THE ANATOMY OF AN ALGOL PROCEDURE

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Abstract

A specific Algol procedure is analyzed in great detail in order to obtain an analysis of exactly what it does. The strategy of analysis is first explained, and the remainder of the paper consists of the written analysis. The main technique is proof by cases.

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5

Table of Contents

The Anatomy of an Algol Procedure

I. Introduction

One of us (A.I.W.) has written an Algol program to bid, to a final contract, both hands of a bridge partnership under the assumption that the opponents are passing.[1] It is the purpose of this paper to analyze in great detail a very small part of this program, namely the opening bid routine. We shall not be concerned here with the problem-solving ability of the program; rather the concern is with a mathematical type analysis of exactly what it does.

It would be extremely pleasant to be able to assert that we have proved the program will open correctly every possible bridge hand, including passing when this is appropriate. Unfortunately, we have no rigorous definition, independent of the program, of what "correctness" means, and so our analysis is better characterized as an attempt to discover what the program will, in fact, bid.[2]

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1. Appendix 7 gives a short introduction to bridge for the uninitiated.

2. Nearly all of the analysis was obtained by the senior author who did not participate in the writing of the original program. In fact, the program was operational long before any analysis was even contemplated. Thus, there is some hope that he has not been so easily deceived, as perhaps the junior author might have been, by knowing the intent of each statement.

Even this type of analysis, however, has been beneficial to the development of the opening bid routine. For by comparing the results of our analysis with our intuitive notion of what we think should be correct, unsatisfactory bidding can be found. This is in addition to the feedback provided by the standard technique of testing and running actual hands. In fact, several flaws in a previous version were uncovered only as a result of our more extensive analysis. This may be typical of such analyses: we may never succeed fully in giving a complete proof of "correctness" but at least the attempt to do so may uncover flaws which can be corrected. In other words, we are systematically searching for a counter-example as well as a complete proof.

The analysis to be given was obtained completely by hand, unaided by the computer in any way. It has been checked, but again only by hand. Obtaining it in this way is a tedious and error-prone process, and we would like to semi-automate the process. In fact, one of the benefits of having carried it out by hand was to gain experience with such detailed analyses to see where the computer could be of assistance.

It might be claimed that, strictly speaking, we would have to prove the correctness of any such computer-provided assistance. This is correct at a certain level. However, as a practical matter, this is perhaps an unfair requirement on two grounds.

First, no such demand is made of human-produced proofs. In this connection we specifically claim that our present analysis is no more or no less accurate than conventional mathematical proofs. Secondly, since people seldom, if ever, attempt to prove their algorithms or programs to be correct, we are at least better off for the attempt at a proof.

The Object of Analysis and the Synopsis.

The analysis will be presented at a certain level of abstraction from the actual Algol statements. We shall be concerned solely with the procedure named OPENBID.[3] Thus, for example, we shall not be concerned with the calling program which invokes OPENBID, with input/output, nor with the internal representation of the hand and the cards. Some of these matters are explained in the appendices, especially appendix 5, but we will make no statements attempting to "prove" anything about them. This will be especially true of an entire procedure, PRELIM, which accepts a hand and computes a series of descriptions of the hand for later use by OPENBID. OPENBID depends exclusively upon these descriptions and upon several Boolean procedures in order to bid; it never consults the actual hand. In appendix 2 we list these descriptions and Boolean procedures together with their English language definitions.

The Algol code of OPENBID appears in appendix 1. As an aid to analyzing the code, we have prepared a synopsis of it. This synopsis appears starting on the next page; it is followed by an explanation, especially of the notation.

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3. Words written in all capital letters correspond to actual identifiers in the program.

SYNOPSIS

Section		The Conditions, etc.	Bids Made	
1.		TOTALPTS ≤ 6	PASS	
2.	a.	PLAYTRICKS ≥ 12 and ACES = 4	6 or 7 NT	a
	b.	PLAYTRICKS ≥ 12 and LONGEST ≥ 8		
		S, H, D, C		
		SUITLENGTH(suit) = LONGEST	6 or 7S, H, D, C	b
3.		HCP ≥ 25 and NOTRUMPTRY and EVENDIST	2D	
4.		22 ≤ HCP ≤ 24 and NOTRUMPTRY and EVENDIST	2NT	
5.		(HCP ≥ 23 or (HCP ≥ 22 and TOTALPTS ≥ 25) or TOTALPTS ≥ 26)		
		and PLAYTRICKS ≥ 9		
	a.	S, H, D, C		
		SUITLENGTH(suit) = LONGEST ≥ 5	2D	a
	b.	S, H, D, C		
		SUITLENGTH(suit) = 4 and SUITPOINTS(suit) = SP(1)	2D	b
6.		LONGEST ≥ 8 and PLAYTRICKS ≥ 9 and HCP ≤ 15		
	a.	S, H		
		SUITLENGTH(suit) = LONGEST	4S, 4H	a
	b.	D, C		
		SUITLENGTH(suit) = LONGEST and PLAYTRICKS ≥ 10	5D, 5C	b

Section	The Conditions, etc.	Bids Made	
7.	15 ≤ HCP ≤ 18 and EVENDIST and STOP = 4 and not [((HCP ≥ 17 and TOTALPTS ≥ 19) or (SUITLENGTH (H or S) = 5 and SUITPOINTS (H or S, respectively) ≥ 5)) and not SOLID]	1NT	
8.	HCP ≥ 17 or (HCP ≥ 16 and TOTALPTS ≥ 20) or TOTALPTS ≥ 21		
a.	18 ≤ HCP ≤ 21 and EVENDIST and SOLID	1C	a
b.	LONGEST ≥ 5	1C	b
c.	HCP ≥ 18 and NOTRUMPTRY	1C	c
d.	LONGEST = 4 and SUITPOINTS (longest suit) ≥ SP(2)	1C	d
e.	HCP ≥ 18 and EVENDIST and STOP = 4	1C	e
9.	SUITLENGTH(C) ≥ 6 and SUITPOINTS(C) ≥ 9 and HONORCOUNT(C) ≥ 4 and HCP ≤ 14	3C	
10.	6 ≤ HCP ≤ 12 and TOTALPTS ≤ 7 + LONGEST and 6 ≤ LONGEST ≤ 7		
	S, H		
	HONORCOUNT(suit) ≥ 9 - SUITLENGTH(suit) and SUITLENGTH(suit)≥6	2S, 2H	
11.	6 ≤ HCP ≤ 10 and LONGEST ≥ 7		
a.	D, H, S		
	SUITLENGTH(suit) = LONGEST and SUITPOINTS(suit) ≥* 18-2x SUITLENGTH(suit) and (VOIDFLAG or LONGEST ≥ 8 or OUTSIDEACE(suit))	3S, 3H, 3D Note 1	a
b.	SUITLENGTH(C) ≥ 8 and PLAYTRICKS ≥ 8	4C	b
	Go to Section 13, Skipping Section 12 altogether Note 2		

*See the definition of SLIDEPOINTS in Appendix 2 for an explanation of this relation.

Section	The Conditions, etc.	Bids Made	
12.	[(HCP ≥ 11 and (TOTALPTS ≥ 13 or FOUR441)) or (HCP ≥ 10 and VOIDFLAG)		
	or (HCP ≥ 12 and DISTP ≠ 0)] and TOTALPTS ≥ 11		
a.	S, H, D		
	SUITLENGTH(suit) = LONGEST ≥ 5 and SUITPOINTS(suit) ≥ 6		
	- SUITLENGTH(suit) and TOTALPTS ≥ 12	1S, 1H, 1D	a
	C = Longest suit		
b.	S, H		
	SUITPOINTS(suit) ≥ 25 - 5x SUITLENGTH(suit) and LONGEST = 5		
	and SUITPOINTS(C) ≤ 6	1S, 1H	b
c.	LONGEST ≥ 5 and CANBEREBID(C)	2C	c
	(C = longest suit and SUITPOINTS(S or H or D) ≥ 31 - 7x SUITLENGTH		
	(S or H or D, respectively)) or LONGEST = 4		
d.	SUITPOINTS(D) ≥ 34 - 8x SUITLENGTH(D) and (SUITLENGTH(H) ≤ 2		
	or (SUITLENGTH(S) ≤ 3 and SUITLENGTH(H) ≤ 3))	1D	d
e.	SUITPOINTS(H) ≥ 34 - 8x SUITLENGTH(H) and (SUITLENGTH(S) ≤ 3		
	or (SUITLENGTH(S) = 4 and SUITPOINTS(S) ≤ 1)) and not (HCP ≤ 12		
	and TOTALPTS ≤ 12 and SUITPOINTS(H) ≤ 3)	1H	e
f.	S, D		
	SUITPOINTS(suit) ≥ 34 - 8x SUITLENGTH(suit) and not (HCP ≤ 12		
	and TOTALPTS ≤ 12 and SUITPOINTS(suit) ≤ 3)	1S, 1D	f

Section	The Conditions, etc.	Bids Made	
g, h.	S, H, D [suit 1] [Note 3]		
	SUITLENGTH(suit1) = LONGEST = 5 and SUITPOINTS(suit1) = 0		
	S, H, D [suit 2] [Note 3]		
	SUITLENGTH(suit2) = 4 and SUITPOINTS(suit2) ≥ 4	1S, 1H, 1D [suit 2]	g
	HCP ≥ 13	1S, 1H, 1D [suit 1]	h
i.	HCP ≥ 13 and EVENDIST and SUITLENGTH(D) ≥ 3	1D	i
j.	SUITLENGTH(C) ≥ 5 and HCP ≥ 13 and SUITPOINTS(C) ≥ 5 -		
	HONORCOUNT(C)	2C	j
13. a.	TOTALPTS ≥ 13 and LONGEST ≥ 6 and HCP ≥ 10		
	S, H, D		
	SUITLENGTH(suit) = LONGEST and SUITPOINTS(suit) ≥ 14 - 2x		
	SUITLENGTH(suit)	1S, 1H, 1D	a
b.	TOTALPTS ≥ 14 and HCP ≥ 10 and CANBEREBID(S or H)	1S, 1H	b
c.	6 ≤ HCP ≤ 12 and LONGEST ≥ 6		
	S, H		
	HONORCOUNT(suit) ≥ 9 - SUITLENGTH(suit) and SUITLENGTH(suit) ≥ 6	2S, 2H	c
d.	HCP ≥ 14		
	S, H, D		
	SUITLENGTH(suit) ≥ 4 and SUITPOINTS(suit) ≥ 1	1S, 1H, 1D	d
14.	No bid found at section 11a [Note 4]	PASS	

NOTES:

1. This bid is subject to replacement: If HCP = 10, control resumes at section 13 rather than stopping with the bid in section 11a. Thus another bid may be found in section 13. This is the only instance where OPENBID continues after finding the first bid. Incidentally, this pathology is the correction of a flaw uncovered by a previous analysis.
2. If control enters section 11 but fails to find a bid there, control goes to section 13. This is the only instance where control does not go to the next section.
3. At section 12g and 12h, there is a loop within a loop, and so we must distinguish between the two implied iteration variables. Suit1 and suit2 serve this purpose. Note that the bid depends on the proper iteration variable.
4. This pseudo-Boolean condition, not part of the code, is a simple way to express the workings of the actual code. In reality, BIDVAL is initially set to zero prior to section 1. If it is not changed, then PASS is the actual bid -- the result of falling through all the sections. The simple Boolean condition, "<u>true</u>", is inaccurate because of note 1.

The synopsis is a representation of the actual Algol code of OPENBID, but in a different and simplified notation. The idea is to retain only the essential logic while eliminating many of the programming details. Some features of the logic have also been eliminated, but these are essential only to other routines, not to OPENBID. Thus, it should be easier for a person to analyze the synopsis rather than the actual code.

We assert, without proof, that this synopsis represents, or is semantically equivalent to, the actual statements of OPENBID. Thus, the analysis can and does proceed directly from the synopsis. Strictly speaking, though, we have analyzed only the synopsis.

Nearly all of the control structure of this representation (<u>if</u>'s, <u>go to</u>'s, <u>for</u>'s) is implicit in the notation itself. Here indentation level plays a critical role and is used instead of <u>begin</u>'s and <u>end</u>'s to indicate the scope or body of Boolean conditions and loops.

A sample part of the synopsis will be used to explain the notation by example. Consider the first part of section 12 up to, but not including, the line marked d . It is reshown on the following page with each of the twelve lines labeled from 1 to 12 .

