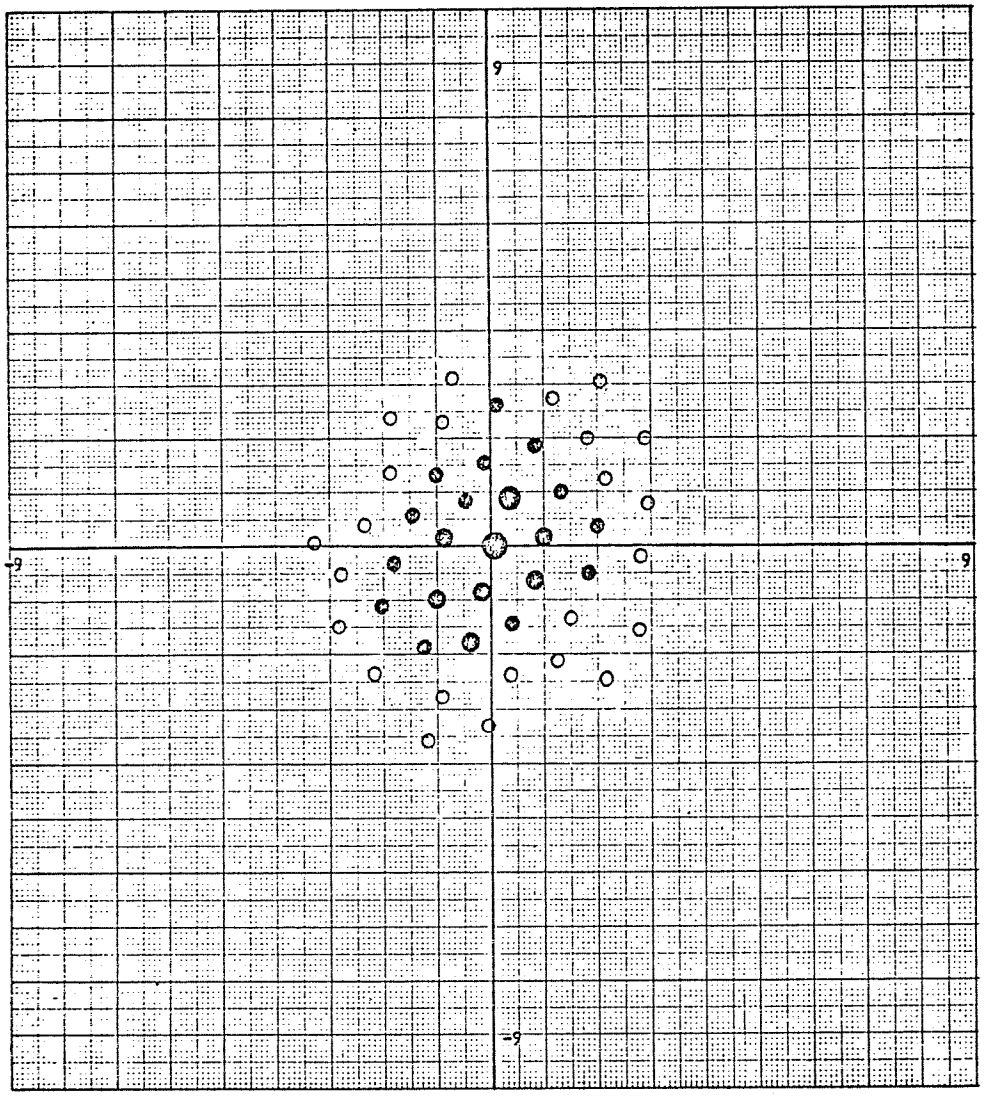


FIGURE 6 (a) t_{30000}

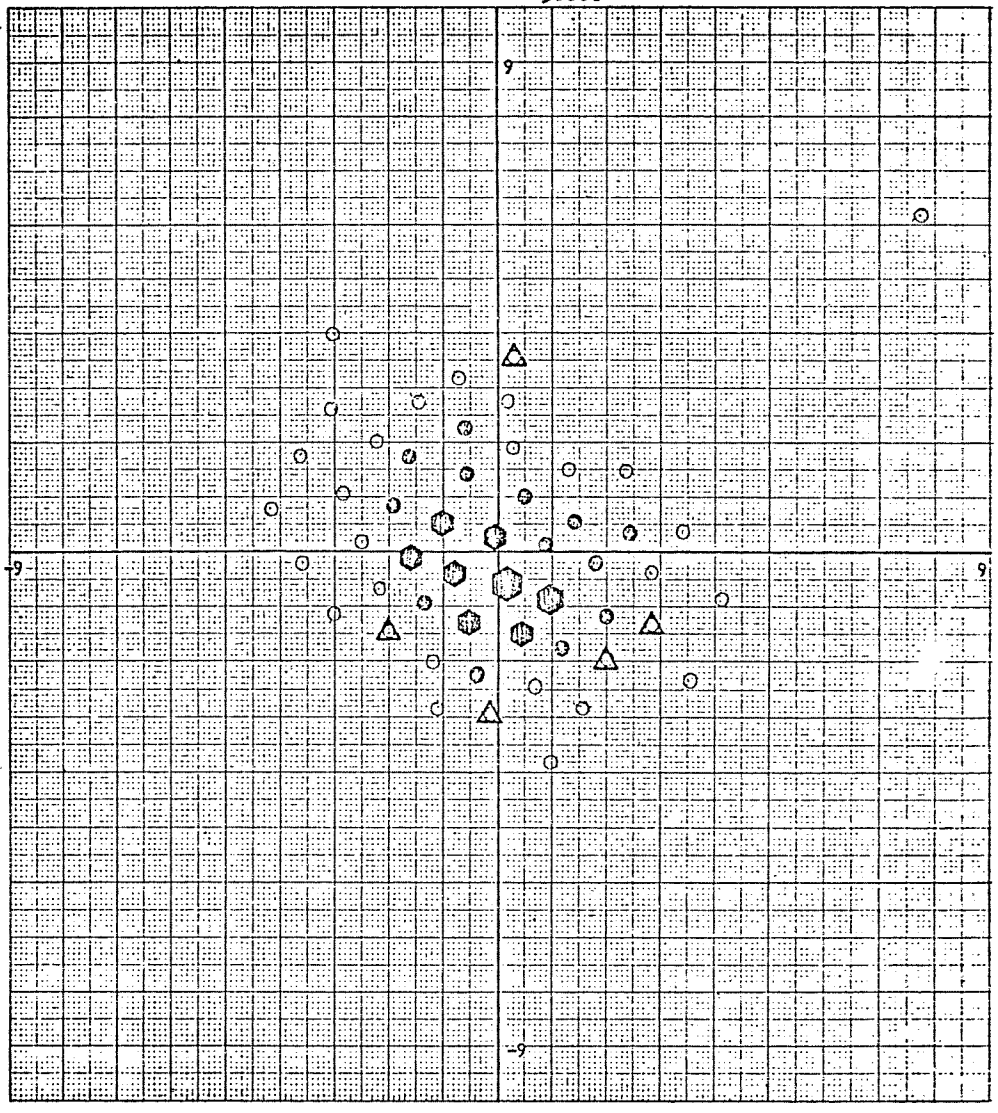


FIGURE 6 (b) t_{40000}

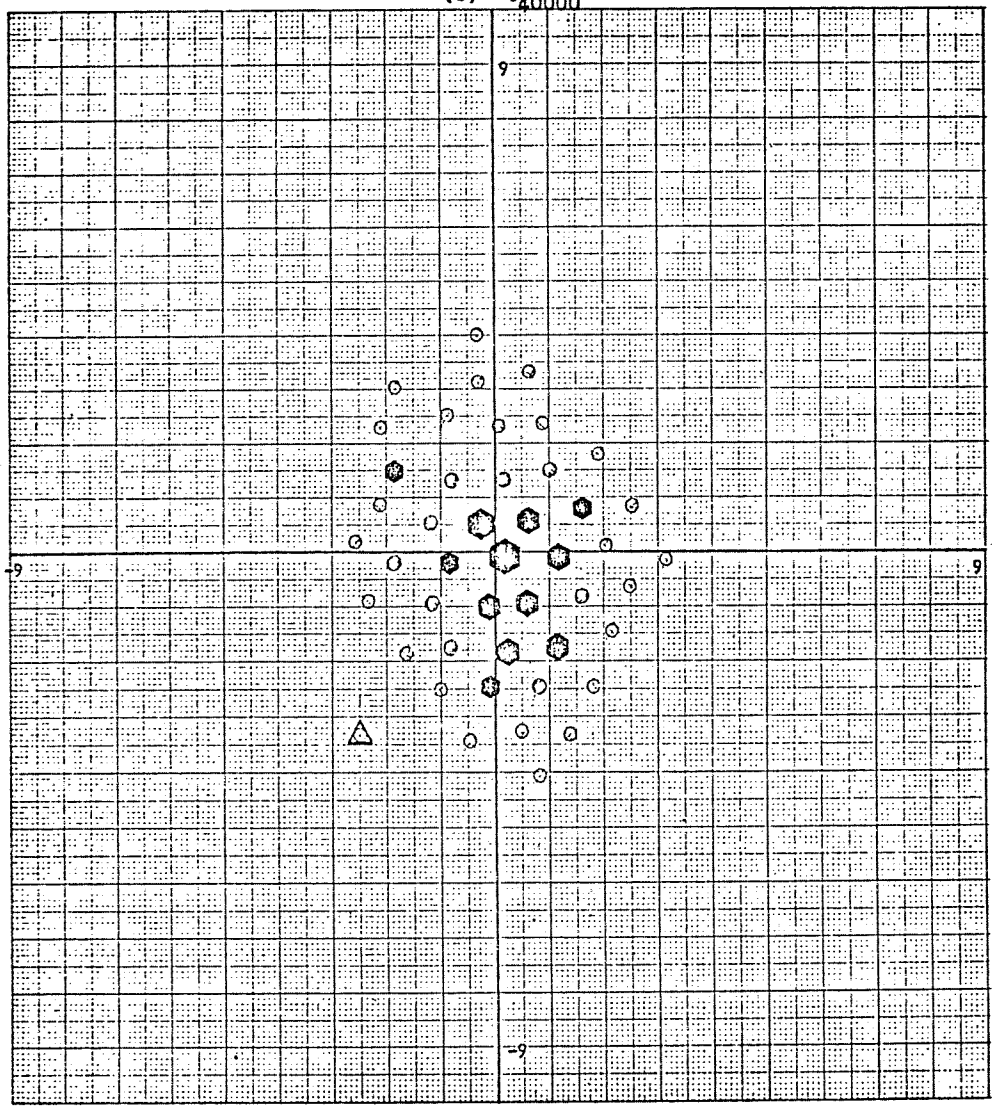
