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DISCRETE SHOCK WAVES

by

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ABSTRACT

A discrete particle approach is developed for the study of shock waves. Prototype problems are discussed for the flow past a wedge and in a shock tube. Computer generated shocks are illustrated graphically.

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1. INTRODUCTION

The study of shock waves, initiated in the fundamental papers of Earnshaw, Hugoniot, Rankine, Rayleigh, Riemann and Stokes, continues to grow in interest and importance (see, e.g., references [1]-[6], [8]-[15] and the additional references contained therein). Traditionally, the models have been continuous and the associated dynamical equations have been nonlinear hyperbolic systems of first order partial differential equations.

In this paper we will develop a finite particle method for constructing shock waves. The formulation will be based on a discrete approach to mechanics [7] in which the dynamical equations are difference equations and the solutions of the equations are discrete functions, that is, functions defined only on finite point sets. In the present paper, applications will be limited to only simple, prototype problems and models. It can be reasonably expected that current limitations on the number of particles allowed in our models can be appreciably relaxed with the development of advanced computer technology.

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2. HEURISTICS

Consider a given fluid, whether it be a gas or a liquid, which consists of a finite number of particles, each of which is in uniform motion in the same direction. To such a fluid, assign a positive measure M of average particle density. Into the particle stream, insert a body B to obstruct the free, uniform flow. Then in certain regions about B, there may occur sets of particles whose densities are not average. A boundary between particles with average density and those with "greater than average" density is called a shock wave.

In this paper we will illustrate the "greater than average" density concept in the construction of shock waves for prototype problems. We will not at present refine the concept to the point where we can differentiate between strong and weak shocks.

3. FLOW PAST A WEDGE

Consider a wedge with inclination angle α, as shown in Figure 3.1. Every Δt seconds, a column of particles, each of radius r and mass m and located at the points $(0, \frac{1}{2} + n)$, $n=0,1,\ldots,9$, on the Y-axis, commences to move horizontally with an initial positive velocity $v_{x,0} = c$. Neglecting gravitational forces, but allowing a repulsive force to similate the effects of collision, the N-body dynamical equations for discrete models [7] implies that the position $(x_{i,k}, y_{i,k})$ of a particle P_i at time $t_k = k\Delta t$ is given by

$$x_{i,1} = x_{i,0} + c\Delta t + \frac{(\Delta t)^2}{2} m \sum_{\substack{j=1 \\ j\neq i}}^{n} \frac{(x_{i,0} - x_{j,0})H_{ij,k}}{(r_{ij,0} + \xi)^{p+1}}$$