Sample Part of the Synopsis

```
[(HCP ≥ 11 and (TOTALPTS ≥ 13 or FOUR441)) or (HCP ≥ 10 and VOIDFLAG) or
 (HCP ≥ 12 and DISTP ≠ 0)] and TOTALPTS ≥ 11
   S, H, D
       SUITLENGTH(suit) = LONGEST ≥ 5 and SUITPOINTS(suit) ≥ 6 - SUITLENGTH
       (suit) and TOTALPTS ≥ 12                                              1S, 1H, 1D
   C = longest suit
       S, H
           SUITPOINTS(suit) ≥ 25 - 5x SUITLENGTH(suit) and LONGEST = 5 and
           SUITPOINTS(C) ≤ 6                                                 1S, 1H
       LONGEST ≥ 5 and CANBEREBID(C)                                         2C
   (C = longest suit and SUITPOINTS(S or H or D) ≥ 31 - 7x SUITLENGTH
   (S or H or D, respectively)) or LONGEST = 4
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If the Boolean condition at lines 1 and 2 is false, control would pass to section 13; otherwise control goes to line 3 . Here is a loop over the three suits S, H, and D. The body of the loop is the Boolean condition at lines 4 and 5 . This loop is executed first with S, then H, then D. If and when the Boolean condition is first true, the corresponding bid is made and OPENBID is finished. If it is false for all three suits, control passes to line 6 .

If this Boolean condition is false, control passes to line 11. If true, the loop at lines 7-9 is executed. If a bid is found, OPENBID is finished. If no bid is found in this loop, control passes to line 10. If this Boolean condition is true, the bid of 2C is made. Otherwise control is at line 11.

In some Boolean conditions, notation such as "SUITPOINTS(suit)" is used. The word "suit" is a dummy variable for the suits over which iteration is occurring. Also, each section is numbered from 1 to 14 so as to be able to refer to a specific section and its bids. If bidding may occur in more than one place within a section, each such place is further noted with a letter, a to j .

One detail which is lacking in the synopsis but which does appear in the actual program requires explanation. This is the variable B, used as an array subscript to indicate which of the two hands (eventually this will become four) is in the process of being bid. Since our analysis is fixed on one hand (the current one that is being bid), B has been completely deleted. Thus two-dimensional arrays used by OPENBID appear as one-dimensional arrays in the synopsis and one-dimensional as simple variables.

Finally, subscript brackets are rendered as parentheses for typographical convenience since the distinction between array elements and procedure calls is unimportant in the analysis.

The Main Technique of Analysis - Case Analysis

The main technique used in the analysis below is case analysis. This is the only way we know to uncover the behavior of the program, and it has proved successful. Since nearly everything of interest about bridge hands is reasonably finite,[4] this technique suffices and, for example, mathematical induction is not needed at all.

There are, however, $\binom{52}{13}$ or about 6.3×10^{11} different bridge hands. More relevant is an estimate, A , of the number of hands that differ by more than just the specific non-honor cards in each suit. For example, ignore whether the two or three is the lowest spade, as OPENBID does. There are twenty honor cards (Ace through 10) in a deck so the honors in a hand may be chosen in $\sum_{i=0}^{13} \binom{20}{i}$ ways. Note that this ignores the actual number of non-honors in each suit which is relevant. Therefore,

$$A > \sum_{i=0}^{13} \binom{20}{i} > \sum_{i=0}^{10} \binom{20}{i} > \frac{1}{2} \sum_{i=0}^{20} \binom{20}{i} = 2^{20}/2 = 2^{19} > 5.2 \times 10^5 .$$

4. Less than, say, 50; for example, 13 cards per hand, 39 distributions by suits, etc.

Even if we consider only the top four honors,

$$A > \sum_{i=0}^{13} \binom{16}{i} > 2^{16}/2 > 3.2 \times 10^4 .$$

Thus, one would never, in practice, analyze the behavior by a brute force analysis of all hands. Using case analysis more selectively, we have done much better although the analysis is still quite lengthy.

The Strategy of Analysis

Our analysis is in two distinct parts. In order to understand what each part contributes toward determining the behavior of OPENBID, it is necessary to explain further the flow of control through OPENBID. As shown by the synopsis, OPENBID is a series of 14 major sections of statements, each representing a bid or family of bids. These sections have been carefully ordered so that OPENBID may make as its bid the first one for which it finds all the required conditions true, and then quit.[5] With the exception of for-loops and procedure calls, the flow of control is always directly toward the end of the OPENBID procedure. If no bid is made in a section, control passes to the next section[6] for the possible finding of a bid in that section. If no bid is ever found, the result is pass, of course.

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5. The one exception to this is shown in Note 1 of the synopsis.

6. The one exception to this general rule is shown in Note 2 of the synopsis.

We note specifically that OPENBID does not choose the best bid of a set of reasonable bids but rather the first valid one. This was done strictly as a matter of programming convenience and before any analysis was even considered. But it has greatly simplified the analysis.

This logic of control through the sections is the key to the analysis strategy, a strategy involving two parts. The first part is of the form: If a hand satisfies the conditions needed to bid at section N, then a bid cannot be made at sections 1, 2, ... N-1. However, there are cases where a hand could bid both at sections N and M, with $M < N$. We have decided, from our knowledge of bridge bidding, that in all such cases the bid at section M is preferable. A clear-cut example of this is that an opening bid of 7NT at section 2 is better than a bid of 1S at section 12.

The second part of the analysis shows, for some hands, what bid the program might make if control would get to that section. Control need not reach that section since a bid may be made in a previous section. We emphasize that this bid may not be the actual bid made. We have, however, shown that the program will make a non-passing bid for these hands.

For the rest of the hands, we assert that the program will pass. This means showing that all hands meeting certain conditions must fail at least one required test of every possible bid.

If we put these two parts together, we obtain the behavior of OPENBID. By the second part, we know which hands pass. Also from the second part, we know which hands meet the conditions of at least one bid in some section. By the first part, the bid of this section is either the actual bid made or else the actual bid is from a previous section, and it is preferable.

No statements will be made showing that the conditions of each bid are correct tests, say, according to some bridge book.[7] For this analysis we consider the definitions of each bid to be given and have asked, and shown, what behavior results.

Now that our problem is defined and the general method of attack is known, we exhibit the analysis of the behavior of OPENBID.

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7. The system which has been implemented is basically the Sckenken system described in Better Bidding in 15 Minutes by Howard Schenken.

II. Part One of the Analysis

We now wish to show the first part of the analysis: If a hand satisfies the conditions needed to bid at section N , then a bid cannot be made in sections 1, 2, ... N-1. However, as already explained, there are exceptions based on our estimate of preferable bids. These are so noted in the analysis.

Frequently exploited is a general relation between HCP and TOTALPTS:

$$\text{TOTALPTS} \geq \text{HCP} - 3.$$

Appendix 6 gives the complete relations between TOTALPTS and HCP for all possible distributions; the above inequality follows from those selections.

A general word about the reasons listed below is in order: These reasons give properites possessed by the hand which follow from assuming it can bid at section N . These properties are used to show why it is impossible to bid at sections 1, 2, ... N-1, respectively. Although the contradiction is not mentioned specifically, it should be obvious.

The first part of the total analysis is now given:

N	Cannot bid at sections 1, 2, ... N-1	
2a.	1.	ACES = 4 implies HCP ≥ 16 implies TOTALPTS ≥ 13.
2b.	1.	LONGEST ≥ 8 implies a contribution toward TOTALPTS of at least 7. Hand contributes an additional 1. Any subtractions for "short honors" is no more than the original contribution by that honor toward HCP. We can subtract, then, at most 1 for no Aces. Thus TOTALPTS ≥ 7.
3.	1.	HCP ≥ 25 implies TOTALPTS ≥ 22.
	2.	preferable bid.
4.	1.	HCP ≥ 22 implies TOTALPTS ≥ 19.
	2.	preferable bid.
	3.	HCP ≤ 24.
5.	1.	either (TOTALPTS ≥ 26) or (HCP ≥ 22 implies TOTALPTS ≥ 20).
	2.	preferable bid.
	3.	same bid.
	4.	preferable bid.
6.	1.	same reasons as 2b, 1.
	2a.	HCP ≤ 15 but 2a requires ACES = 4 implies HCP ≥ 16.
	2b.	preferable bid.
	3, 4.	HCP ≤ 15
	5.	if TOTALPTS ≤ 25, then section 5 fails because HCP ≤ 15.
		if TOTALPTS ≥ 26, then 2D is a preferable bid.

N	Cannot bid at sections 1, 2, ... N-1	
7.	1.	HCP ≥ 15 implies TOTALPTS ≥ 12.
	2.	preferable bid.
	3, 4.	HCP ≤ 18.
	5.	either (HCP ≤ 18) or (EVENDIST implies TOTALPTS ≤ 18 + 3 = 21).
	6.	EVENDIST implies LONGEST ≤ 5.
8.	1.	either (TOTALPTS ≥ 21) or (HCP ≥ 16 implies TOTALPTS ≥ 13).
	2, 3, 4, 5, 7.	preferable bid.
	6.	if TOTALPTS ≤ 20, then section 6 fails because HCP ≥ 16.
		if TOTALPTS ≥ 21, then 4H, 4S, 5C, 5D are satisfactory bids.
9.	1.	(SUITPOINTS(Clubs) ≥ 9 and SUITLENGTH(Clubs) ≥ 6) implies TOTALPTS ≥ 8, since if LONGEST ≥ 6, TOTALPTS ≥ HCP - 1.
	2, 6.	preferable bid.
	3, 4, 7.	HCP ≤ 14.
	5, 8.	either (HCP ≤ 14) or (2D, 1C are preferable bids).
10.	1.	If LONGEST ≥ 6, TOTALPTS ≥ HCP - 1. Thus, if HCP ≥ 8, TOTALPTS ≥ 7. But if HCP = 6 or 7, it is more involved:

N	Cannot bid at sections 1, 2, ... N-1
10.	1. (continued) (A). if LONGEST = 7, the longest suit must have at least 2 honors implies at least J 10. Let us now compute TOTALPTS.

The eight factors and their values are:

HCP ≥ 6	K -2 (can be ignored since Q is the same)
Hand 1	Q -2
Length ≥ 5*	Qx -1
No Aces -1	Jx -1

Therefore, TOTALPTS ≥ 11 - short honor terms.

Consider the possible short honor terms:

(1). if 2Q, subtract 4, but no more, so TOTALPTS ≥ 7.

(2). if 1Q (-2), there could be at most one other short suit. QJ would subtract -2, the maximum possible, so again TOTALPTS ≥ 7.

(3). if 0 Q, there could be 3 short suits.

(a). if all are Qx or Jx, x ≠ Q, J, subtract 3 implies TOTALPTS ≥ 8.

(b). if one QJ (-2), at most -2 for other two suits implies TOTALPTS ≥ 7.

*The 7-card suit contributes 5. An additional 5- or 6-card suit would contribute more although here we may safely ignore this.

N	Cannot bid at sections 1, 2, ... N-1
10	1. (continued) (c). if two QJ (-4), now third suit has SUITPOINTS = 0 else HCP ≥ 8, since the 7-card suit has at least a J. Again TOTALPTS ≥ 7. (d). if three QJ, then HCP ≥ 10 which is impossible since we are assuming HCP = 6 or 7. (B). if LONGEST = 6, the longest suit must have at least 3 honors implies at least QJ 10. Now TOTALPTS's factors are: HCP ≥ 6 Hand 1 Length ≥ 3 (could also have 1 for a second 5 card suit) No Aces -1 Therefore, TOTALPTS ≥ 9 - short honor terms + (length for second long suit) (1). if 2Q (-4), but now HCP ≥ 7 and there is a second suit. Thus TOTALPTS = 7. (2). if 1Q (-2), there could be at most one other short suit. The hand, S:QJ10xxx, H:Q, D:Jx, C:xxxx has TOTALPTS = 6 and so it will pass by section 1 even though it meets the conditions of section 10. We

N	Cannot bid at sections 1, 2, ... N-1	
10.	1. (continued)	have decided that pass is the correct bid. However, if the second short suit has A, K or Q in it, or if the long suit has more SUITPOINTS, then TOTALPTS $\geq$ 7. (3). if 0 Q, there are at most two short suits. (a). if both are Qx or Jx, $x \neq$ Q, J, subtract 2 and so TOTALPTS $\geq$ 7. (b). if one QJ (-2), we now have HCP = 6. Therefore the second short suit also adds to HCP so there is no net subtraction implies TOTALPTS $\geq$ 7. (c). if two QJ, then HCP $\geq$ 9 which is impossible.
