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NUMERICAL SOLUTION OF A CLASS OF NONSTEADY CAVITY FLOW PROBLEMS

by Donald Greenspan

APPENDIX: PROGRAMMING VISCOUS INCOMPRESSIBLE FLOW PROBLEMS

by David Schultz

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1. INTRODUCTION.

In recent years a large variety of viscous flow problems have been studied with the aid of modern, high speed digital computers (see, e.g., references [1] - [9] and the additional references contained therein). Those problems which are time dependent are usually treated by initial-value, or step-ahead, techniques, while those which are steady state are usually treated by boundary value techniques. But because step-ahead methods always contain an accumulation with time of both round-off and truncation errors, while boundary value methods tend to smooth out such errors, we will show in this paper how to apply a boundary value technique to a large class of prototype, time dependent problems.

2. THE ANALYTICAL PROBLEMS.

We will consider two-dimensional, incompressible flow problems in a cavity, which for simplicity will be taken to be a unit square. Analytically we seek to find two functions $\psi(x, y, t)$ and $\omega(x, y, t)$, $0 \le x \le 1$,

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$0 \le y \le 1$, $0 \le t$ such that for a given non-negative number $\Re$ the following are valid:

$$\Delta\psi \equiv \frac{\partial^2\psi}{\partial x^2} + \frac{\partial^2\psi}{\partial y^2} = -\omega\,; \quad 0 < x < 1, \quad 0 < y < 1, \quad 0 < t \tag{2.1}$$

$$\frac{\partial\omega}{\partial t} = \frac{1}{\Re}\,\Delta\omega + \frac{\partial\psi}{\partial x}\,\frac{\partial\omega}{\partial y} - \frac{\partial\psi}{\partial y}\,\frac{\partial\omega}{\partial x}\,; \quad 0 < x < 1, \quad 0 < y < 1, \quad 0 < t \tag{2.2}$$

$$\psi(x, y, 0) = \omega(x, y, 0) = 0\,; \qquad 0 \le x \le 1, \quad 0 \le y \le 1 \tag{2.3}$$

$$\psi(x, 0, t) = \frac{\partial\psi(x, 0, t)}{\partial y} = 0\,; \quad 0 \le x \le 1, \quad 0 \le t \tag{2.4}$$

$$\psi(0, y, t) = \frac{\partial\psi(0, y, t)}{\partial x} = \psi(1, y, t) = \frac{\partial\psi(1, y, t)}{\partial x} = 0; \quad 0 \le y \le 1, \quad 0 \le t \tag{2.5}$$

and

$$\psi(x, 1, t) = 0, \quad \frac{\partial\psi}{\partial y}(x, 1, t) = -1; \quad 0 \le x \le 1\ , \quad 0 \le t\ . \tag{2.6}$$

In initial value problem (2.1) - (2.6), ψ is the stream function, ω is the vorticity and $\Re$ is the Reynolds number. We will consider (2.1) - (2.6) under the assumption that a steady state solution exists and proceed numerically to show how the **nonsteady** flow can be reconstructed from the steady one.

3. THE NUMERICAL METHOD.

In this section, let us describe in complete generality a finite difference method for solving (2.1) - (2.6) numerically on a digital computer. Specific examples will be presented in Section 4.

The first step in the method is to solve numerically with grid size h the steady state problem associated with (2.1) - (2.6), and shown in Figure 3.1, in any of the presently available ways (for small $\mathcal{R}$, see [3] and [5]; for large $\mathcal{R}$, see [5]). This numerical solution will be denoted by $\psi_h(x, y, \infty)$ and $\omega_h(x, y, \infty)$.