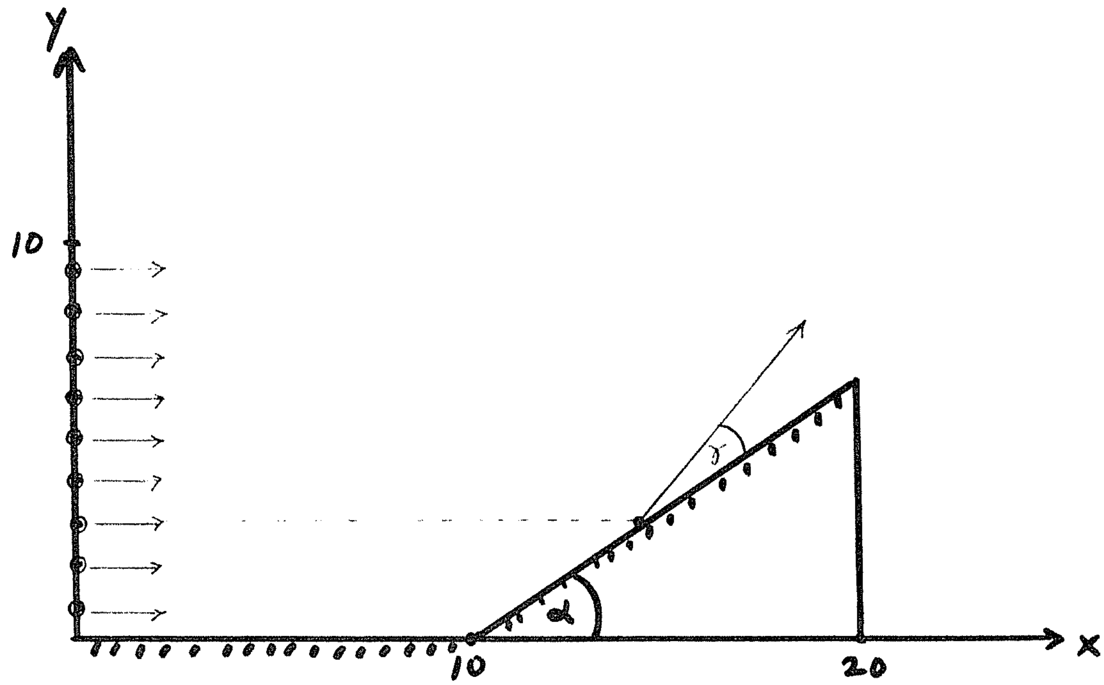

FIGURE 3.1

$$y_{i,1} = y_{i,0} + \frac{(\Delta t)^2}{2} m \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{(y_{i,0} - y_{j,0}) H_{ij,k}}{(r_{ij,0} + \xi)^{p+1}}$$

$$x_{i,2} = 3x_{i,1} - 2x_{i,0} - c(\Delta t) + \frac{(\Delta t)^2}{2} m \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{(x_{i,1} - x_{j,1}) H_{ij,k}}{(r_{ij,1} + \xi)^{p+1}}$$

$$y_{i,2} = 3y_{i,1} - 2y_{i,0} + \frac{(\Delta t)^2}{2} m \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{(y_{i,1} - y_{j,1}) H_{ij,k}}{(r_{ij,1} + \xi)^{p+1}}$$

and, for $k \geq 2$,

$$x_{i,k+1} = 3x_{i,k} + 2(-1)^k x_{i,0} - 4 \sum_{j=2}^{k} [(-1)^j x_{i,k-j+1}] + (-1)^k c \Delta t$$

$$+ \frac{(\Delta t)^2 m}{2} \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{(x_{i,k} - x_{j,k}) H_{ij,k}}{(r_{ij,k} + \xi)^{p+1}}$$

$$y_{i,k+1} = 3y_{i,k} + 2(-1)^k y_{i,0} - 4 \sum_{j=2}^{k} [(-1)^j y_{i,k-j+1}]$$

$$+ \frac{(\Delta t)^2}{2} m \sum_{\substack{j=1 \\ j \neq i}}^{n} \frac{(y_{i,k} - y_{j,k}) H_{ij,k}}{(r_{ij,k} + \xi)^{p+1}} ,$$

where

$r_{ij,k}$ = distance between P_i and P_j at t_k

$$H_{ij,k} = \begin{cases} 0 , & \text{if } r_{ij,k} \geq 2r \\ H , & \text{if } r_{ij,k} < 2r, \text{ where } H > 0 , \end{cases}$$

and where ξ and p are positive parameters associated with the repulsive force between two particles which have collided.

In the particular examples to be discussed, the choices $\xi = 0.1$, $p = 1$, $r = \frac{1}{2}$, $H = 1$, $c = 100$, $\Delta t = .01$ are used consistently in both this and the next sections.

If a particle impacts on the inclined face of the wedge with a speed $|v|$, assume that it rebounds at an angle γ, as shown in Figure 3.1, at a speed $|v_r|$ given by

$$|v_r| = \delta |v_i| ,$$

where $o < \delta \leq 1$. Finally, when the coordinates $(x_{i,k}, y_{i,k})$ of a particle satisfy either

$$x > 20 - r$$

$$y > 10 - r ,$$

assume that the particle no longer has an effect on other particles and hence can be discarded.