10.	2a.	HCP $\leq$ 12 but 2a requires ACES = 4 implies HCP $\geq$ 16.
	2b, 6.	LONGEST $\leq$ 7.
	3, 4, 7.	HCP $\leq$ 12.
	5, 8.	either (HCP $\leq$ 12) or (2D, 1C are preferable bids).
	9.	In order also to meet 9, we would need SUITPOINTS (Clubs) $\geq$ 9 and we have SUITPOINTS (MAJOR) $\geq$ 1 implies HCP $\geq$ 10. There are thus two 6-card suits implies TOTALPTS $\geq$ 17. But TOTALPTS $\leq$ 14 if the hand meets 10.

N	Cannot bid at sections 1, 2, ... N-1	
11.	1.	If LONGEST = 7, TOTALPTS $\geq$ HCP − 1, while if LONGEST $\geq$ 8, TOTALPTS $\geq$ HCP + 1. Thus, if LONGEST $\geq$ 8 or if HCP $\geq$ 8, then TOTALPTS $\geq$ 7. Thus consider LONGEST = 7 and (HCP = 6 or 7). We have SUITPOINTS (longest suit) $\geq$ 4. If there is an OUTSIDEACE, then HCP $\geq$ 8 which violates our assumption that HCP = 6 or 7. Thus there must be a void in order to bid at section 11. Again, TOTALPTS's factors are: HCP $\geq$ 6 Hand 1 Length $\geq$ 5 No Aces −1 Therefore, TOTALPTS $\geq$ 11 - short honor terms. Since there is a void, there is at most one short suit which implies that TOTALPTS $\geq$ 9.
	2a.	HCP $\leq$ 10 but 2a requires ACES = 4 implies HCP $\geq$ 16.
	2b, 6.	preferable bid.
	3, 4, 7.	HCP $\leq$ 10.
	5, 8.	either (HCP $\leq$ 10) or (2D, 1C are acceptable bids).

N	Cannot bid at sections 1, 2, ... N-1	
	9.	If clubs is longest suit, then 3C is a preferable bid. If clubs is not the longest suit, then in order also to meet 9, we would need SUITPOINTS (Clubs) ≥ 9 and SUITLENGTH (Clubs) ≥ 6. Since the hand meets 11, SUITLENGTH (longest suit) = 7 and thus SUITPOINTS (longest suit) ≥ 4. Therefore HCP ≥ 13. But HCP ≤ 10 if the hand meets 11.
	10.	satisfactory bid.
12.	1.	TOTALPTS ≥ 11
	2-9, 11.	preferable bids.
	10.	satisfactory bid.
13a, b, d.	1.	either (TOTALPTS ≥ 13) or (HCP ≥ 14 implies TOTALPTS ≥ 11)
	2-12.	preferable or same bids.
13c.	1.	if LONGEST ≥ 8, then TOTALPTS ≥ 7 since TOTALPTS ≥ HCP + 1; if LONGEST = 6 or 7, then as in 10, 1.
	2a.	HCP ≤ 12 but 2a requires ACES = 4 implies HCP ≥ 16.
	2b, 6, 9, 11a.	preferable bids.
	3, 4, 5, 7, 8.	as in 10.
	10.	same bid.
	11b.	to meet 11b, SUITLENGTH(Clubs) ≥ 8 but SUITLENGTH(MAJOR Suit) ≥ 6 implies 14 cards in hand.

N	Cannot bid at sections 1, 2, ... N-1	
	12.	now a preferable bid since the hand failed at section 10.
	13a, b.	preferable bid.

Strictly speaking, there is more to this part of the analysis. We have shown, in some sense, that the ordering of the sections among themselves is correct. It remains to show that the ordering within each section is correct. For the first 11 sections, this is easy:

1, 3, 4, 5, 7, 8, 9.	only one bid is possible.
2.	NT is preferable to a suit bid. The level is the same in either case. The suit order of S, H, D, C is correct, but irrelevant since there is only one suit with length ≥ 8.
6.	the suit order of S, H, then D, C is correct.
10.	the suit order of S, H is correct.
11.	the suit order of S, H, D then C is correct.

For section 12 we do not have good justifications at all. The following comments may be helpful. There are only 4 bids made in this section: 1S, 1H, 1D, 2C. Overall and at each stage, in general, they are checked for in this order which is correct. Even at 12d, e, f this order is not completely violated since, e.g., it will not bid 1D if either spades or hearts has minimal promise. We think it handles correctly rules involving two suits of the same length although we have no deep analysis of this fact. Also at 12g, h it will bid a strong 4-card suit in preference to a 5-card suit with no SUITPOINTS; this, too, is in accordance with accepted bidding strategy. 12i and 12j are special situations and properly belong last.

Section 13 is concerned with S, H, D bids only. These are checked in the correct order. The only question is whether a 2S or 2H at 13c bid is now to be preferred over 1S or 1H at 13a or b. We have decided no in view of the hand's having failed at section 10. Note that 13c (HCP $\leq$ 12) and 13d (HCP $\geq$ 14) are mutually exclusive so that no ordering question is involved.

III. Part Two of the Analysis

The second part of the analysis is divided into subparts according as hands meet the following conditions:

a. HCP $\geq$ 17

b. 13 $\leq$ HCP $\leq$ 16

c. HCP $\leq$ 12 and the length of the longest suit = 4

d. HCP $\leq$ 12 and LONGEST = 5

e. LONGEST = i, $6 \leq i \leq 13$

Note that e. overlaps with both a. and b. This could easily be avoided by adding the condition, HCP $\leq$ 12, to e. but at a cost of generality. This would, however, only trivially alter the corresponding analysis. No contradition is caused by the overlap.

Each of the above conditions is analyzed as a separate proposition. Note that the proposition is only stated implicitly during the analysis or proof of the proposition.

In this part of the analysis we are concerned with what bid the program might make if control would get to that section. This non-passing bid may not be the actual bid made, but this happens only if it bids at a previous section. However, when the bid of pass is indicated, we are asserting that all hands so delimited by the case analysis will fail to bid in any section.

Our unconcern about whether a bid is the actual bid or not is made possible by the first part of the analysis. To invoke that part, we must only show, as we have done, that some bid might be made in some section, say M. The first part thus frees us from having to show, in addition, that hands do not meet the conditions of sections 1, 2, ... M-1. We must, however, do the latter in order to assert that all hands of a class will pass. This is quite lengthy although none of these details are presented in the analysis. The value of the first part of the analysis is now clear: It saves an extensive amount of checking and verifying, and at the risk of overstatement, makes a nearly hopeless task possible.

The analysis involves extensive use of case analysis sometimes to a depth of eleven. By using indentation and a hierarchial numbering scheme, we have attempted to make it intelligible. We use the strategy of making simple or small assumptions at each level since this tends to simplify the statement of complementary cases. Such assumptions would also be easier to manipulate by a machine in a man-machine environment.

In order to assert that OPENBID might at least make a stated bid, it is necessary to show that all the tests for that bid have been met. A test is usually met by an explicit assumption as part of the case analysis. Indeed, the case analysis is generally guided by this need to have certain conditions definitely true or definitely false. Complementary assumptions are considered, of course, in order that the case analysis be complete.

Tests may also be shown to be true by inference from the assumptions. These inferences are prefaced by the "therefore" symbol (∴) to distinguish them from the assumptions of the case analysis. Some verification is, however, left to the reader in order to improve readability at the expense of absolute completeness.

Specific bridge hands are sometimes given when a pass is indicated. This is to show that the conditions delimited by the case analysis are not impossible to attain. The hands are given in standard bridge notation where "x" is here taken to mean some non-honor card. In this and other parts of the analysis, "S," "H," "D" and "C" stand for Spades, Hearts, Diamonds and Clubs, respectively.

One other convention is that the inequalities, all of which involve integers, always are stated using "$\geq$" or "$\leq$," never just "$>$" or "$<$."

The reader will note some similarity in various parts of the analysis. Sometimes we have given such analyses only once and then referred to it at a later time. Mainly, however, we have found it easier to repeat similar, but not identical, parts rather than to attempt to explain the needed differences. This also guards against a glib statement of "similarly" when, in fact, a subtle, but important difference is ignored.

Proposition 1: HCP $\geq$ 17

I. LONGEST $\geq$ 5 — 1C at 8b

II. LONGEST = 4

A. Distribution is 4441

$\therefore$ At least 13 HCP over the three 4-card suits implies one 4-suit has the highest ($\geq$5) SUITPOINTS — 1C at 8d

B. Distribution is 4432

$\therefore$ EVENDIST

1. HCP $\geq$ 18

a. STOP = 4 — 1C at 8e

b. STOP $\neq$ 4

$\therefore$ the 2-card suit is at best QJ else STOP = 4. Thus SUITPOINTS (first 4-card suit) + SUITPOINTS (second 4-card suit) $\geq$ 18 - (9+3) = 6 implies SUITPOINTS (one 4-card suit) $\geq$ 3. But this is at least the second best SUITPOINTS. — 1C at 8d

2. HCP = 17

a. ACES = 4

$\therefore$ SUITPOINTS (one 4-card suit) $\geq$ 4, at least second best SUITPOINTS — 1C at 8d

b. ACES $\neq$ 4

i. STOP = 4

$\therefore$ TOTALPTS $\leq$ 18 since only add 1 for Hand — 1NT at 7

ii. STOP $\neq$ 4

as in 1b. SUITPOINTS of the two 4-card suits $\geq$ 5 implies SUITPOINTS (one 4-card suit) $\geq$ 3 — 1C at 8d

C. Distribution is 4333

$\therefore$ STOP = 4 and $\therefore$ EVENDIST

1. HCP $\geq$ 18 — 1C at 8e

2. HCP = 17

$\therefore$ TOTALPTS $\leq$ 18 since add only 1 for Hand, possible 1 for ACES = 4 but we do subtract 1 for 4333. — 1NT at 7

end of Proposition 1.

Proposition 2: $13 \leq HCP \leq 16$

I. LONGEST $\geq$ 6

$\therefore$ DISTP $\neq$ 0 and $\therefore$ TOTALPTS $\geq$ 12

A. Longest suit = S or H or D — 1S, 1H, 1D at 12a

B. Longest suit = C — 2C at 12c

II. LONGEST = 5

$\therefore$ DISTP $\neq$ 0

A. TOTALPTS $\leq$ 10

S:KJxxx, H:QJ, D:QJ, C:Kxxx — Pass

B. TOTALPTS $\geq$ 11

1. Longest suit = S or H or D

a. SUITPOINTS (longest suit) $\geq$ 1

i. TOTALPTS $\geq$ 12 — 1S, 1H, 1D at 12a

ii. TOTALPTS = 11

$\therefore$ Distribution is not 5332 else

TOTALPTS $\geq$ 12

$\therefore$ Not EVENDIST

α. SUITLENGTH (C) $\geq$ 5 and SUITPOINTS

(C) $\geq$ 5 - HONORCOUNT (C) — 2C at 12j

β. Not α.

(I). HCP $\geq$ 14 — 1S, 1H, 1D at 13d

(II). HCP = 13

(A). Longest suit is not also C

S:KQxxx, H:QJ, D:Jx, C:KJxx — Pass

(B). Longest suit is also C

(1). Longest suit = S or H

∴ by β. SUITPOINTS (C)

≤ 4 — 1S, 1H at 12b

(2). Longest suit = D — 1D at 12d

b. SUITPOINTS (longest suit) = 0 — 1S, 1H, 1D at 12h

2. Longest suit = C

a. Longest suit is also S or H or D

As in 1. but now Pass is impossible at (A).

b. Not a., i.e., no second 5-card suit

i. SUITPOINTS (C) ≥ 4 — 2C at 12j

ii. SUITPOINTS (C) = 3

α. HONORCOUNT (C) ≥ 2 — 2C at 12j

β. HONORCOUNT (C) = 1

∴ C suit is Kxxxx, x ≠ 10

(I). Distribution = 5332

∴ EVENDIST

(A). SUITLENGTH (D) ≥ 3 — 1D at 12i

(B). SUITLENGTH (D) = 2

(1). D suit is SAFE

∴ STOP = 4

(a). HCP ≥ 15 1NT at 7

(b). HCP ≤ 14

S:AKx, H:Jxx, D:Kx, C:Kxxxx Pass

(2). D suit is not SAFE

∴ STOP ≠ 4

S:AKJ, J:QJx, D:Qx, C:Kxxxx Pass

(II). Distribution ≠ 5332

∴ Not EVENDIST

(A). Distribution = 5440 1S, 1D at 12f

(B). Distribution = 5431 or 5422

(1). SUITPOINTS (4-card suit) ≥ 3

(a). 4-card suit = S or D 1S, 1D at 12f

(b). 4-card suit = H 1H at 12e

(2). SUITPOINTS (4-card suit) ≤ 2

(a). HCP ≥ 14

(i). SUITPOINTS (4-card suit) ≥ 1 1S, 1H, 1D at 13d

(ii). SUITPOINTS (4-card suit) = 0

S:xxxx, H:AQ, D:AJ, C:Kxxxx Pass

(b). HCP = 13

S:Qxxx, H:AQ, D:Qx,

C:Kxxx — Pass

iii. SUITPOINTS (C) ≤ 2

as in ii. ignoring the assumptions made at α. and β. since HONORCOUNT (C) ≤ 2 now and thus cannot bid at 12j. The examples of pass will be insignificantly different.

III. LONGEST = 4

$\therefore$ TOTALPTS ≥ 11

A. Distribution = 4441

$\therefore$ Not EVENDIST

$\therefore$ at least two suits of S , H , D have length 4

1. HCP ≥ 14

a. One 4-card suit of S , H , D has SUITPOINTS ≥ 1 — 1S, 1H, 1D at 13d

b. All 4-card suits of S , H , D have SUITPOINTS = 0

S:xxxx, H:xxxx, D:A, C:AKQJ — Pass

2. HCP = 13

a. One 4-card suit of S , H , D has SUITPOINTS ≥ 2

i. that suit is S or D — 1S, 1D at 12f

ii. H is only such suit — 1H at 12e

b. All 4-card suits of S , H , D have SUITPOINTS ≤ 1

S:Jxxx, H:Jxxx, D:A, C:AKxx — Pass

B. Distribution = 4333

∴ EVENDIST

1. TOTALPTS ≥ 13 — 1D at 12i

2. TOTALPTS = 12

∴ HCP = 13

S:KJxx, H:Kxx, D:Kxx, C:Kxx — Pass

C. Distribution = 4432

∴ EVENDIST

1. SUITLENGTH (D) ≥ 13 — 1D at 12i

2. SUITLENGTH (D) = 2