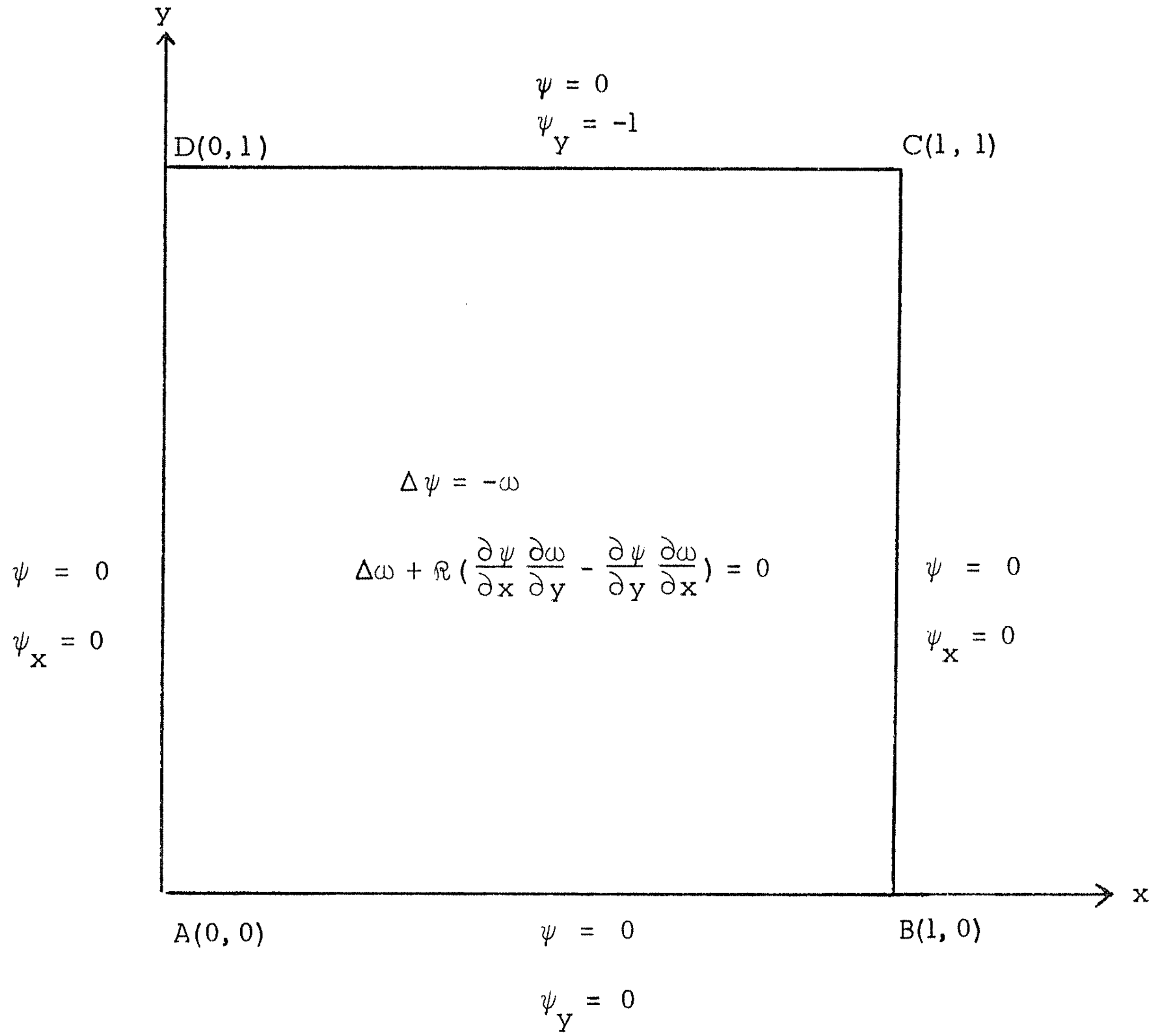

FIGURE 3.1

Next, for fixed $t = T$, subdivide $0 \le t \le T$ into n equal parts, each of length Δt, by means of the points $t_k = k\Delta t$; $k = 0, 1, \ldots, n$, and, as shown in Figure 3.2, let P be the rectangular parallelopiped defined by $P = \{(x, y, t): 0 \le x \le 1, \ 0 \le y \le 1, \ 0 \le t \le T\}$. Define R to be the interior and S to be the boundary of P.

Using space grid size h and time grid size Δt, then construct in the usual way the three dimensional sets of interior grid points $R_{h, \Delta t}$ and boundary grid points $S_{h, \Delta t}$. At the boundary grid points $S_{h, \Delta t}$, set

(3.1) $$\psi(x, y, 0) = 0$$

(3.2) $$\psi(x, y, T) = \psi(x, y, \infty)$$

(3.3) $$\psi(0, y, t) = \psi(1, y, t) = \psi(x, 0, t) = \psi(x, 1, t) = 0 .$$

At each point (x, y, t) of $R_{h, \Delta t}$ define

(3.4) $$\psi^{(0)}(x, y, t) = 0, \quad \omega^{(0)}(x, y, t) = 0 .$$

We will now show how to construct from (3.4) on $R_{h, \Delta t}$ a sequence of discrete functions