Under the assumption that a particle has greater than average density if the distance between it and at least three other particles is less than unity, typical UNIVAC 1108 computer examples of the development of shock waves are shown in Figures 3.2, 3.3, and 3.4, where only the centers of particles are plotted. In Figure 3.2, the parameter choices are $\delta = .6$, $\alpha = \gamma = \frac{\pi}{8}$, $m = 1$ and the time steps shown are t_{19}, t_{26} and t_{31}. The development simulates the shock development in moving from a vacuum into a fluid medium. In Figure 3.3, the parameter choices are $\delta = .1$, $\alpha = \gamma = \frac{\pi}{8}$, $m = 10$, and the time steps shown are

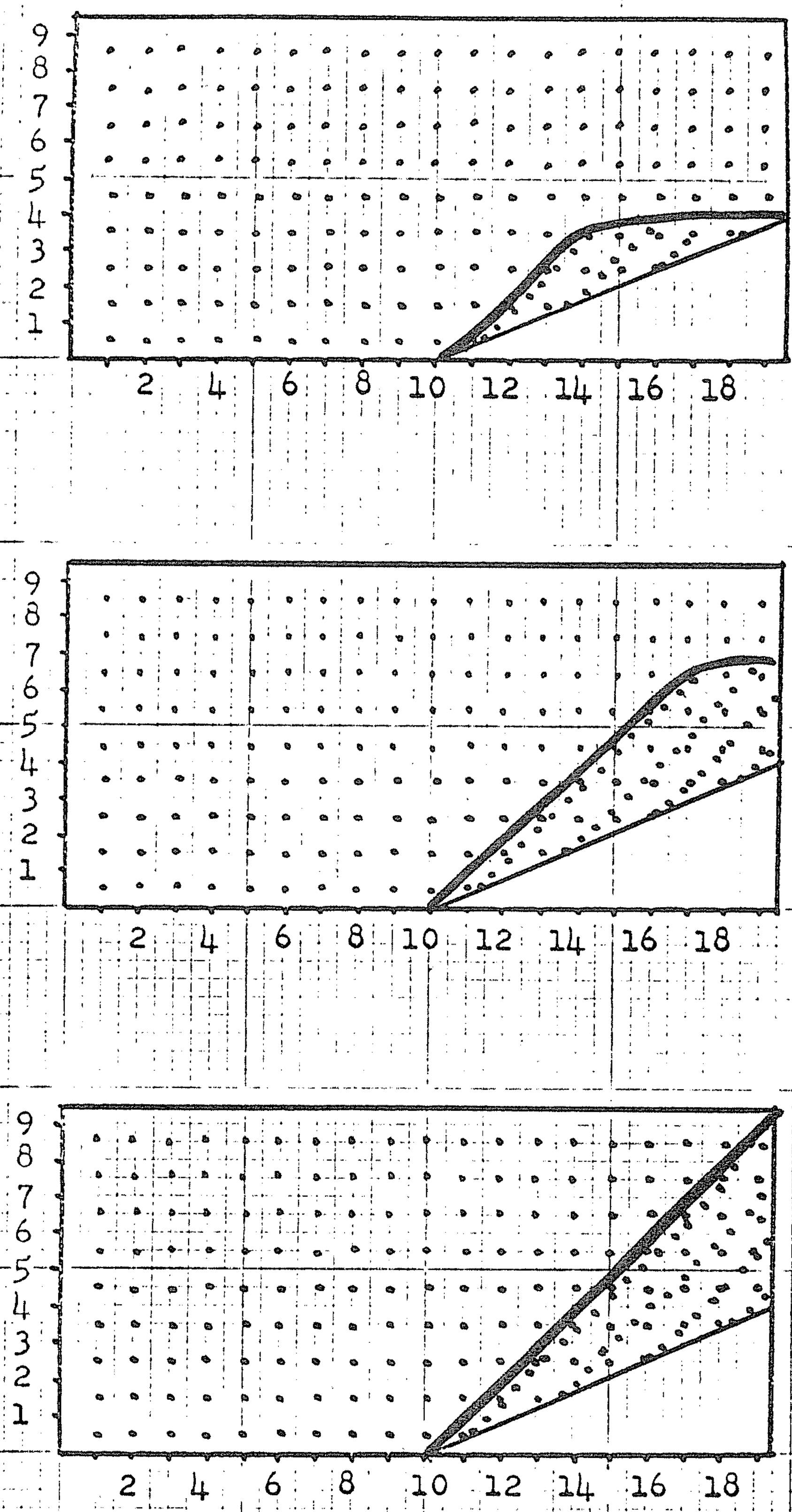

FIGURE 3.2