a. HCP ≥ 15

i. STOP = 4 — 1NT at 7

ii. STOP ≠ 4

∴ D suit is at best QJ

α. SUITLENGTH (C) = 3

∴ SUITPOINTS (C) + SUITPOINTS

(D) $\leq$ 12

∴ SUITLENGTH (S) = SUITLENGTH

(H) = 4 and (SUITPOINTS (S) $\geq$ 2 or

SUITPOINTS (H) $\geq$ 2) — 1S, 1H at 13d

β. SUITLENGTH (C) = 4

(I). Non-club 4-card suit has

SUITPOINTS $\geq$ 1 — 1S, 1H at 13d

(II). That suit has SUITPOINTS = 0

S:xxxx, H:Kxx, D:QJ, C:AKQJ — Pass

b. HCP = 14

i. One 4-card suit, S or H, has

SUITPOINTS $\geq$ 1 — 1S, 1H at 13d

ii. All 4-card suits, S or H, have

SUITPOINTS = 0

S:xxxx, H:Jxx, D:QJ, C:AKQJ — Pass

c. HCP = 13

i. One 4-card suit, S or H, has

SUITPOINTS $\geq$ 2

α. That suit is S — 1S at 12f

β. That suit is H only — 1H at 12e

ii. All 4-card suits, S or H, have

SUITPOINTS ≤ 1

S:Jxxx, H:Jxxx, D:Ax, C:AKx Pass

end of Proposition 2.

Proposition 3: We have omitted this number in order that Proposition i corresponds to LONGEST = i for $4 \leq i \leq 13$.

Proposition 4: LONGEST = 4 and HCP ≤ 12

I. HCP ≤ 10 — Pass

II. HCP = 11

A. TOTALPTS ≤ 10 — Pass

B. TOTALPTS ≥ 11

1. Distribution = 4333 or 4432

∴ TOTALPTS ≤ 12 — Pass

2. Distribution = 4441

a. Singleton is Ace

∴ TOTALPTS ≥ 13

i. One 4-card suit of S , H , D has SUITPOINTS ≥ 2

α. That suit is S or D — 1S, 1D at 12f

β. H is only such suit — 1H at 12e

ii. All 4-card suits of S , H , D have SUITPOINTS ≤ 1

S:Jxxx, H:Jxxx, D:A, C:AJxx — Pass

b. Singleton is not Ace

∴ TOTALPTS ≤ 12

i. One 4-card suit of S , H , D has SUITPOINTS ≥ 4

α. That suit is S or D — 1S, 1D at 12f

β. H is only such suit

(I). Singleton is S or SUITPOINTS

(S) ≤ 1 — 1H at 12e

(II). Not (I).: Singleton ≠ S and 2 ≤ SUITPOINTS

(S) ≤ 3

S:Qxxx, H:AKxx, D:x, C:Qxxx — Pass

ii. All 4-card suits of S , H , D have

SUITPOINTS ≤ 3

α. SUITPOINTS (D) ≥ 2 and Singleton = H — 1D at 12d

β. Not α.: SUITPOINTS (D) ≤ 1 or

Singleton ≠ H

S:x, H:Kxxx, D:Qxxx, C:AQxx — Pass

III. HCP = 12

A. TOTALPTS ≤ 10 — Pass

B. TOTALPTS ≥ 11

1. Distribution = 4333

∴ DISTP = 0 and TOTALPTS ≤ 12 — Pass

2. Distribution = 4432

∴ TOTALPTS ≤ 13

a. TOTALPTS = 13

i. One 4-card suit of S , H , D has

SUITPOINTS ≥ 2

α. That suit is S or D — 1S, 1D at 12f

β. H is only such suit — 1H at 12e

ii. All 4-card suits of S , H , D have SUITPOINTS ≤ 1

S:Jxxx, H:Jxxx, D:Ax, C:AQx — Pass

b. TOTALPTS ≤ 12

i. One 4-card suit of S , H , D has SUITPOINTS ≥ 4

α. That suit is S or D — 1S, 1D at 12f

β. H is only such suit

(I). SUITLENGTH (S) ≤ 3 or (SUITLENGTH (S) = 4 and SUITPOINTS (S) ≤ 1 — 1H at 12e

(II). Not (I).: SUITLENGTH (S) = 4 and 2 ≤ SUITPOINTS (S) ≤ 3

S:Qxxx, H:AKxx, D:Qx, C:Jxx — Pass

ii. All 4-card suits of S , H , D have SUITPOINTS ≤ 3

α. SUITPOINTS (D) ≥ 2 and (SUITLENGTH (H) ≤ 2 or (SUITLENGTH (S) ≤ 3 and (SUITLENGTH (H) ≤ 3))

β. Not α.

S:Qx, H:Kxx, D:Jxxx, C:AQxx — Pass

3. Distribution is 4441

   a. TOTALPTS ≥ 13

      as in II., B., 2., a.

   b. TOTALPTS ≤ 12

      as in II., B., 2., b.

end of Proposition 4.

Proposition 5: LONGEST = 5 and HCP ≤ 12

I. TOTALPTS ≤ 10 — Pass

II. TOTALPTS ≥ 11

A. HCP ≤ 9 — Pass

B. HCP = 10

1. TOTALPTS ≥ 14 and CANBEREBID (S or H) — 1S, 1H at 13b

2. Not 1.: TOTALPTS ≤ 13 or (no rebid of S or H)

a. Not VOIDFLAG — Pass

b. VOIDFLAG

i. Distribution = 5530

∴ TOTALPTS ≥ 12 and (S or H or D is a longest suit)

α. One 5-card suit of S , H , D has SUITPOINTS ≥ 1 — 1S, 1H, 1D at 12a

β. All 5-card suits of S , H , D have SUITPOINTS = 0

∴ SUITLENGTH (C) = 5 (i.e., C is also a longest suit) since SUITPOINTS (3-card suit) ≤ 9 implies at least one HCP is in the 5-card suits

(I). S or H is a longest suit

(A). SUITPOINTS (C) ≤ 6 1S, 1H at 12b

(B). SUITPOINTS (C) ≥ 7 2C at 12c

(II). D is a longest suit 1D at 12d

ii. Distribution = 5440

α. ACES ≥ 1

$\therefore$ TOTALPTS = 12

(I). S or H or D is the longest suit

(A). SUITPOINTS (longest suit)

≥ 1 1S, 1H, 1D at 12a

(B). SUITPOINTS (longest suit)

= 0

(1). One 4-card suit of

S or H or D has

SUITPOINTS ≥ 4 1S, 1H, 1D at 12g

(2). Not (1).: non-club,

4-card suit has SUITPOINTS

≤ 3. Note that SUITLENGTH

(C) $\neq 0$ else SUITPOINTS

(one 4-card suit) ≥ 5

S:xxxxx, H:Kxxx, D:---,

C:AKxx Pass

(II). C is the longest suit

(A). CANBEREBID (C) 2C at 12c

(B). Not CANBEREBID (C)

$\therefore$ SUITPOINTS (C) ≤ 4

(1). S or D has SUITPOINTS

≥ 4 1S, 1D at 12f

(2). SUITPOINTS (H) ≥ 4 and

Not (1).

(a). SUITPOINTS (H)

≥ 5 1H at 12b

(b). SUITPOINTS (H)

$= 4$

(i). S = void or

SUITPOINTS (S)

≤ 1 1H at 12e

(ii). Not (i).

S:Qxxx, H:Axxx,

D:---, C:Axxxx Pass

(3). All suits of S , H , D

have SUITPOINTS ≤ 3

(a). SUITPOINTS (D)

$= 3$

(i). H = void 1D at 12d

(ii). H $\neq$ void

S:---, H:Kxxx,

D:Kxxx, C:Axxxx Pass

(b). SUITPOINTS (D) ≤ 2

∴ D = Void since otherwise SUITPOINTS (C) ≤ 4 would imply a violation of (3).

S:Kxxx, H:Kxxx,

D:---, C:Axxxx Pass

β. ACES = 0

∴ TOTALPTS = 11

(I). S or H or D is the longest suit

(A). SUITPOINTS (longest suit) ≥ 1

S:Jxxxx, H:KQJx, D:Kxxx, C:--- Pass

(B). SUITPOINTS (longest suit) = 0

∴ One 4-card suit of S , H , D , has SUITPOINTS ≥ 4 since ACES = 0 implies SUITPOINTS (each 4-card suit) ≤ 6 implies SUITPOINTS (each 4-card suit) ≥ 4 1S, 1H, 1D at 12g

(II). C is the longest suit

as in α. (II). except the cases

(2).(b).(ii). new example of Pass:

S:Kxxx, H:KJxx, D:---, C:Kxxxx

(3). this is now impossible since

ACES = 0 implies SUITPOINTS (C)

≤ 3 else CANBEREBID (C). Now

one 4-card suit of S , H , D has

SUITPOINTS ≥ 4.

C. HCP ≥ 11

1. TOTALPTS ≥ 14 and CANBEREBID (S or H) — 1S, 1H at 13b

2. Not 1.: TOTALPTS ≤ 13 or (no rebid of S or H)

a. Distribution = 5530

∴ TOTALPTS ≥ 13

an in B.2.b.i.

b. Distribution = 5440

∴ TOTALPTS ≥ 12

i. S or H or D is the longest suit

α. SUITPOINTS (longest suit) ≥ 1 — 1S, 1H, 1D at 12a

β. SUITPOINTS (longest suit) = 0

(I). One 4-card suit of S or H

or D has SUITPOINTS ≥ 4 — 1S, 1H, 1D at 12g

(II). Not (I).: non-club, 4-card

suit has SUITPOINTS ≤ 3.

Note that SUITLENGTH (C) $\neq 0$ else SUITPOINTS (one 4-card suit) ≥ 6

S:xxxxx, H:Kxxx, D:---, C:AKJx — Pass

ii. C is the longest suit

α. CANBEREBID (C) — 2C at 12c

β. Not CANBEREBID (C)

$\therefore$ SUITPOINTS (C) ≤ 4

(I). HCP = 12 or ACES ≥ 1

$\therefore$ TOTALPTS ≥ 13

(A). S or D has SUITPOINTS ≥ 3 — 1S, 1D at 12f

(B). Not (A).: Both S and D have SUITPOINTS ≤ 2

$\therefore$ H $\neq$ void else HCP ≤ 8

$\therefore$ H , the "third" suit has SUITPOINTS ≥ 5 — 1H at 12b

(II). Not (I).: HCP = 11 and ACES = 0

$\therefore$ TOTALPTS = 12

(A). S or D has SUITPOINTS ≥ 4 — 1S, 1D at 12f

(B) Not (A).: Both S and D have SUITPOINTS ≤ 3

∴ ACES = 0 implies SUIT-
POINTS (C) ≤ 3 else
CANBEREBID (C). H ≠ void
else HCP ≤ 9

∴ SUITPOINTS (H) ≥ 5 — 1H at 12b

c. Distribution = 5521

∴ FOUR441 and (S or H or D is a longest suit)

i. TOTALPTS ≥ 12

α. SUITPOINTS (longest suit) ≥ 1 — 1S, 1H, 1D at 12a

β. SUITPOINTS (longest suit) = 0

(I). C is also a longest suit

(A). S or H is a longest suit

(1). SUITPOINTS (C) ≤ 6 — 1S, 1H at 12b

(2). SUITPOINTS (C) ≥ 7 — 2C at 12c

(B). D is a longest suit — 1D at 12d

(II). C is not also a longest suit

S:xxxxx, H:xxxxx, D:AK, C:A — Pass

ii. TOTALPTS = 11

α. C is also a longest suit

as in (I). under i.

β. C is not also a longest suit

S:Axxxx, H:Kxxxx, D:Qx, C:Q — Pass

d. Distribution = 5431

∴ FOUR441

i. TOTALPTS ≥ 12

α. S or H or D is the longest suit

(I). SUITPOINTS (longest suit) ≥ 1 — 1S, 1H, 1D at 12a

(II). SUITPOINTS(longest suit) = 0

(A). S or H or D is the 4-card suit

(1). SUITPOINTS (4-card suit) ≥ 4 — 1S, 1H, 1D at 12g

(2). SUITPOINTS (4-card suit) ≤ 3

[1] S:xxxxx, H:Kxxx, D:AKJ, D:x — Pass

[1] These bracketed numbers will serve as labels for case e.

(B). C is the 4-card suit

[2] S:xxxxx, H:x, D:AKJ, C:Kxxx — Pass

β. C is the longest suit

(I). CANBEREBID (C) — 2C at 12c

(II). Not CANBEREBID (C)

∴ SUITPOINTS (C) ≤ 4

(A). S or D is the 4-card suit

(1). SUITPOINTS (4-card suit) ≥ 4 — 1S, 1D at 12f

(2). SUITPOINTS (4-card
suit) = 3

(a). TOTALPTS ≥ 13 — 1S, 1D at 12f

(b). TOTALPTS = 12

(i). D is the
4-card suit — 1D at 12d

(ii). S is the
4-card suit

[3] S:Kxxx, H:Kxx,
D:Q, C:Axxxx — Pass

(3). SUITPOINTS (4-card
suit) ≤ 2

[4] S:Qxxx, H:KQx,
D:x, C:Axxxx — Pass

(B). H is the 4-card suit

(1). SUITPOINTS (H) ≥ 5 — 1H at 12b

(2). SUITPOINTS (H) = 3 or 4

(a). TOTALPTS ≥ 13 or
SUITPOINTS (H) ≥ 4 — 1H at 12e

(b). Not (a).: TOTALPTS
= 12 and SUITPOINTS
(H) = 3

[5] S:Kxx, H:Kxxx,

D:Q, C:Axxxx Pass

(3). SUITPOINTS (H) ≤ 2

[6] S:Axx, H:Qxxx, D:Q, C:Axxxx Pass

ii. TOTALPTS = 11

α. S or H or D is the longest suit

(I). SUITPOINTS (longest suit) ≥ 1

[7] S:Jxxxx, H:AQx, D:Qxxx, C:Q Pass

(II). SUITPOINTS (longest suit) = 0

(A). S or H or D is the 4-card suit

(1). SUITPOINTS (4-card suit) ≥ 4 1S, 1H, 1D at 12g

(2). SUITPOINTS (4-card suit) ≤ 3

[8] S:xxxxx, H:AQx,

D:Kxxx, C:Q Pass

(B). C is the 4-card suit

[9] S:xxxxx, H:AQx, D:Q, C:Kxxx Pass

β. C is the longest suit

(I). CANBEREBID (C) 2C at 12c

(II). Not CANBEREBID (C)

∴ SUITPOINTS (C) ≤ 4

(A). S or D is the 4-card suit

(1). SUITPOINTS (4-card suit) ≥ 4 — 1S, 1D at 12f

(2). SUITPOINTS (4-card suit) = 3

(a). D is the 4-card suit — 1D at 12d

(b). S is the 4-card suit — Pass

[10] S:Kxxx, H:AQx, D:Q, C:xxxxx

(3). SUITPOINTS (4-card suit) ≤ 2

[11] S:Qxxx, H:AQJ, D:Q, C:xxxxx — Pass

(B). H is the 4-card suit

(1). SUITPOINTS (H) ≥ 4 — 1H at 12e

(2). SUITPOINTS (H) ≤ 3

[12] S:AQx, H:Kxxx, D:Q, C:xxxxx — Pass

e. Distribution = 5422

i. HCP = 12 or (HCP = 11 and TOTALPTS ≥ 13)

This now follows as in d., distribution = 5431, with new examples of Pass at each labeled point in d.

label	S	H	D	C
[1]	xxxxx	Kxxx	Ax	Ax
[2]	xxxxx	Ax	Ax	Kxxx
[3]	Kxxx	KJ	AJ	xxxxx
[4]	Qxxx	KJ	AQ	xxxxx
[5]	KJ	Kxxx	Jx	Axxxx
[6]	KQ	Qxxx	Jx	Axxxx
[7]	Jxxxx	AQxx	QJ	Qx
[8]	xxxxx	AQ	Kxxx	QJ
[9]	xxxxx	AQ	QJ	Kxxx
[10]	Kxxx	AQ	QJ	xxxxx
[11]	Qxxx	AQ	QJ	Jxxxx
[12]	AQ	Kxxx	QJ	xxxxx

ii. Not i.: HCP = 11 and TOTALPTS ≤ 12 Pass
S: KQJxx, H: KQxx, D: xx, C: xx

f. Distribution = 5332

i. HCP = 12 or (HCP = 11 and TOTALPTS ≥ 13)

α. S or H or D is the longest suit

(I). SUITPOINTS (longest suit) $\geq$ 1

(A). TOTALPTS $\geq$ 12 — 1S, 1H, 1D at 12a

(B). TOTALPTS = 11

S:Jxxxx, H:Qxx, D:KQJ,
C:QJ — Pass

(II). SUITPOINTS (longest suit) = 0

S:xxxxx, H:AKQ, D:Qxx, C:xx — Pass

β. C is longest suit

(I). CANBEREBID (C) — 2C at 12c

(II). Not CANBEREBID (C)

S:KQJ. H:Qxx, D:xx, C:Axxxx — Pass

ii. Not i.: HCP = 11 and TOTALPTS $\leq$ 12 — Pass

S:AKQxx, H:xxx, D:xxx, C:Qx

end of Proposition 5.

Proposition 6: LONGEST = 6

Note: I. is the same as Proposition 2.I.

I. HCP ≥ 13

∴ TOTALPTS ≥ 12

A. S or H or D is a longest suit — 1S, 1H, 1D at 12a

B. C is (the only) longest suit — 2C at 12c

II. HCP ≤ 12

A. HCP ≤ 5 — Pass

B. HCP ≥ 6

1. 3C bid conditions met — 3C at 9

2. Not 1.: not 3C bid

a. HCP ≤ 9

i. S or H is a longest suit

α. HONORCOUNT (longest suit) ≥ 3 — 2S, 2H at 13c

β. HONORCOUNT (longest suit) ≤ 2 — Pass

ii. Not i. — Pass

b. HCP = 10

i. VOIDFLAG

∴ TOTALPTS ≥ 12

as in I.

ii. Not VOIDFLAG

α. S or H is a longest suit

(I). HONORCOUNT (longest suit) ≥ 3 — 2S, 2H at 13c

(II). HONORCOUNT (longest suit) ≤ 2

(A). TOTALPTS ≥ 14 — 1S, 1H at 13b

(B). TOTALPTS = 13

(1). SUITPOINTS (longest suit) ≥ 2 — 1S, 1H at 13a

(2). SUITPOINTS (longest suit) ≤ 1

S:Jxxxxx, H:KQJ, D:Kx, C:xx — Pass

(C). TOTALPTS ≤ 12

S:Jxxxxx, H:KQx, D:KJ, C:xx — Pass

β. D is a longest suit but both S and H are not

(I). TOTALPTS ≥ 13 and SUITPOINTS (D) ≥ 2 — 1D at 13a

(II). Not (I).: TOTALPTS ≤ 12 or SUITPOINTS (D) ≤ 1

S:KJ, H:KQx, D:Jxxxxx, C:xx — Pass

γ. C is the only longest suit

(I). TOTALPTS ≥ 14 and (S or H CANBEREBID) — 2S, 2H at 13b

(II). Not (I). — Pass

c. HCP = 11

i. VOIDFLAG or TOTALPTS ≥ 13

∴ TOTALPTS ≥ 13

as in I.

ii. Not i.: not VOIDFLAG AND TOTALPTS ≤ 12

as in a.

d. HCP = 12

∴ TOTALPTS ≥ 11

i. C is a longest suit — 2C at 12c

ii. C is not a longest suit

α. TOTALPTS ≥ 12 — 1S, 1H, 1D at 12a

β. TOTALPTS = 11

as in a.

end of Proposition 6.

Proposition 7: LONGEST = 7

I. HCP ≤ 5

∴ PLAYTRICKS ≤ 6 — Pass

II. HCP ≥ 6

A. 3C bid — 3C at 9

B. Not A.: Not 3C bid

1. HCP ≤ 10

∴ Section 12 is skipped

a. Distribution = 7600

∴ TOTALPTS ≥ 14

i. Longest suit = S or H or D