(3.5) $$\psi^{(1)}, \psi^{(2)}, \psi^{(3)}, \ldots, \psi^{(k)} ,$$

and on $R_{h, \Delta t} + S_{h,\Delta t}$ a sequence of discrete functions

(3.6) $$\omega^{(1)}, \omega^{(2)}, \omega^{(3)}, \ldots, \omega^{(k)}$$

which will both converge. For this purpose, at each point of $R_{h, \Delta t}$

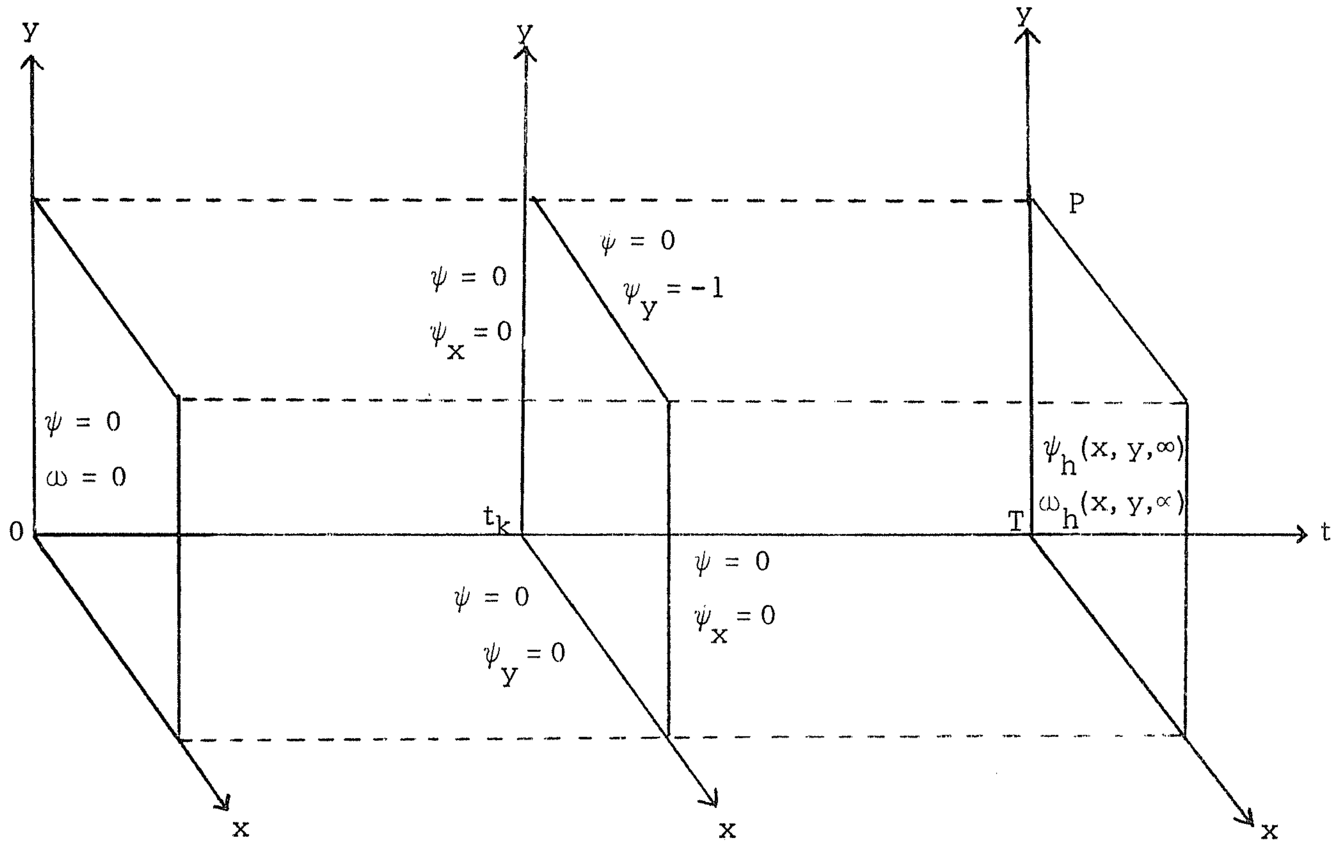

FIGURE 3.2

write down the difference equation

$$(3.7) \qquad -4\psi(x, y, t_k) + \psi(x+h, y, t_k) + \psi(x, y+h, t_k) + \psi(x-h, y, t_k) + \psi(x, y-h, t_k) = -h^2\omega^{(0)}(x, y, t_k), \quad k = 1, 2, \ldots, n-1.$$

Inserting the known values (3.1) - (3.3) wherever possible into (3.7), one then solves the resulting linear algebraic system by successive-over-relaxation with over-relaxation factor r_ψ. If the resulting solution is denoted by $\overline{\psi}^{(1)}$, then on $R_{h,\Delta t}$ the element $\psi^{(1)}$ of (3.5) is defined by the smoothing formula

$$\psi^{(1)} = \rho\psi^{(0)} + (1 - \rho)\overline{\psi}^{(1)}, \qquad 0 \le \rho \le 1. \tag{3.8}$$

Next, at each point of $S_{h,\Delta t}$ set

$$\overline{\omega}^{(1)} = \frac{2\xi}{h} - \frac{2\psi^{(1)}}{h^2}, \tag{3.9}$$

where $\psi^{(1)}$ is evaluated at the point of $R_{h,\Delta t}$ which is nearest to the given point and where

$$\xi = \begin{cases} 1, & \text{if } y = 1 \\ 0, & \text{if } y \ne 1 \end{cases}. \tag{3.10}$$

At each point of $R_{h,\Delta t}$, write down the difference equation

$$\begin{aligned} &\frac{1}{2h}\left[-3\omega(x, y, t-\Delta t) + 4\omega(x, y, t) - \omega(x, y, t+\Delta t)\right] \\ &= \frac{1}{h^2 \mathcal{R}}\left[-4\omega(x, y, t) + \omega(x+h, y, t) + \omega(x, y+h, t)\right. \\ &\left. + \omega(x-h, y, t) + \omega(x, y-h, t)\right] + \alpha \cdot \beta - \gamma \cdot \delta, \end{aligned} \tag{3.11}$$

where

$$\alpha = \frac{\psi^{(1)}(x+h, y, t) - \psi^{(1)}(x-h, y, t)}{2h}$$

$$\gamma = \frac{\psi^{(1)}(x, y+h, t) - \psi^{(1)}(x, y-h, t)}{2h}$$

and

$$\beta = \frac{\omega(x, y+h, t) - \omega(x, y, t)}{h}, \qquad \text{if } \alpha \geq 0$$

$$\beta = \frac{\omega(x, y, t) - \omega(x, y-h, t)}{h}, \qquad \text{if } \alpha < 0$$

$$\delta = \frac{\omega(x, y, t) - \omega(x-h, y, t)}{h}, \qquad \text{if } \gamma \geq 0$$

$$\delta = \frac{\omega(x+h, y, t) - \omega(x, y, t)}{h}, \qquad \text{if } \gamma < 0 .$$

Inserting the values (3.9) wherever possible into (3.11), one then solves the system generated by (3.11) by successive over-relaxation with over-relaxation factor r_ω. The solution is denoted by $\bar{\omega}^{(1)}$. Then, on $R_{h,\Delta t} + S_{h,\Delta t}$ one defines

$$\omega^{(1)} = \mu\omega^{(0)} + (1-\mu)\bar{\omega}^{(1)}, \qquad 0 \leq \mu \leq 1,$$

which completes the construction of the element $\omega^{(1)}$ in sequence (3.6).