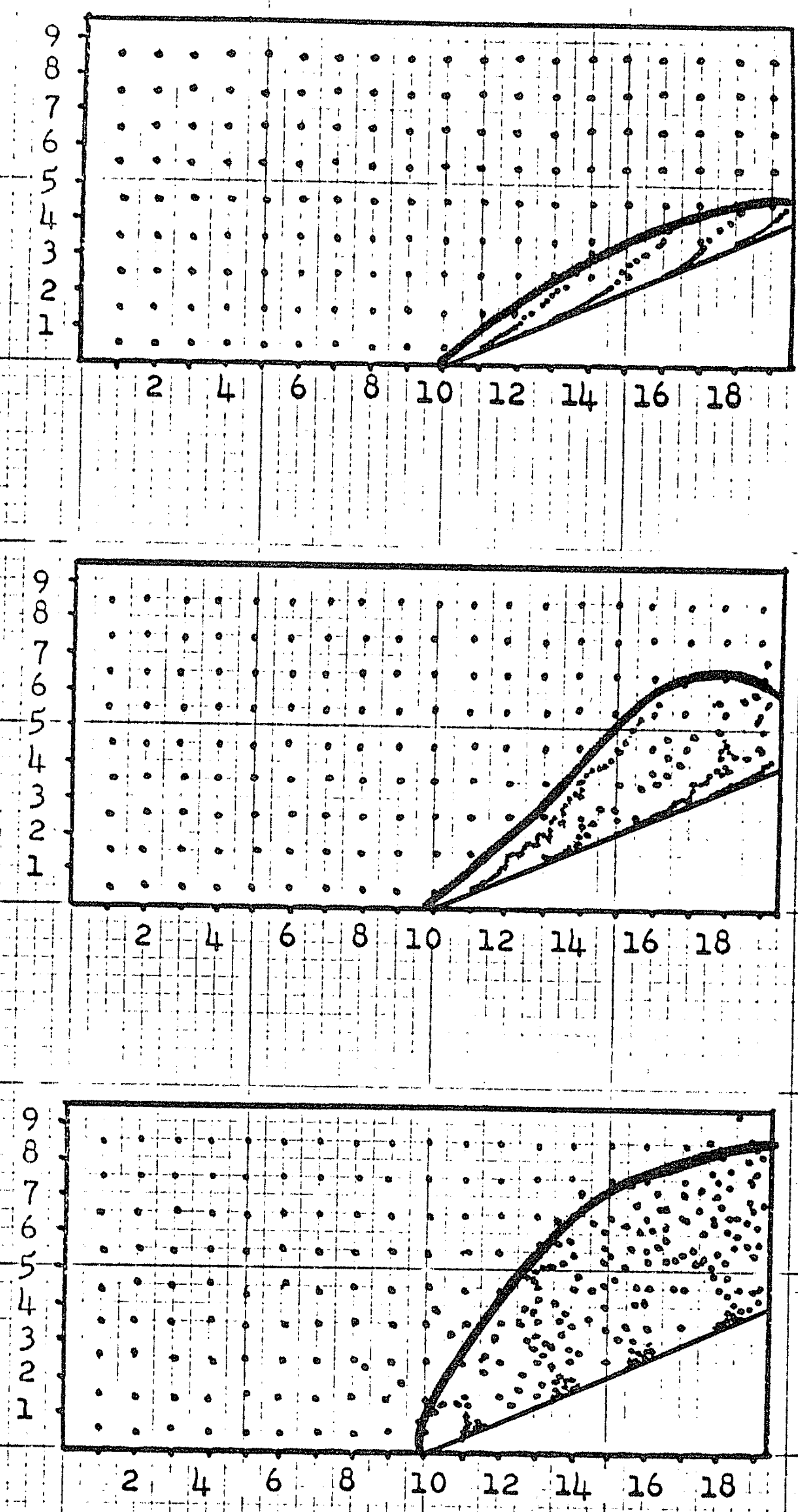

FIGURE 3.3

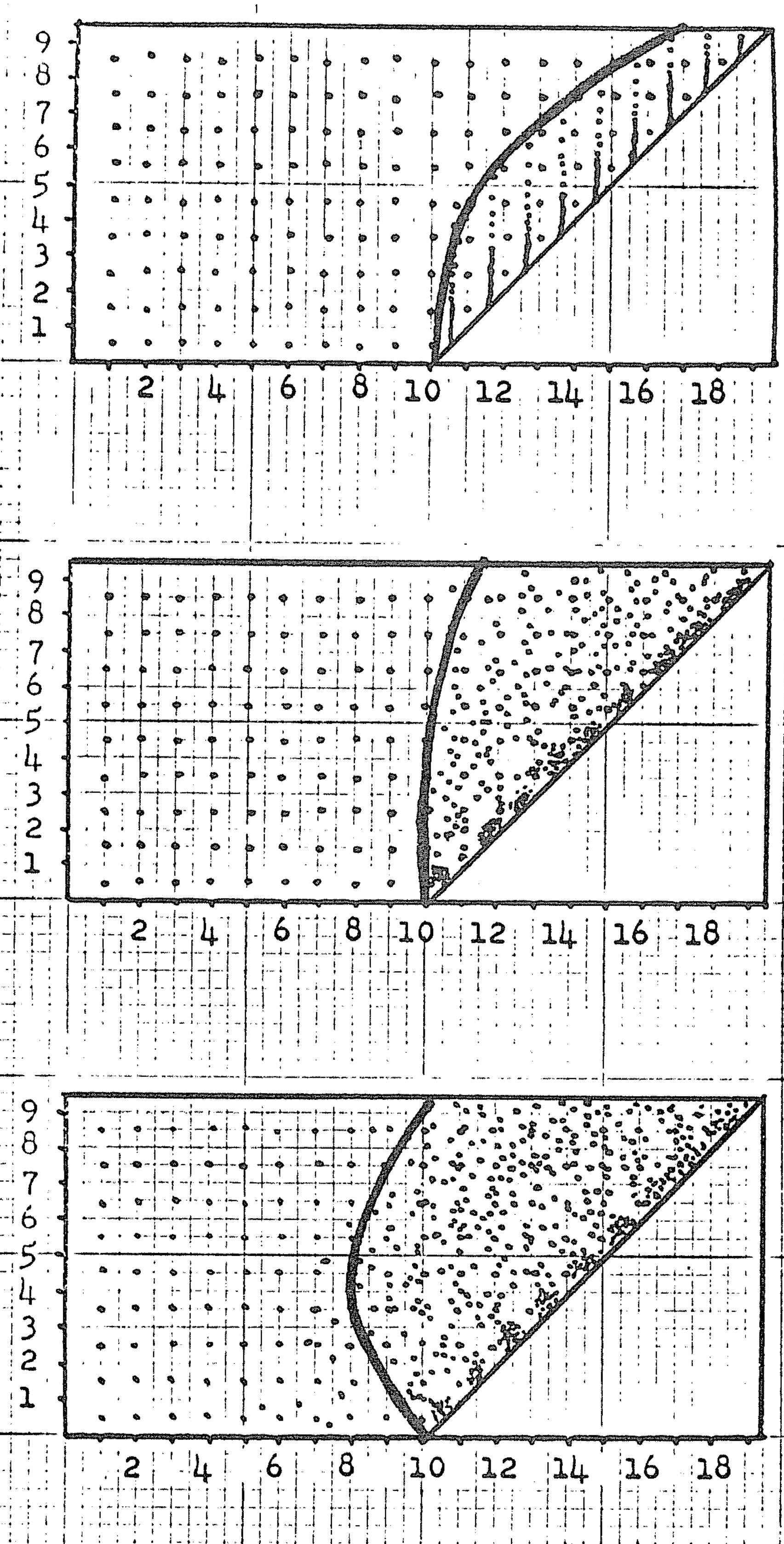

FIGURE 3.4

t_{35}, t_{55}, and t_{75}. In Figure 3.4 the parameter choices are $\delta = .1$, $\alpha = \gamma = \frac{\pi}{4}$, $m = 10$ and the time steps shown are t_{35}, t_{55} and t_{75}.

Varying m and allowing a more random particle input distribution led to modifications in Figures 3.3-3.4 in which straight line trajectories became curved and isolated accumulations of particles were spread more uniformly.

4. THE SHOCK TUBE.

With only minor modifications to the discussion in Section 3, the development extends readily to the construction of a shock by a piston moving rapidly in a cylinder. To do this, one need only take $\alpha = \frac{\pi}{2}$, insert a ceiling at $y = 10$, and force all particles to remain in the resulting chamber by reflecting them from the ceiling and the floor in such a fashion that the angles of reflection are well defined,while the speed on being reflected has been reduced by a factor δ_1 at a ceiling point and δ_2 at a floor point. Of the several examples run, Figure 4.1 shows the shock development at t_{20}, t_{40} and t_{60} for parameter choices $m = 1$ and $\delta = \delta_1 = \delta_2 = .1$ under the assumption that all incidence angles equal all angles of reflection. Also, the free stream fluid particles have been staggered with five equally spaced particles in odd numbered columns and with four equally spaced particles in even numbered columns. For Figure 4.1, the coordinate axes have been fixed relative to the particles so that the piston is in relative motion.

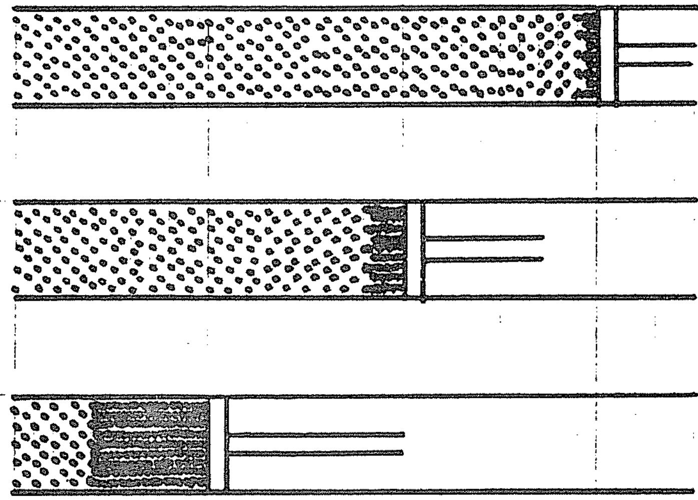

FIGURE 4.1

5. REMARKS

Considering the various bodyshapes which are of interest and the vast number of parameter choices which would correspond to factors like temperature, surface hardness, and so forth, the number of interesting problems which should be done next is exceptionally large. In this first paper, however, we have attempted only to establish the feasability of a computer oriented, particle approach to shock development for only the simplest of shapes and the simplest of physical assumptions.

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