α. SUITPOINTS (longest suit) ≥ 4 — 3S, 3H, 3D at 11a

β. SUITPOINTS (longest suit) ≤ 3

(I). S or H has SUITLENGTH ≥ 6

(A). HONORCOUNT (S or H)

≥ 9 - SUITLENGTH (S or

H, respectively) = 2 or 3 — 2S, 2H at 13c

(B). Not (A).

(1). HCP ≤ 9

S:Kxxxxxx, H:---,

D:AQxxxx, C:--- — Pass

(2). HCP = 10 — 1S, 1H at 13b

(II). S and H have SUITLENGTH

≤ 5

$\therefore$ SUITLENGTH (S) = SUITLENGTH

(H) = 0 implies SUITLENGTH (D)

= 7 and SUITLENGTH (C) = 6

(A). HCP ≤ 9

S:---, H:---, D:Kxxxxxx,

C:AQxxxx — Pass

(B). HCP = 10 — 1D at 13a

ii. Longest suit = C

α. 6-card suit is D

S:---, H:---, D:AQxxxx, C:KJxxxxx — Pass

β. 6-card suit is S or H

(I). HONORCOUNT (6-card suit)

≥ 3 — 2S, 2H at 13c

(II). HONORCOUNT (6-card suit)

≤ 2

(A). HCP ≤ 9

S:AQxxxx, H:---, D:---,

C:Qxxxxxx — Pass

(B). HCP = 10 — 1S, 1H at 13b

b. Distribution $\neq$ 7600

∴ No second 6-card suit

i. Longest suit = S or H or D

α. SUITPOINTS (longest suit) ≥ 4 and (VOIDFLAG or OUTSIDEACE) — 3S, 3H, 3D at 11a

β. Not α.: SUITPOINTS (longest suit) ≤ 3 or (Not VOIDFLAG and not OUTSIDEACE)

(I). Longest suit = S or H

(A). HONORCOUNT (longest suit) ≥ 2 — 2S, 2H at 13c

(B). HONORCOUNT (longest suit) ≤ 1

(1). HCP ≤ 9

S:Kxxxxxx, H:AQxx, D:x, C:x — Pass

(2). HCP = 10

(a). TOTALPTS ≥ 13 — 1S, 1H at 13a

(b). TOTALPTS ≤ 12

S:Kxxxxxx, H:Qxx, D:QJ, C:Q — Pass

(II). Longest suit = D

(A). HCP ≤ 9

S:AQxx, H:x,

D:Kxxxxxx, C:x — Pass

(B). HCP = 10

(1). TOTALPTS $\geq$ 13 — 1D at 13a

(2). TOTALPTS $\leq$ 12

S:QJ, H:Qxx,

D:Kxxxxxx, C:Q — Pass

ii. Longest suit = C

α. Distribution $\neq$ 7510

S:AQxx, H:x, D:x, C:Axxxxxx — Pass

β. Distribution = 7510

(I). TOTALPTS $\geq$ 14 and HCP = 10

and (S or H CANBEREBID) — 1S, 1H at 13b

(II). Not (I).

S:x, H:---, D:AQxxx,

C:Axxxxxx — Pass

2. HCP $\geq$ 11

a. VOIDFLAG

$\therefore$ TOTALPTS $\geq$ 14

i. Longest suit = S or H or D — 1S, 1H, 1D at 12a

ii. Longest suit = C — 2C at 12c

b. Not VOIDFLAG

i. Distribution $\neq$ 7222

$\therefore$ TOTALPTS $\geq$ 12

α. TOTALPTS $\geq$ 13 or HCP $\geq$ 12

$\therefore$ TOTALPTS $\geq$ 13

(I). Longest suit = S or H or D — 1S, 1H, 1D at 12a

(II). Longest suit = C — 2C at 12c

β. Not α.: TOTALPTS = 12 and

HCP = 11

(I). (Longest suit = S or H) and

HONORCOUNT (longest suit)

$\geq$ 2 — 2S, 2H at 13c

(II). Not (I).

S:Q, H:Q, D:AQJxxxx,

C:xxxx — Pass

ii. Distribution = 7222

α. HCP = 11

(I). TOTALPTS $\geq$ 13

(A). Longest suit = S or

H or D — 1S, 1H, 1D at 12a

(B). Longest suit = C — 2C at 12c

(II). TOTALPTS $\leq$ 12

(A). (Longest suit = S or H)

and HONORCOUNT (longest

suit) ≥ 2 — 2S, 2H at 13c

(B). Not (A).

S:QJ, H:Qxxxxxx, D:QJ,

C:Kx — Pass

β. HCP ≥ 12

∴ TOTALPTS ≥ 11

(I). Longest suit = C — 2C at 12c

(II). Longest suit = S or H or D

(A). TOTALPTS ≥ 12 — 1S, 1H, 1D at 12a

(B). TOTALPTS = 11

∴ HCP = 12

as in α.(II).

end of Proposition 7.

Proposition 8: LONGEST = 8

I. Longest suit = S or H or D

A. HCP $\geq$ 6

1. HCP $\leq$ 10

$\therefore$ Section 12 is skipped

a. SUITPOINTS (longest suit) $\geq$ 2 — 3S, 3H, 3D at 11a

b. SUITPOINTS (longest suit) $\leq$ 1

$\therefore$ PLAYTRICKS $\leq$ 5

i. Longest suit = S or H

α. HONORCOUNT (longest suit) $\geq$ 1 — 2S, 2H at 13c

β. HONORCOUNT (longest suit) = 0

(I). HCP $\leq$ 9 — Pass

(II). HCP = 10

(A). Distribution $\neq$ 8221 or TOTALPTS $\geq$ 13

$\therefore$ TOTALPTS $\geq$ 13 — 1S, 1H at 13a

(B). Not (A).: Distribution = 8221 and TOTALPTS $\leq$ 12

S:xxxxxxxx, H:KQ, D:QJ, C:Q — Pass

ii. Longest suit = D

α. HCP ≤ 9 — Pass

β. HCP = 10

(I). Distribution ≠ 8221 or TOTALPTS ≥ 13

∴ TOTALPTS ≥ 13 — 1D at 13a

(II). Not (I).: Distribution = 8221 and TOTALPTS ≤ 12

S:QJ, H:KQ, D:xxxxxxxx, C:Q — Pass

2. HCP ≥ 11

∴ TOTALPTS ≥ 12

a. Distribution ≠ 8221 or TOTALPTS ≥ 13

∴ TOTALPTS ≥ 13 — 1S, 1H, 1D at 12a

b. Not a.: Distribution = 8221 and TOTALPTS = 12

∴ HCP = 11 and the 2,2,1-card suits are QJ , QJ , Q(or K) , respectively, implies SUITPOINTS (8-card suit) ≤ 3 implies PLAYTRICKS ≤ 5

i. (Longest suit = S or H) and (HONORCOUNT (longest suit) ≥ 1) — 2S, 2H at 13c

ii. Not i.

S:QJ, H:QJ, D:Kxxxxxxx, C:Q — Pass

B. HCP $\leq$ 5

∴ PLAYTRICKS $\leq$ 6 — Pass

II. Longest suit = C

A. HCP $\geq$ 11

∴ TOTALPTS $\geq$ 12

1. Distribution $\neq$ 8221 or TOTALPTS $\geq$ 13

∴ TOTALPTS $\geq$ 13 — 2C at 12c

2. Not 1.: Distribution = 8221 and TOTALPTS = 12

∴ as in I.A.2.b., PLAYTRICKS $\leq$ 5 and HCP = 11

S:QJ, H:QJ, D:Q, C:Kxxxxxxx — Pass

B. HCP $\leq$ 10

1. HCP $\geq$ 6

a. PLAYTRICKS $\geq$ 8 — 4C at 11b

b. PLAYTRICKS $\leq$ 7

i. Distribution $\neq$ 8500 — Pass

ii. Distribution = 8500

α. TOTALPTS $\geq$ 14 and HCP = 10 and

(S or H CANBEREBID) — 2S, 2H at 13b

β. Not α. — Pass

2. HCP $\leq$ 5

∴ PLAYTRICKS $\leq$ 6 — Pass

end of Proposition 8.

Proposition 9: LONGEST = 9

I. Longest suit = S or H or D

A. HCP ≥ 6

1. HCP ≤ 10 — 3S, 3H, 3D at 11a

2. HCP ≥ 11

∴ TOTALPTS ≥ 14 — 1S, 1H, 1D at 12a

B. HCP ≤ 5

∴ PLAYTRICKS ≤ 8 — Pass

II. Longest suit = C

A. HCP ≥ 11

∴ TOTALPTS ≥ 14 — 2C at 12c

B. HCP ≤ 10

1. HCP ≥ 6

a. PLAYTRICKS ≥ 8 — 4C at 11b

b. PLAYTRICKS ≤ 7

i. Distribution = 9211 or 9310

α. TOTALPTS ≥ 21 — 1C at 8b

β. TOTALPTS ≤ 20

S:x, H:A, D:Ax, C:Jxxxxxxxx — Pass

ii. Distribution = 9400 or 9220

∴ TOTALPTS ≤ 20 — Pass

2. HCP ≤ 5

∴ PLAYTRICKS ≤ 8 — Pass

end of Proposition 9.

Proposition 10: LONGEST = 10

I. Longest suit = S or H or D

A. HCP ≥ 6

1. HCP ≤ 10 — 3S, 3H, 3D at 11a

2. HCP ≥ 11

∴ TOTALPTS ≥ 16 — 1S, 1H, 1D at 12a

B. HCP = 5

1. Longest suit = S or H

a. SUITPOINTS (longest suit) = 5

∴ PLAYTRICKS = 9 — 4S, 4H at 6

b. SUITPOINTS (longest suit) ≤ 4

∴ PLAYTRICKS ≤ 8 — Pass

2. Longest suit = D — Pass

C. HCP ≤ 4

∴ PLAYTRICKS ≤ 8 — Pass

II. Longest suit = C

A. HCP ≥ 11

∴ TOTALPTS ≥ 16 — 2C at 12c

B. HCP ≤ 10

1. HCP ≥ 6

∴ TOTALPTS ≥ 11

a. PLAYTRICKS ≥ 8 — 4C at 11b

b. PLAYTRICKS ≤ 7

i. TOTALPTS ≥ 21 — 1C at 8b

ii. TOTALPTS ≤ 20 — Pass

2. HCP ≤ 5

∴ PLAYTRICKS ≤ 9 — Pass

end of Proposition 10.

Proposition 11: LONGEST = 11

∴ SUITPOINTS (longest suit) ≥ 3

I. Longest suit = S or H or D

A. HCP ≥ 6

1. HCP ≤ 10 — 3S, 3H, 3D at 11a

2. HCP ≥ 11

∴ TOTALPTS ≥ 20 — 1S, 1H, 1D at 12a

B. HCP ≤ 5

∴ TOTALPTS ≤ 20

1. Longest suit = S or H

∴ PLAYTRICKS ≥ 9 — 4S, 4H at 6

2. Longest suit = D

a. SUITPOINTS (D) = 5

∴ PLAYTRICKS = 10 — 5D at 6

b. SUITPOINTS (D) ≤ 4

∴ PLAYTRICKS = 9 — Pass

II. Longest suit = C

A. HCP ≥ 11

∴ TOTALPTS ≥ 20 — 2C at 12c

B. HCP ≤ 10

1. HCP ≥ 6

∴ PLAYTRICKS ≥ 9 — 4C at 11b

2. HCP ≤ 5

∴ TOTALPTS ≤ 20

a. SUITPOINTS (C) = 5

∴ PLAYTRICKS = 10 — 5C at 6

b. SUITPOINTS (C) ≤ 4

∴ PLAYTRICKS = 9 — Pass

end of Proposition 11.

Proposition 12: LONGEST = 12

$\therefore$ HCP $\leq$ 14

I. The Ace is missing from the 12-card suit

$\therefore$ PLAYTRICKS $\geq$ 11 — 4S, 4H, 5D, 5C at 6

II. Not I.: The Ace is present in the 12-card suit,

i.e., some other card is missing

$\therefore$ PLAYTRICKS $\geq$ 12 — 6 or 7 SUIT at 2b

end of Proposition 12.

Proposition 13: LONGEST = 13

∴ PLAYTRICKS = 13 7S, 7H, 7D, 7C at 2b

end of Proposition 13.

IV. Conclusion

Few programs of the complexity of OPENBID have been subjected to the detailed treatment that OPENBID has received. We have thus demonstrated the possibility of completing such a large and complicated case analysis to determine the behavior of non-trivial computer programs.

It matters little whether one views the analysis as a formal mathematical proof about the behavior of OPENBID, or whether one views it as a very formal debugging process in which all the many possibilities have been considered and even some errors uncovered. The end result is the same: increased confidence in the correctness of OPENBID and a better understanding of how it does and does not work.

We have been aided in our analysis by the special structure of OPENBID:

1. There are few loops in the code.

2. The loops which are present are especially simple and have short bodies. In particular, all the for-statements could be written with for-list elements consisting solely of constants (e.g., <u>for</u> L ← 4, 3, 2, 1 <u>do</u>).

3. The program deals, for our purposes, totally with small integers.

4. The flow of control is particularly well suited to analysis.

5. The use of memory by OPENBID is nearly all "read-only" in that it consults the previously computed hand descriptions but does not change them. In other words, the essence of OPENBID is in the flow of control, and this flow does not depend on values of variables set or changed by OPENBID itself. Thus we basically analyze flow with little concern for the changing values of variables since the values are all constant.

However, all was not as easy as the above points might imply. Indeed, the number and complexity of the conditional statements governing the flow of control made the analysis quite formidable. Basically, successive conditional statements often deal with different descriptions of a bridge hand and in such a way that the various descriptions are not easily comparable. Examples of this are the comparison of PLAYTRICKS with HCP or of distributions with TOTALPTS. Put another way, it is certainly easy to show that a hand meeting the conditions of given bid will make that bid (assuming control reaches the bid). But what happens if the hand does not meet those conditions? Any one of several conditions might fail, and it is now necessary to consider whether the hand now meets other bid conditions. At this point we must be able to interrelate the different descriptions of the bids in question.

This requirement necessitates having available a large amount of semantic information such as tables of distributions, relations, representations and facts. Several of the appendices include the needed information.

We have used two methods or strategies to complete the analysis. Each should have application in future analyses. The first is the already explained strategy of breaking the analysis into the two parts. This has enabled us to avoid interrelating many descriptions that might otherwise be necessary if, for example, the second part of the analysis had to show, in addition, that the bid is the first one possible. This strategy should work best on those programs whose control structure and possibly memory utilization is similar to those of OPENBID.

The second method is the use of systematic case analysis. This assures that all possibilities have been covered. The case analysis makes many small assumptions at each case. This makes it easier to compare descriptions since in general cases differ only slightly from their predecessors (parents). Small assumptions also enable us to follow the flow of control since often the additional assumptions are made with this in mind.

Case analysis is applicable to other program analyses, since much of programming is of the form: if this then such and such else if that then ... . Case analysis provides a way to ascertain the completeness of the conditional expressions and, to a lesser extent, the resulting actions.

It has already been noted that this analysis was obtained completely by hand. Clearly the computer should be able to aid a human in obtaining these analyses. Work is now proceeding in this direction.

Appendix 1. The Algol Code (Burroughs B5500 Extended Algol)

This version of OPENBID (and the other procedures) is the code which existed on April 3, 1967. It is out-of-date, partly because the analysis presented in this paper has caused changes and corrections to be made. Until an analysis uncovers no additional flaws, the analyzed code will always pre-date the current version. This is so because any change can void much of the analysis (and it is hard to discover which parts!) and because analyses are very time-consuming to obtain.

The Algol code is nearly standard Algol 60 except for input/output, of course. Note that "x" denotes multiplication and "*" is exponentiation. All labels must be declared. A <u>define</u> feature is also used which causes specified Algol text to be inserted at compile time by the compiler in place of a specified identifier. The following definitions are involved:

Identifier	Substitution
GOTO	GO TO
SUITS	L← 4 STEP -1 UNTIL 1
SHD	L← 4 STEP -1 UNTIL 2
MAJORS	L← 4 STEP -1 UNTIL 3

For example,