One proceeds next to construct $\psi^{(2)}$ from $\psi^{(1)}$ and $\omega^{(1)}$ in the same spirit as $\psi^{(1)}$ was constructed from $\psi^{(0)}$ and $\omega^{(0)}$, and to generate $\omega^{(2)}$ from $\psi^{(2)}$ just as $\omega^{(1)}$ was generated from $\psi^{(1)}$. And in the indicated fashion, the iteration continues until for some preassigned tolerance ε one finds that uniformly

$$|\psi^{(k)} - \psi^{(k+1)}| < \varepsilon$$

$$|\omega^{(k)} - \omega^{(k+1)}| < \varepsilon .$$

The discrete functions $\psi^{(k)}$ and $\omega^{(k)}$ are then taken, on their points of definition, to be numerical approximations of $\psi(x, y, t)$ and $\omega(x, y, t)$, respectively.

4. EXAMPLES.

Various examples for $0 \leq \Re \leq 500$ were run on the CDC 3600 at the University of Wisconsin. Since all behaved similarly, we shall discuss in detail only the case $\Re = 500$, with which other authors have found exceptional difficulties (see, e.g., [3]).

The steady state problem shown in Figure 3.1 was solved by the method in [5] for $h = \frac{1}{20}$. These results are shown graphically in Figures 4.1 and 4.2. Then, with $T = 5$, $\Delta t = \frac{1}{2}$, $r_\psi = 1.8$, $r_\omega = 1.0$, $\rho = .3$, $\mu = .7$ and $\varepsilon = 10^{-3}$, the method of Section 3 converged in 54 iterations, which took 22 minutes of running time. The stream curves $\psi = .09, .07, .05, .03, .01$ and equivorticity curves $\omega = 4, 1.6, 0$ are given for $t = 1, 2, 3, 4$ in Figures 4.3 - 4.10.

It should be observed from this example that one gets excellent results for large time steps, which makes the method most attractive and exceptionally fast.

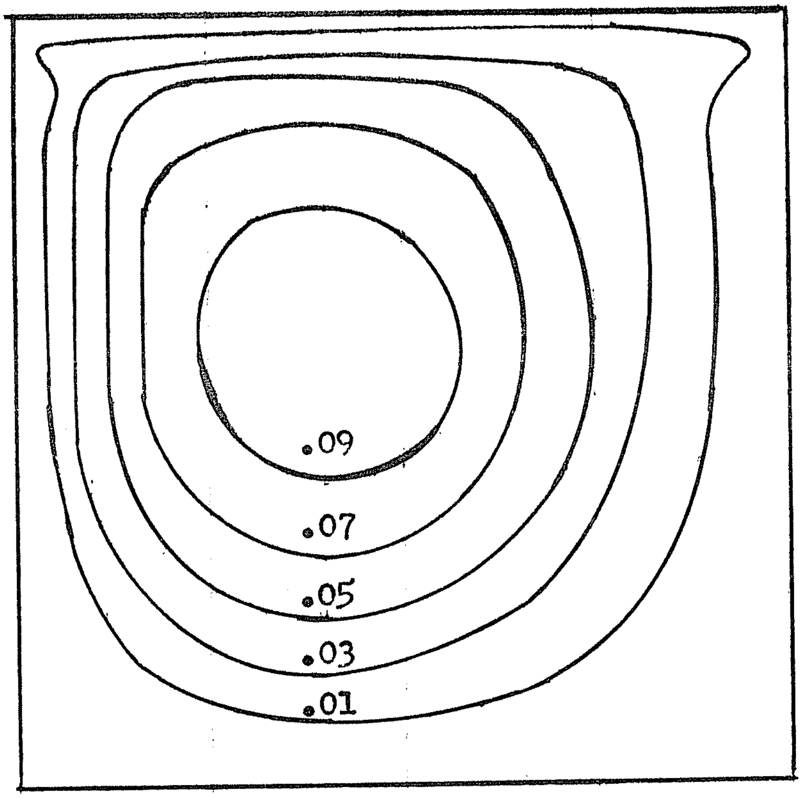

FIGURE 4.1 Steady state stream curves.