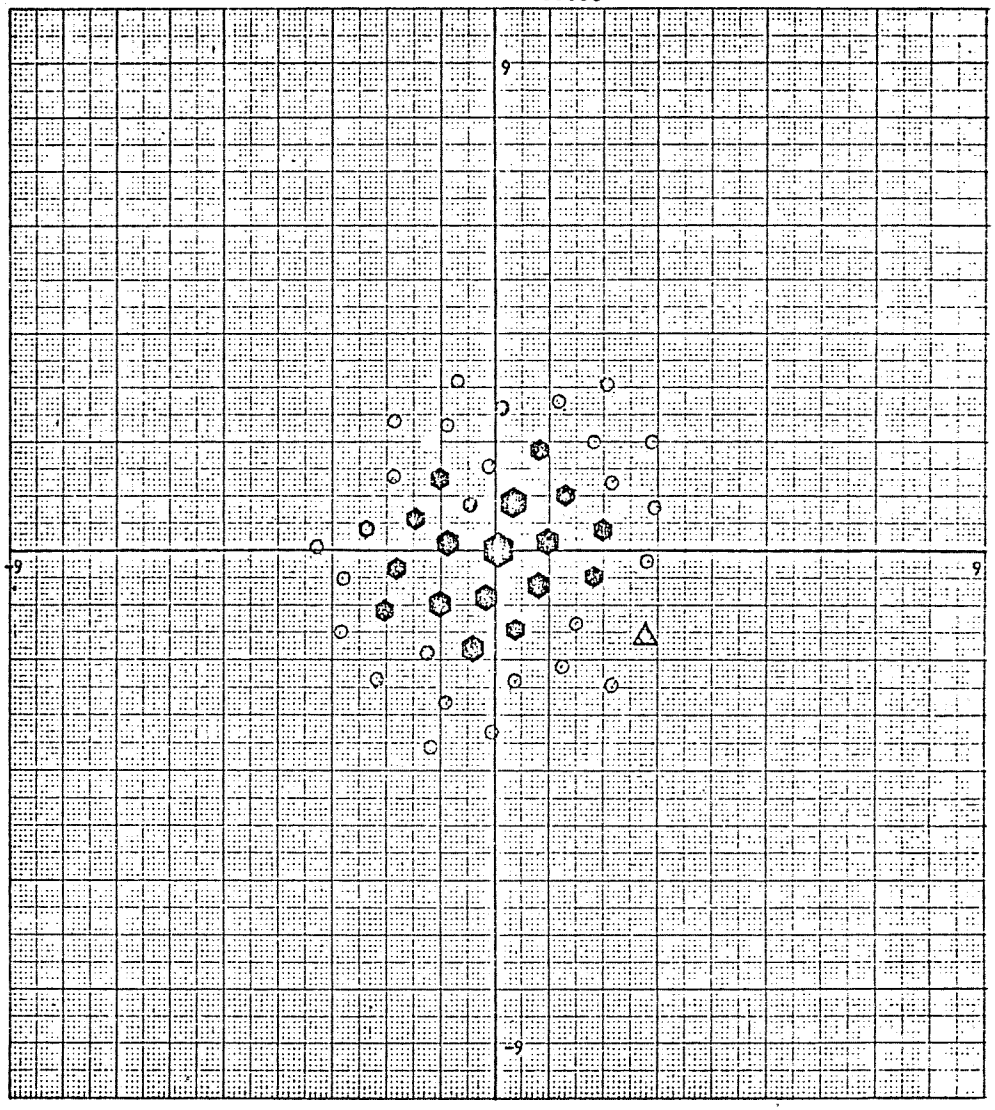


FIGURE 6 (c) t_{60000}



in galaxy arm formation.

Finally, note that initial calculations with a 239 particle configuration were begun, but these had to be discontinued due to a lack of funds.

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