```
FOR SHD DO
```

expands into

```
FOR L← 4 STEP -1 UNTIL 2 DO
```

giving the desired effect of iterating over S, then H, and finally D.

The *fill* statement fills a specified array row with the given value starting at the lower bound. The *alpha* declarations allows variables to assume string values of at most six characters.

The code, which starts on the next page, includes only those essential declarations and procedures which are needed to understand OPENBID. The procedures which print the hand and the bid, shuffle and deal the deck, and sort the hand, are all excluded, as is the driver program.

```
INTEGER B,I,J,K,L,M,N,OPENVAL;
INTEGER ARRAY A[1:52],HCP,DISTP,TOTALPTS,SUMP,ACES,KINGS,LONGEST,
   STOP,MINPOINTS,MAXPOINTS,SP,BIDVAL,PLAYTRICKS[1:4],
   SUITLENGTH,SUITPOINTS,HONORCOUNT[1:4,0:5],HAND[1:4,1:13],
   RANK[1:4,1:15],SEQ[1:32];
BOOLEAN ARRAY VOIDFLAG,EVENDIST,REBIDDABLE,BWOOD[1:4],
   NOTR[1:4,1:4],EXES[1:15];
BOOLEAN OPEN,ACE,KING; ALPHA ARRAY PRINTHAND[1:4,0:16];
DEFINE GOTO= GO TO#, MAJORS = L← 4 STEP -1 UNTIL 3#,
   SHD = L←4 STEP -1 UNTIL 2#,SUITS = L← 4 STEP -1 UNTIL 1#;
PROCEDURE ASORT(A,M); VALUE M;
INTEGER M; INTEGER ARRAY A[1];
BEGIN INTEGER TEMP;
   FOR I← 1 STEP 1 UNTIL M-1 DO
   BEGIN L←I; FOR J← I+1 STEP 1 UNTIL M DO
      IF A[J]> A[L] THEN L←J;
      TEMP←A[I]; A[I]←A[L]; A[L]←TEMP;
   END;
END;
INTEGER PROCEDURE PLAYINGTRICKS (SUIT); VALUE SUIT; INTEGER SUIT;
BEGIN INTEGER LNG;
   LABEL NONE,WUN,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,NINE,Z;
   SWITCH POINTS←NONE,WUN,TWO,THREE,FOUR,FIVE,SIX,SEVEN,EIGHT,NINE,Z;
   LNG← SUITLENGTH[B,SUIT];  N←0;
   GO TO POINTS[SUITPOINTS[B,SUIT]+1] ;
 NINE: IF LNG≥4 AND LNG≤7 THEN N←1; GO TO Z;
 EIGHT: IF LNG≤10 THEN N←1; GO TO Z;
 SEVEN: IF KING THEN
   BEGIN IF LNG≤10 THEN N←1; IF LNG≤7 THEN N←2;
      IF LNG≤4 THEN N←LNG-2;
   END   ELSE
   IF LNG≥4 AND LNG≤7 THEN N←2 ELSE IF LNG≤11 THEN N←1; GO TO Z;
 SIX: IF ACE THEN
   BEGIN IF LNG≤3 THEN N←LNG-1 ELSE IF LNG≤5 THEN N←3
      ELSE IF LNG≤8 THEN N←2 ELSE IF LNG≤11 THEN N←1;
   END    ELSE IF LNG≥4 AND LNG≤6 THEN N←2 ELSE N←1;  GO TO Z;
FIVE:N←1; IF ACE THEN
   BEGIN IF LNG≤9 THEN N←2; IF LNG≤5 THEN N←3;
      IF LNG=3 THEN N←2;
   END    ELSE
```

```
      BEGIN IF LNG≤8 THEN N←2; IF LNG≤6 THEN N←3;
      END;
      IF LNG≤2 THEN N←1; GO TO Z;
    FOUR:N←2; IF ACE THEN
      BEGIN IF LNG≤8 THEN N←3; IF LNG=6 THEN N←4; IF LNG≤5 THEN N←LNG-1;
      END     ELSE
      BEGIN IF LNG≤7 THEN N←3; IF LNG=3 OR LNG=4 THEN N←LNG-1;
      END;
      GO TO Z;
    THREE: N←2; IF KING THEN
      BEGIN IF LNG≤7 THEN N←4 ELSE IF LNG≤10 THEN N←3; END     ELSE
         IF LNG≤6 THEN N←4 ELSE IF LNG≤8 THEN N←3;
      IF LNG≤4 THEN N←LNG; GO TO Z;
    TWO: IF LNG≤5 THEN N←LNG ELSE IF LNG≤8 THEN N←4 ELSE N←3; GO TO Z;
    WUN: IF LNG≥8 THEN N←4 ELSE N←LNG; GO TO Z;
    NONE: N←LNG;
    Z: PLAYINGTRICKS← LNG-N;
   END;
    PROCEDURE PRELIM;
    BEGIN
      INTEGER INDEX; INTEGER ARRAY SL,SAFE[1:4],SUB,XTRP[0:4];
      ALPHA ARRAY LETTERS[1:4],NUMBERS[1:13]; LABEL BOUT,XIT,OMT;
      FILL LETTERS[*] WITH " C", " D"," H"," S";
      FILL NUMBERS[*] WITH " A"," 2"," 3"," 4"," 5"," 6"," 7"," 8",
         " 9","10"," J"," Q"," K";
      J ← 1; SUB [B] ← 0; STOP [B]  ← 0; INDEX ← 0;
      SUITLENGTH [B,0] ←0; SUITPOINTS [B,0] ←0; HONORCOUNT [B,0] ← 0;
      SUITLENGTH [B,5] ←2; SUITPOINTS [B,5] ←0; HONORCOUNT [B,5] ← 0;
      ACES [B]  ← 0; KINGS [B]  ← 0; HCP [B]  ← 0;
      DISTP [B] ←0;XTRP [B] ←1;PLAYTRICKS [B] ←0;
      VOIDFLAG[B]←FALSE; EVENDIST[B]←FALSE; BWOOD[B]←FALSE;
       FOR  I ← 1  STEP  1  UNTIL  4  DO
       BEGIN  SAFE [I]  ← 0; SUITPOINTS [B,I] ←0; HONORCOUNT [B,I] ←0;
         SL [I] ←0; SUITLENGTH [B,I] ←0;NOTR [B,I] ← FALSE;
       END ;
       FOR  I ← 1  STEP  1  UNTIL  15  DO  RANK  [B,I]  ← 0;
       FOR  K ← 3  STEP  -1  UNTIL  0  DO
       BEGIN  L ← K+1;
          FOR  I ← J  STEP  1  UNTIL  13  DO
          BEGIN  RANK [B,I]  ← HAND [B,I]  - 13×K;
```

```
        IF  RANK  [B,I]    ≤    0  THEN  GO TO  BOUT;
     END ;
     I ← 14;
BOUT: SL [L] ← I-J;
     SUITLENGTH [B,L] ← SL [L] ;
     PRINTHAND[B,INDEX]←LETTERS[L];  INDEX←INDEX+1;
     IF SL[L]=0 THEN GO TO OMT;
     ACE←KING←FALSE;
      IF  RANK [B,I-1]    =    1   THEN
     BEGIN SAFE[L]←1; ACE←TRUE; PRINTHAND[B,INDEX]←" A";
        INDEX←INDEX+1; SUITPOINTS[B,L]←SUITPOINTS[B,L]+4;
        ACES[B]←ACES[B]+1; IF SL[L]=1 THEN SUB[B]←SUB[B]-1;
      END ;
     IF RANK[B,J]=13 THEN
     BEGIN KING←TRUE;  SUITPOINTS[B,L]←SUITPOINTS[B,L]+1;
        KINGS[B]←KINGS[B]+1; IF SL[L]=1 THEN
        BEGIN SUB [B] ← SUB [B] +2; NOTR [B,L] ← TRUE; END
                                   ELSE  SAFE [L]  ← 1;
      END ;
     IF RANK[B,J]=12 OR RANK[B,J+1]=12 THEN
     BEGIN SUITPOINTS[B,L]←SUITPOINTS[B,L]+2;
        IF SL[L] < 3 THEN SUB [B] ← SUB [B] + 3 - SL[L];
      END ;
      IF   RANK   [B,J]    =    11   OR   RANK   [B,J+1]    =    11
         OR   RANK   [B,J+2]    =    11   THEN
     BEGIN
        SUITPOINTS[B,L]←SUITPOINTS[B,L]+1;
         IF   SL  [L]    <    3   THEN   SUB [B] ←SUB [B] +1;
      END ;
     IF SL[L] = 2 AND RANK [B,J] = 12 AND RANK [B,J+1] = 11 THEN
        NOTR [B,L] ← TRUE;
      IF   SL  [L]   ≥    3   THEN   SAFE  [L]  ←1;
      FOR   M ← J   STEP   1   UNTIL   I-1     DO
     BEGIN N←RANK[B,M]; IF N≠1 THEN
        BEGIN PRINTHAND[B,INDEX]←NUMBERS[N];INDEX←INDEX+1; END;
        IF N≥10 OR N=1 THEN HONORCOUNTER[B,L]←HONORCOUNTER[B,L]+1;
        IF N≥2 AND N≤9 AND EYES[M] THEN PRINTHAND[B,INDEX-1]←" X";
      END ;
     PLAYTRICKS[B]← PLAYTRICKS[B] + PLAYINGTRICKS(L);
     J←I;
```

```
OMT:  IF J >  13 THEN  GOTO XIT;
   END ;
XIT:  FOR  I ← 1  STEP  1  UNTIL  4  DO
   BEGIN  STOP [B]  ← STOP [B]  + SAFE [I] ;
     HCP [B]  ← HCP [B]  + SUITPOINTS  [B,I] ;
   END ;
  IF HAND[B,13]>13 THEN
  FOR J←16,J-1 WHILE PRINTHAND[B,J]="  " DO
  PRINTHAND[B,J]←LETTERS[17-J];
  ASORT(SL,4); LONGEST [B] ← SL[1];
   FOR  I ← 2,3,4  DO
   BEGIN  IF  SL [I]  <  3 THEN DISTP [B] ←DISTP [B] -SL [I] +3;
      IF  SL [I]  =  0  THEN  VOIDFLAG [B] ←  TRUE ;
   END ;
   IF  DISTP [B]  ≤  1  THEN  EVENDIST [B] ←  TRUE ;
  IF DISTP[B]= 0 THEN XTRP[B]← XTRP[B]-1;
  IF ACES[B] = 0 THEN XTRP[B] ← XTRP[B]-1;
   IF  ACES [B]  =  4  THEN  XTRP [B] ←XTRP [B] +1;
  FOR I← 1 STEP 1 UNTIL 2 DO
   IF  SL [I]  ≥  5  THEN XTRP [B] ←XTRP [B] +2×(SL [I] -5)+1;
  SUMP [B] ←HCP [B] +DISTP [B] -SUB [B] ;
  TOTALPTS [B]  ← HCP [B]  + XTRP [B]  - SUB [B] ;
END;
BOOLEAN PROCEDURE MAJOR(SUIT); VALUE SUIT; INTEGER SUIT;
MAJOR← SUIT≥3;
BOOLEAN PROCEDURE MINOR(SUIT); VALUE SUIT; INTEGER SUIT;
MINOR← SUIT≤2;
BOOLEAN PROCEDURE OUTSIDEACE(SUIT); VALUE SUIT; INTEGER SUIT;
BEGIN OUTSIDEACE←FALSE;
  FOR I← 1 STEP 1 UNTIL 13 DO
  FOR K← 3 STEP -1 UNTIL 0 DO
  IF HAND[B,I]=13×K+1 AND K≠SUIT-1 THEN OUTSIDEACE←TRUE;
END;
BOOLEAN PROCEDURE CANBEREBID(SUIT); VALUE SUIT;
INTEGER SUIT;
CANBEREBID←(SUITLENGTH[B,SUIT]≥5 AND SUITPOINTS[B,SUIT]≥4 AND
  HONORCOUNT[B,SUIT]≥2) OR SUITLENGTH[B,SUIT]≥6;
BOOLEAN PROCEDURE SOLID;
BEGIN SOLID←TRUE;
  FOR M←1 STEP 1 UNTIL 4 DO
```