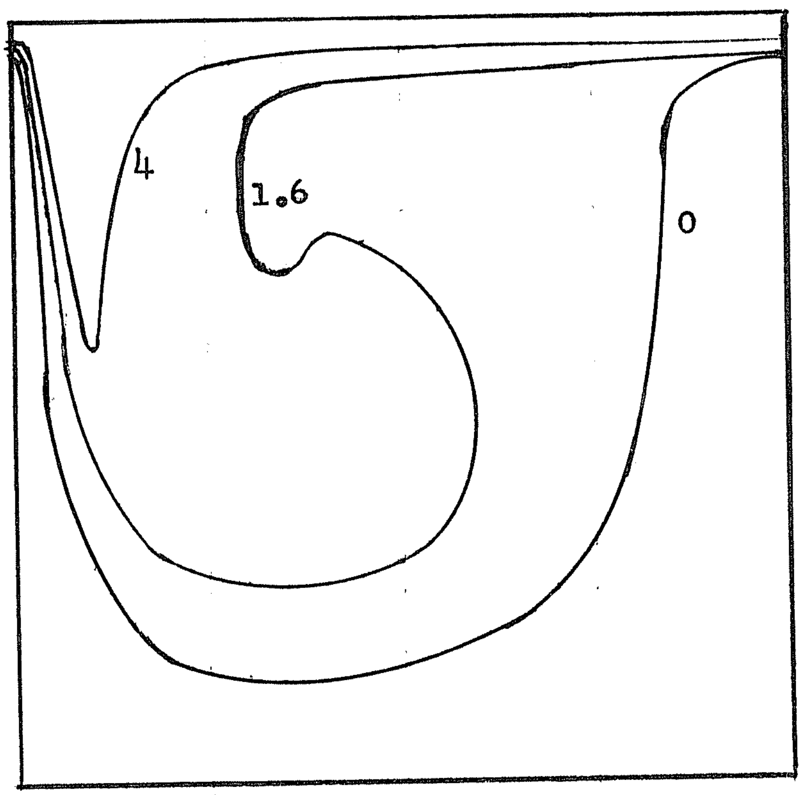

FIGURE 4.2 Steady state equivorticity curves.

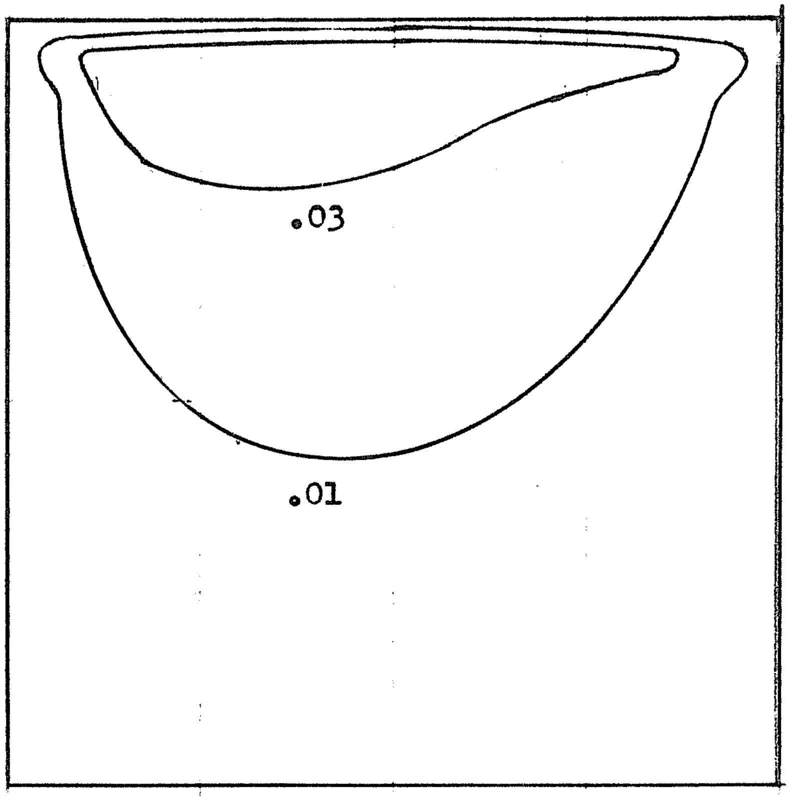

FIGURE 4.3 Streamlines at t = 1.

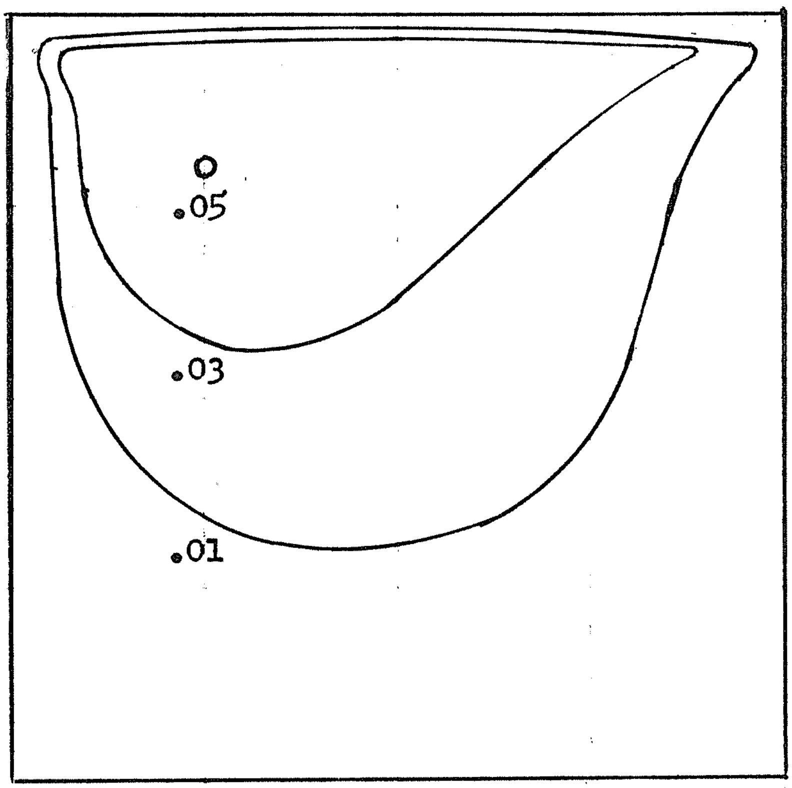

FIGURE 4.4 Streamlines at t = 2.

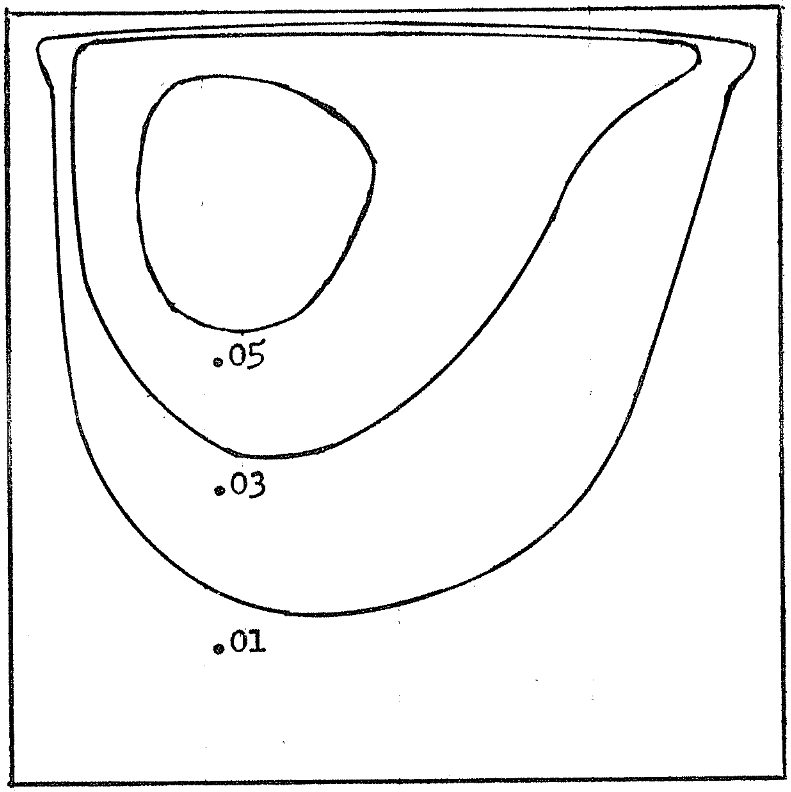

FIGURE 4.5 Streamlines at t = 3.

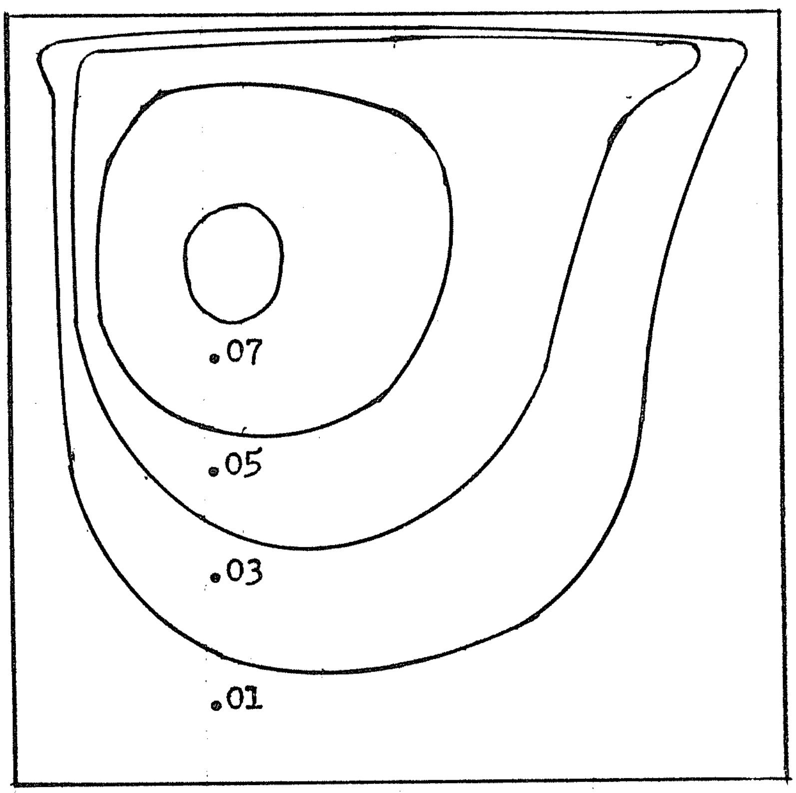

FIGURE 4.6 Streamlines at t = 4.