```
        IF (SUITPOINTS[B,M]≥2 AND SUITLENGTH[B,M]≤4) OR
          NOTR[B,M] THEN SOLID←FALSE;
    END;
    BOOLEAN PROCEDURE NOTRUMPTRY;
    BEGIN LABEL DONE; NOTRUMPTRY←TRUE;
        FOR SUITS DO
        IF (SUITPOINTS[B,L]<3 AND SUITLENGTH[B,L]=2) OR
        (SUITPOINTS[B,L]<2 AND SUITLENGTH[B,L]=3) OR
        SUITLENGTH[B,L]≤1 OR NOTR[B,L] THEN
        BEGIN NOTRUMPTRY←FALSE; GO TO DONE; END;
  DONE:END;
    BOOLEAN PROCEDURE SLIDEPOINTS(SUIT,NUM1,NUM2);
    VALUE SUIT,NUM1,NUM2; INTEGER SUIT,NUM1,NUM2;
    SLIDEPOINTS←SUITPOINTS[B,SUIT]≥NUM1-NUM2×SUITLENGTH[B,SUIT];
    PROCEDURE OPENBID;
    BEGIN BOOLEAN FOUR441;
        LABEL J4,PREEMPT,WEAK3,OPENER,LASTCHANCE,NOBID,TEXT,AWAY,SKIP;
        REBIDDABLE[B]←FALSE; BIDVAL[B]←0;
        OPEN←FALSE; FOUR441←FALSE;
        FOR SUITS DO SP[L]←SUITPOINTS[B,L]; ASORT(SP,4);
        IF LONGEST[B]≤5 THEN FOR SUITS DO
        IF SUITLENGTH[B,L]=1 THEN FOUR441←TRUE;
        IF TOTALPTS[B] < 7 THEN GO TO NOBID;
        IF PLAYTRICKS[B]≥12 AND ACES[B]=4 THEN
        BEGIN BIDVAL[B]←(PLAYTRICKS[B]-6)×10+5; GO TO TEXT; END;
        IF PLAYTRICKS[B]≥12 AND LONGEST[B]≥8 THEN FOR SUITS DO
        IF SUITLENGTH[B,L]=LONGEST[B] THEN
        BEGIN BIDVAL[B]←(PLAYTRICKS[B]-6)×10+L; GO TO TEXT; END;
        IF HCP[B]≥25 AND EVENDIST[B] AND NOTRUMPTRY THEN
        BEGIN BIDVAL[B]←10; GO TO TEXT; END;
        IF HCP[B]≥22 AND HCP[B]≤24 AND EVENDIST[B] AND NOTRUMPTRY THEN
        BEGIN BIDVAL[B]←25; GO TO TEXT; END;
        IF (HCP[B]≥23 OR (HCP[B]≥22 AND TOTALPTS[B]≥25) OR
        TOTALPTS[B]≥26) AND PLAYTRICKS[B]≥9 THEN
        BEGIN FOR SUITS DO
            IF SUITLENGTH[B,L]=LONGEST[B] AND LONGEST[B]≥5 THEN
            BEGIN BIDVAL[B]←5+L; GO TO TEXT; END;
            FOR SUITS DO
            IF SUITLENGTH[B,L]=4 AND SUITPOINTS[B,L]=SP[1] THEN
            BEGIN BIDVAL[B]←5+L; GO TO TEXT; END;
```

```
END;
IF LONGEST[B]≥8 AND PLAYTRICKS[B]≥9 AND HCP[B]≤15 THEN
FOR SUITS DO IF SUITLENGTH[B,L]= LONGEST[B] THEN
BEGIN IF MINOR(L) AND PLAYTRICKS[B]≥10 THEN
   BEGIN BIDVAL[B]←50+L; GO TO TEXT; END;
   IF MAJOR(L) THEN
   BEGIN BIDVAL[B]←40+L; GO TO TEXT; END;
END;
IF HCP[B]≥15 AND HCP[B]≤18 AND EVENDIST[B] AND STOP[B]=4 THEN
BEGIN IF ((HCP[B]≥17 AND TOTALPTS[B]≥19) OR
   (SUITLENGTH[B,3]=5 AND SUITPOINTS[B,3]≥5) OR
   (SUITLENGTH[B,4]=5 AND SUITPOINTS[B,4]≥5)) AND NOT SOLID THEN
   GO TO J4  ELSE BEGIN BIDVAL[B]←15; GO TO TEXT; END;
END;
J4: IF HCP[B]≥17 OR (HCP[B]≥16 AND TOTALPTS[B]≥20) OR
TOTALPTS[B]≥21 THEN
BEGIN IF HCP[B]≥18 AND HCP[B]≤21 AND EVENDIST[B] AND SOLID THEN
   BEGIN FOR MAJORS DO IF CANBEREBID(L) THEN
      BEGIN BIDVAL[B]←L; GO TO TEXT; END;
      BIDVAL[B]←5; GO TO TEXT;
   END;
   FOR SUITS DO
   IF SUITLENGTH[B,L]=LONGEST[B] AND LONGEST[B]≥5 THEN
   BEGIN BIDVAL[B]←L; GO TO TEXT; END;
   IF HCP[B]≥18 AND NOTRUMPTRY THEN
   BEGIN BIDVAL[B]←5; GO TO TEXT; END;
   FOR SUITS DO
   IF SUITLENGTH[B,L]=4 AND SUITPOINTS[B,L]=SP[1] THEN
   BEGIN BIDVAL[B]←L; GO TO TEXT; END;
   FOR SUITS DO
   IF SUITLENGTH[B,L]=4 AND SUITPOINTS[B,L]=SP[2] THEN
   BEGIN BIDVAL[B]←L; GO TO TEXT; END;
   IF HCP[B]≥18 AND EVENDIST[B] AND STOP[B]=4 THEN
   BEGIN BIDVAL[B]←5; GO TO TEXT; END;
END;
IF SUITLENGTH[B,1]≥6 AND SUITPOINTS[B,1]≥9 AND
HONORCOUNT[B,1]≥4 AND HCP[B]≤14 THEN
BEGIN BIDVAL[B]←31; GO TO TEXT; END;
IF HCP[B]≥6 AND HCP[B]≤12 AND LONGEST[B]≥6 AND
TOTALPTS[B]≤7 + LONGEST[B] THEN
```

```
  BEGIN FOR MAJORS DO
     IF HONORCOUNT[B,L]≥9-SUITLENGTH[B,L] AND SUITLENGTH[B,L]≥6 THEN
     BEGIN IF LONGEST[B]≥8 THEN GO TO PREEMPT  ELSE
        BEGIN BIDVAL[B]←20+L; GO TO TEXT; END;
     END;
  END;
PREEMPT: IF LONGEST[B]≥7 AND HCP[B]≥6 AND HCP[B]≤10 THEN
  BEGIN FOR SHD DO
     IF SUITLENGTH[B,L] = LONGEST[B] AND SLIDEPOINTS(L,18,2) THEN
     BEGIN IF VOIDFLAG[B] OR LONGEST[B]≥8 OR OUTSIDEACE(L) THEN
        GO TO WEAK3;
     END;
     IF SUITLENGTH[B,1]≥8 AND PLAYTRICKS[B]≥8 THEN
     BEGIN BIDVAL[B]←41; GO TO TEXT; END;
     GO TO LASTCHANCE;
WEAK3: BIDVAL[B]←30+L; GO TO IF HCP[B]≥10 THEN LASTCHANCE ELSE TEXT;
  END;
  IF ((HCP[B]≥11 AND (TOTALPTS[B]≥13 OR FOUR441)) OR
     (HCP[B]≥10 AND VOIDFLAG[B]) OR (HCP[B]≥12 AND DISTP[B]≠0))
     AND TOTALPTS[B]≥11 THEN
  BEGIN FOR SHD DO
     BEGIN IF SUITLENGTH[B,L]=LONGEST[B] AND
        LONGEST[B]≥5 AND SLIDEPOINTS(L,6,1) AND TOTALPTS[B]≥12 THEN
        GO TO OPENER;
     END;
     IF SUITLENGTH[B,1]=LONGEST[B] THEN
     BEGIN FOR MAJORS DO IF SLIDEPOINTS(L,25,5) AND LONGEST[B]=5
        AND SUITPOINTS[B,1]≤6 THEN GO TO OPENER;
        IF LONGEST[B]≥5 AND CANBEREBID(1) THEN
        BEGIN BIDVAL[B]←21; GO TO TEXT; END;
        FOR SHD DO
        IF SLIDEPOINTS (L,31,7) THEN GO TO SKIP;
     END;
     IF LONGEST[B]=4 THEN
   SKIP: BEGIN IF SLIDEPOINTS(2,34,8) AND (SUITLENGTH[B,3]≤2 OR
        (SUITLENGTH[B,4]≤3 AND SUITLENGTH[B,3]≤3)) THEN
        BEGIN BIDVAL[B]←12; GO TO TEXT; END;
        IF SLIDEPOINTS(3,34,8) AND (SUITLENGTH[B,4]≤3 OR
        (SUITLENGTH[B,4]=4 AND SUITPOINTS[B,4]≤1)) THEN
        BEGIN IF NOT (HCP[B]≤12 AND TOTALPTS[B]≤12 AND
```