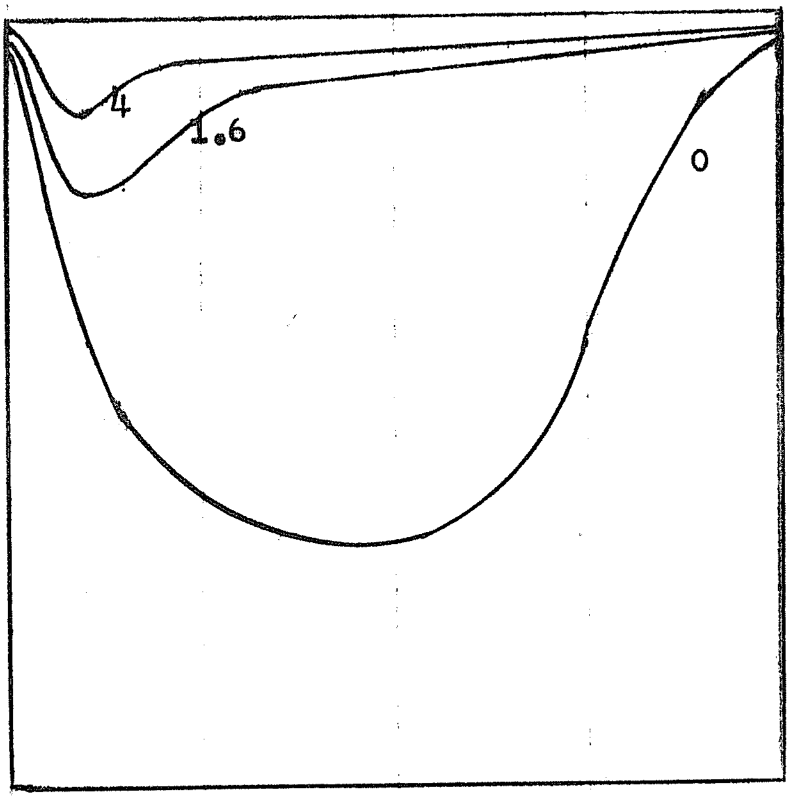

FIGURE 4.7 Equivorticity curves at t = 1.

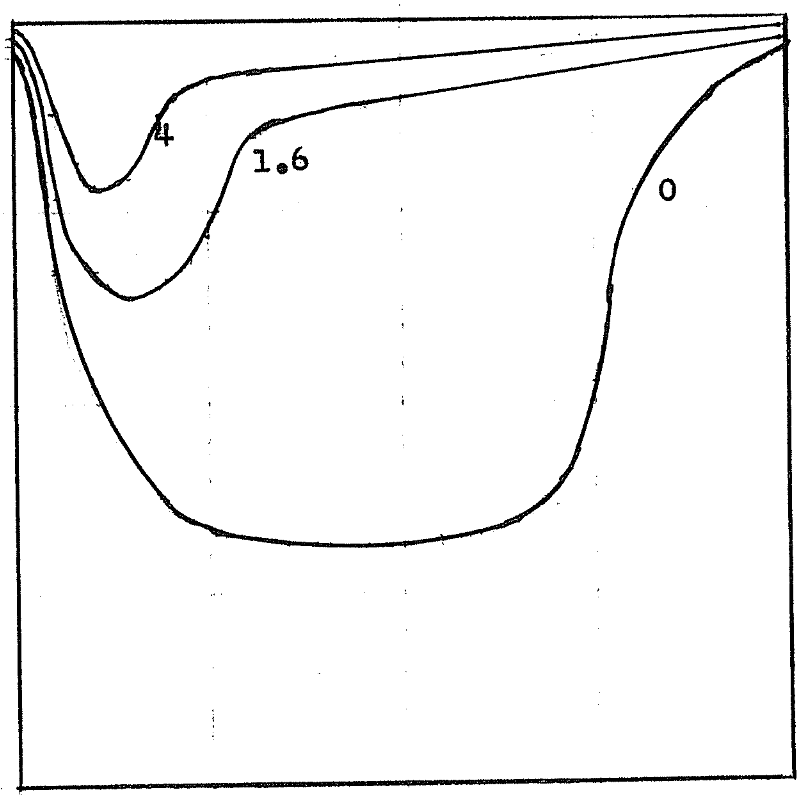

FIGURE 4.8 Equivorticity curves at t = 2.