```
            SUITPOINTS[B,3]≤3 ) THEN
            BEGIN BIDVAL[B]←13; GO TO TEXT; END;
         END;
         FOR L←4,2 DO IF SLIDEPOINTS(L,34,8) THEN
         BEGIN IF NOT (HCP[B]≤12 AND TOTALPTS[B]≤12 AND
            SUITPOINTS[B,L]≤3) THEN GO TO OPENER;
         END;
      END;
      FOR SHD DO
      IF SUITLENGTH[B,L]=LONGEST[B] AND LONGEST[B]=5 AND
      SUITPOINTS[B,L]=0 THEN
      BEGIN FOR K←4 STEP -1 UNTIL 2 DO
         IF SUITLENGTH[B,K]=4 AND SUITPOINTS[B,K]≥4 THEN
         BEGIN BIDVAL[B]←10+K; GO TO TEXT; END;
         IF HCP[B]≥13 THEN GO TO OPENER;
      END;
      IF HCP[B]≥13 AND EVENDIST[B] AND SUITLENGTH[B,2]≥3 THEN
      BEGIN BIDVAL[B]←12; GO TO TEXT; END;
      IF SUITLENGTH[B,1]≥5 AND HCP[B]≥13 AND
      SUITPOINTS[B,1]≥5-HONORCOUNT[B,1] THEN
      BEGIN BIDVAL[B]←21; GO TO TEXT; END;
   END;
 LASTCHANCE: IF TOTALPTS[B]≥13 AND LONGEST[B]≥6 AND HCP[B]≥10 THEN
   FOR SHD DO IF SUITLENGTH[B,L]=LONGEST[B] AND
      SLIDEPOINTS(L,14,2) THEN GO TO OPENER;
   IF TOTALPTS[B]≥14 AND HCP[B]≥10 THEN
   FOR MAJORS DO IF CANBEREBID(L) THEN GO TO OPENER;
   IF HCP[B]≥6 AND HCP[B]≤12 AND LONGEST[B]≥6 THEN
   FOR MAJORS DO IF HONORCOUNT[B,L]≥9-SUITLENGTH[B,L] AND
      SUITLENGTH[B,L]≥6 THEN
   BEGIN BIDVAL[B]←20+L; GO TO TEXT; END;
   IF HCP[B]≥14 THEN FOR SHD DO
   IF SUITLENGTH[B,L]≥4 AND SUITPOINTS[B,L]≥1 THEN GO TO OPENER;
   GO TO TEXT;
 OPENER: BIDVAL[B]←10+L;
 NOBID:TEXT:IF BIDVAL[B]≠0 THEN OPEN ← TRUE; OPENVAL←BIDVAL[B];
   REBIDDABLE[B]← CANBEREBID(OPENVAL MOD 5);
AWAY: END;
```

Appendix 2. Definitions of Program Identifiers

Distinctions between procedures and arrays are not shown here but may be found in the code in Appendix 1.

1. ACES - the number of Aces in the hand.

2. CANBEREBID(SUIT) - is true for any suit of length at least 6 and for any 5-card suit with at least 4 SUITPOINTS and 2 honors. Same as REBIDDABLE.

3. DISTP - the number of distributional points. Each void counts 3, each singleton 2, and each doubleton 1.

4. EVENDIST - true if DISTP $\leq$ 1.

5. FOUR441 - true if the distribution is either 4441, 5521, or 5431.

6. HCP - the total of the high card points, where each Ace counts 4, each King 3, each Queen 2, and each Jack 1.

7. HONORCOUNT(SUIT) - the number of honors in the suit, where an honor is an Ace, King, Queen, Jack, or Ten.

8. LONGEST - the length of the longest suit. Note that the value is unique, although there may be two or even three such longest suits.

9. MAJOR(SUIT) - true if the suit is hearts or spades.

10. MINOR(SUIT) - true if the suit is clubs or diamonds.

11. NOTR(SUIT) - true if suitholding is K only or QJ only.

12. NOTRUMPTRY - true if each suit in the hand is of at least length 2 and 2-card suits are at least Kx and 3-card suits at least Qxx.

13. OUTSIDEACE(SUIT) - true if the hand has an Ace that is not in the specified suit.

14. PLAYTRICKS - an estimate of the number of playing tricks that the hand contains.* It is the sum of the PLAYTRICKS for the four suits. The number of PLAYTRICKS for each suitholding is given in Appendix 3.

15. SAFE - see STOP.

16. SLIDEPOINTS(SUIT, N1, N2) - true if

SUITPOINTS(SUIT) ≥ N1 - N2 x SUITLENGTH(SUIT).

This relation is used to implement the bridge adage "length before strength," since the greater the SUITLENGTH, the fewer SUITPOINTS are needed for the inequality to be true. Consider the case SLIDEPOINTS(SUIT, 31, 7): For a 5-card or longer suit, the inequality is true regardless of SUITPOINTS. For a 4-card suit, SUITPOINTS must be at least 3. For a 3-card suit, SUITPOINTS would have to be at least 10. But this is impossible, since 9 points is the maximum with only 3 cards. This prevents bidding less than a 4-card suit.

17. SOLID - true if every suit is of length 2 or greater and has at least 3 SUITPOINTS. It is also true if a suitholding is Ace only.

18. SP(1), SP(2), SP(3), SP(4) - the four SUITPOINTS ordered such that

SP(1) ≥ SP(2) ≥ SP(3) ≥ SP(4).

---

*For the knowledgeable bridge player, this is the approximate number of tricks that can be taken in the bidder's hand alone without finesses, squeezes, etc.

19. STOP - the number of suits considered to be SAFE. A suit is SAFE if

    SUITLENGTH(SUIT) ≥ 3

    or SUITLENGTH(SUIT) = 2 and Ace or King present

    or singleton Ace.

20. SUITLENGTH(SUIT) - the number of cards in the suit.

21. SUITPOINTS(SUIT) - the number of HCP in the suit.

22. TOTALPTS - HCP suitably modified.

    Add to HCP:

    For each suit with at least 5 cards, 2 x (SUITLENGTH - 5) + 1

    For each singleton Ace, 1

    For having all 4 Aces, 1

    For the hand, 1 (i.e., TOTALPTS is initially 1, not 0)

    Subract from HCP:

    For having no Aces, 1

    For 4333 distribution, 1

    For each singleton King, 2

    For each singleton Queen, 2

    For each doubleton Queen, i.e., Qx, 1

    For each doubleton Jack or singleton J, 1

    (note that a QJ doubleton subracts 2)

23. VOIDFLAG - true if the hand has a void, i.e., if one or more suits has SUITLENGTH of zero.

Appendix 3. PLAYTRICKS in a Suit

SUITPOINTS &	suitholdings	SUITLENGTH													
		0	1	2	3	4	5	6	7	8	9	10	11	12	13
10	AKQJ	-	-	-	-	4	5	6	7	8	9	10	11	12	13
9	AKQ	-	-	-	3	3	4	5	6	8	9	10	11	12	-
8	AKJ	-	-	-	2	3	4	5	6	7	8	9	11	12	-
7	AK	-	-	2	2	2	3	4	5	7	8	9	11	-	-
	AQJ	-	-	-	2	2	3	4	5	7	8	9	10	12	-
6	AQ	-	-	1	1	1	2	4	5	6	8	9	10	-	-
	KQJ	-	-	-	2	2	3	4	6	7	8	9	10	11	-
5	AJ	-	-	1	1	1	2	4	5	6	7	9	10	-	-
	KQ	-	-	1	0*	1	2	3	5	6	8	9	10	-	-
4	A	-	1	1	1	1	1	2	4	5	7	8	-	-	-
	KJ	-	-	1	1	1	2	3	4	6	7	8	9	-	-
3	K	-	0	0	0	0	1	2	3	5	6	7	-	-	-
	QJ	-	-	0	0	0	1	2	4	5	7	8	9	-	-
2	Q	-	0	0	0	0	0	2	3	4	6	7	-	-	-
1	J	-	0	0	0	0	0	0	0	0	0	0	-	-	-
0	-	0	0	0	0	0	0	0	0	0	0	-	-	-	-

*This is a bug which has been corrected. The proper value is 1. If the analysis used any value, it was the incorrect 0 .

Appendix 4. Descriptive Names of Each Section of OPENBID

Section	Descriptive Name
1.	Hopeless Pass
2.	Slam in own hand
3.	Strong NT, bid conventional 2D, not 2NT
4.	2NT
5.	Strong suit bid, bid conventional 2D
6.	Strong preempt
7.	1NT
8.	1C, a highly conventional bid
9.	3C, a very special bid
10.	Weak two, S or H only
11.	Weak preempt (club is 4C, others are at 3 level)
12.	Standard Opening: 1S, 1H, 1D, 2C including short 1D
13.	Last chances
14.	Pass, by falling through

Appendix 5. Internal Representations

Bids

The bid of the hand is recorded as the value of an integer variable BIDVAL. A value of 0 indicates pass, and the other possible bids are as shown in the table:

Suit or No Trump

		Clubs	Diamonds	Hearts	Spades	No Trump
	1	1-5[A], 11*	12	13	14	15
L	2	21	6-10[B], 22*	23	24	25
E	3	31	32	33	34	35*
V	4	41	42*	43	44	45*
E	5	51	52	53*	54*	55*
L	6	61	62	63	64	65
	7	71	72	73	74	75

*This value is impossible as an opening bid.

[A] 1 Club is a strong conventional bid in the bidding system being implemented. All of these five values print as 1C. The exact value indicates to later procedures the source of strength, i.e., which suit or no trump.

[B] Same as note A for the very strong conventional bid of 2D.

Suits and No Trump

Clubs	Diamonds	Hearts	Spades	No Trump
1	2	3	4	5

Cards

	Clubs	Diamonds	Hearts	Spades
K	13	26	39	52
Q	12	25	38	51
J	11	24	37	50
10	10	23	36	49
9	9	22	35	48
8	8	21	34	47
7	7	20	33	46
6	6	19	32	45
5	5	18	31	44
4	4	17	30	43
3	3	16	29	42
2	2	15	28	41
A	1	14	27	40

Appendix 6. Distributions and TOTALPTS

If we do not distinguish between the suits, the thirteen cards of a bridge hand may be distributed over the four suits in (only) thirty-nine different ways. Thus, for example, four distinct hands consisting, respectively, of thirteen spades, thirteen hearts, thirteen diamonds or thirteen clubs are all instances of the single distribution, 13 0 0 0. This table lists all such distributions together with the relation between HCP and TOTALPTS for that distribution. Specifically, it gives the minimum and maximum possible changes that can be made to HCP in obtaining TOTALPTS. In symbols, HCP + minimum change ≤TOTALPTS ≤ HCP + maximum change.

This table is obtained by considering all the possible modifications to HCP within the constraint of the given distribution.

Distribution	TOTALPTS	
	minimum change	maximum change*
13 0 0 0	18	18
12 1 0 0	13	17
11 2 0 0	11	14
11 1 1 0	9	16
10 3 0 0	11	12
10 2 1 0	7	13
10 1 1 1	5	15
9 4 0 0	9	10

Distribution	TOTALPTS	
	minimum change	maximum change*
9 3 1 0	7	11
9 2 2 0	5	10
9 2 1 1	3	12
8 5 0 0	8	9
8 4 1 0	5	9
8 3 2 0	5	8
8 3 1 1	3	10
8 2 2 1	1	9
7 6 0 0	8	9
7 5 1 0	4	8
7 4 2 0	3	6
7 4 1 1	1	8
7 3 3 0	5	6
7 3 2 1	1	7
7 2 2 2	-1	6
6 6 1 0	4	8
6 5 2 0	2	5
6 5 1 1	0	7
6 4 3 0	3	4
6 4 2 1	-1	5

Distribution	TOTALPTS	
	minimum change	maximum change*
6 3 3 1	1	5
6 3 2 2	-1	4
5 5 3 0	2	3
5 5 2 1	-2	4
5 4 4 0	1	2
5 4 3 1	-1	3
5 4 2 2	-3	2
5 3 3 2	-1	2
4 4 4 1	-2	2
4 4 3 2	-2	1
4 3 3 3	-1	0

*The entry under maximum change assumes HCP ≤ 15, i.e., ACES ≤ 3. If HCP ≥ 16, the entry would be increased by one for having four aces unless the distribution has a void.

Appendix 7. A Brief Introduction to Bridge and Bidding

Bridge is played by four persons with an ordinary playing deck of 52 cards. Players sitting opposite each other at the table are partners. Each player receives 13 cards. There are two phases to each deal of the cards--the bidding and the play.

The bidding is a type of "conversation" among the four players which precedes the play. One player (the dealer) begins the bidding by making one of a number of possible bids. Each person in turn then bids (in a clockwise order) until there have been three consecutive bids of "pass." The last bid made before the three passes becomes the "contract" and the second phase of the game is a play of the hand using this contract in which one side attempts to make the contract and the other side attempts to defeat it.

What are the legal bids and what do they mean? There are 38 bids in all -- "pass", "double", "redouble" and an ordered set of 35 others consisting of five "suits" at seven levels. The five suits, in descending order, are NoTrump, Spades, Hearts, Diamonds, and Clubs, and the levels are one through seven. The level dominates the suit, i.e., 4 Hearts is higher than 3 Spades, and 3 Diamonds is higher than 3 Clubs. Each of these 35 bids is legal only if it or a higher bid has not previously been made. Double is legal only following an opponent's bid of one of these 35. Redouble is legal only following an opponent's double. Pass is legal at any time.

There are 13 "tricks" to a hand, each consisting of one card from each player's hand. A bid of n of some suit promises to take 6+n tricks, with that suit as the trump suit. Thus, a player may contract to take anywhere from 7 to 13 tricks. A bid of 3 diamonds, for example, promises to take 9 tricks with diamonds as the trump suit. A bid of 6 no trump promises to take 12 tricks with no trump suit.

What is the trump suit? Within any particular suit, the cards have a ranking, with Ace the highest and 2 the lowest. The distinguishing characteristic of a suit trump is that it has greater trick-taking power than any other suit. Any card of the trump suit wins the trick to which it is played provided that no higher trump is played to that trick. Whereas at no trump, the highest card of the suit led wins the trick, at suit play, the highest card of the suit led wins except when one or more trumps are played, when the highest trump wins. The trump suit, therefore, becomes of extreme importance in suit play. At a suit contract, length in the trump suit is of overwhelming importance. Thus, one important bidding consideration is to end up in the suit in which you and your partner have the most cards.

How do you find out each other's strength? Charles Goren, one of the best known bridge players, devised a point count system for bridge hands, which is now the basis for many bidding systems. The point count is as follows:

For a no-trump or a suit contract: (cf. HCP in Appendix 2)

Add 4 points for each ACE.
Add 3 points for each KING.
Add 2 points for each QUEEN.
Add 1 point for each JACK.

In a suit contract: (cf. DISTP in Appendix 2)

Add 3 points if you possess no cards in some suit (void)
Add 2 points if you possess only 1 card (singleton)
Add 1 point if you possess only 2 cards (doubleton)

Numerous modifications in the point count are generally made depending upon your particular hand and upon the distribution--e.g., a singleton queen is not generally valued at 4 points. Other modifications may be made as a result of your partner's bidding.

As far as making an opening bid goes, a general rule of thumb is that you must have 13 points. (It is important to note here that you may make any legal bid with any number of points, but this may confuse your partner as well as the opposition.) While 13 points is standard, this may be reduced to 12 points in many cases, particularly with a rebiddable suit (one which contains five or more cards with considerable strength). The following hand is a typical hand that most bridge players would open with a bid of one heart:

SPADES K 9 7 HEARTS A K 10 4 2 DIAMONDS Q 6 3 CLUBS J 5

Note that there are 13 high card points and 1 distributional point. Bidding generally starts at the lowest level except in the case of unusual hands. An opening bid of two or three of a suit has a special meaning. As the bidding proceeds, you partner's bidding will give you information about his strength (or weakness) and tell you if he has support for your suit, or a good suit of his own, or a very strong hand, etc. When you and your partner possess most of the points, the opponents will most likely pass. When the points are more evenly distributed, there is likely to be bidding by both partnerships, and it will be more difficult for each to arrive at the optimum final contract.

The score received by bidding and making a particular contract can be divided into three major categories:

1) SLAM -- You and your partner have contracted to take 12 or 13 tricks. This usually requires a minimum of 33 points in the combined hands and is relatively infrequent. It is extremely important, however, to bid the slam if at all possible.

2) GAME -- Each trick (above the first 6) has a value. In notrump it is 40 for the first trick and 30 for each succeeding trick, i.e. 1 NT is worth 40 points and 3 NT is worth 100 points. In spades and hearts, the so-called "major" suits, each trick is worth 30 points. In diamonds and clubs, the "minor suits", each trick is worth 20 points. A game bid is any bid worth 100 or more points. 3NT, 4S, 4H, 5C, and 5D are all examples of game bids. An extra bonus is given for making a game bid and then fulfilling the contract, although this is smaller than the slam bonus.

3) SUB GAME or PARTIAL -- scored as above with a smaller bonus for the contract. Any partial contract is inferior to any game contract.

It is very important to get to game or slam if at all possible. It is equally important to realize when game or slam is impossible and stop short of there. For game in a major suit or no trump, a combined holding of 26 points is usually necessary; in a minor suit, this number is increased to 29, since more tricks must be taken. Thus the preferred contracts are in no trump, followed by the majors, followed by the minor suits.

There are a number of bidding systems in use today. Standard American is the term applied to most bidding systems which adhere rather closely to Charles Goren's bidding system. Its greatest weakness seems to be in the wide point count range indicated by the opening bid and the response. An opening bid of 1 heart may indicate anywhere from about 12 to 22 points. A responder who says 1 spade over 1 heart may hold any number of points from about 6 to 18. Thus the partnership may have all 40 points in the deck or may have only 18 or so. Second bids by each player are necessary to clarify the holding. Goren's system does seem to provide a good basis from which to work and many players have modified Goren's system by incorporating some special bids, designed mainly to clarify the opening bid. The system closest to Goren's in this respect is the Schenken system, in which the opening bid of 1 club is reserved for those hands containing a minimum of 17 points. The standard opening bid of one club is replaced for the most part by an opening bid of 2 clubs. In response to this conventional one club opening, partner will respond 1 diamond if he has fewer than 9 points and naturally otherwise. Thus an opening bid of 1 D, 1 H, 1 S, or 2 C indicates a maximum of 16 points by the opener. The combined holding may be more closely ascertained on the first round of bidding.

Bridge, then, is a partnership game, in which you and your partner must combine to communicate accurately the strength of your hands in order to reach an optimal final contract and then play the hand to take the specified number of tricks.