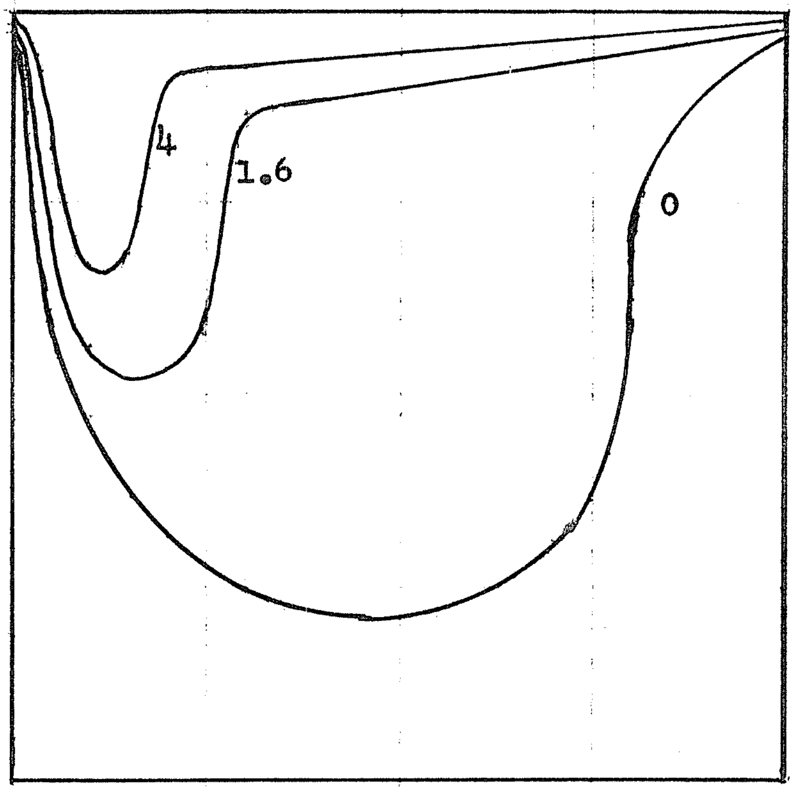

FIGURE 4.9 Equivorticity curves at t = 3.

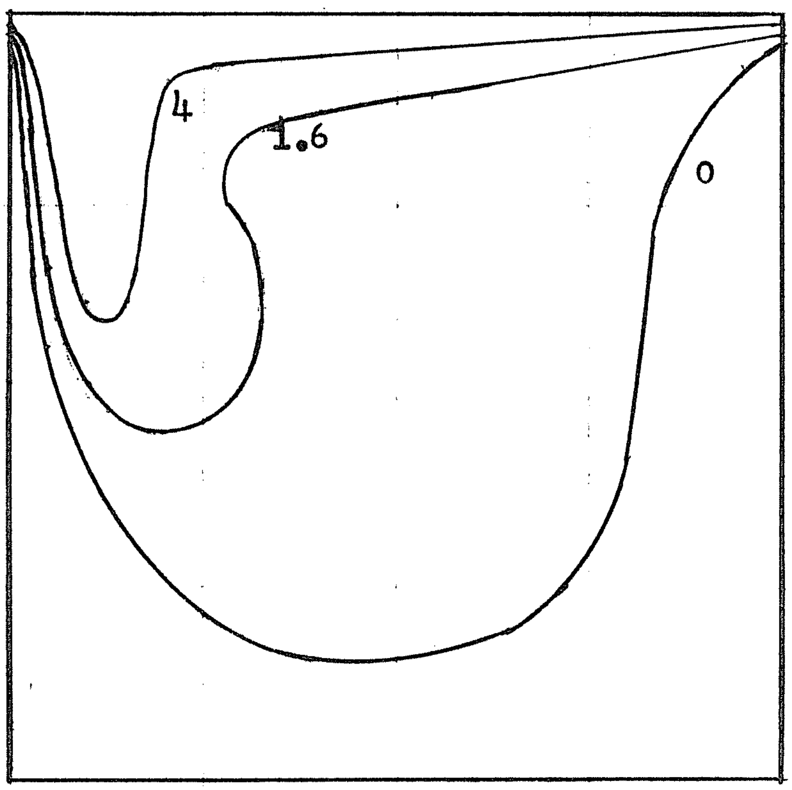

FIGURE 4.10 Equivorticity curves at t= 4.

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APPENDIX

PROGRAMMING VISCOUS, INCOMPRESSIBLE FLOW PROBLEMS

D. SCHULTZ

DEFINITIONS OF PROGRAM VARIABLES

OMA = VORTICITY VALUES

PSI = STREAM VALUES

N = NUMBER OF VERTICAL SPACES IN THE GRID

M = NUMBER OF HORIZONTAL SPACES IN THE GRID

NT = NUMBER OF TIME STEPS

R = REYNOLD'S NUMBER

H = GRID SIZE

EPS = TOLERANCE FOR INNER-AND OUTER-ITERATIONS

C1 = WEIGHTING FACTOR FOR OMA

F1 = WEIGHTING FACTOR FOR PSI

RW = RELAXATION FACTOR FOR OMA EQUATIONS

NM = NUMBER OF OUTER-ITERATIONS

NCOUNT = NUMBER OF INNER-ITERATIONS

W0,W1,W2,W3,W4 = COEFFICIENTS FOR THE VORTICITY EQUATION

ISTOP = SWITCH TO INDICATE CONVERGENCE

```
      PROGRAM NAVSTK
     BANK,(0),NAVSTK,/2/
     BANK,(1),/1/
      COMMON/1/PSI(21,21,21),SVOUT(21,21,21),SVPSI(21,21,21)
      COMMON/2/OMA(21,21,21)
      COMMON N,NPLUS1,M,MPLUS1,NZ,MP,NT,NTPLUS1
      DIMENSION DT(21)
      READ 300,N,M,NT
300  FORMAT(3I2)
      MPLUS1=M+1
      MMESH=M-1
      JM=0
      NPLUS1=N+1
      NMESH=N-1
      H=1./N
      H2=H*H
      DDT=1./2.
     NTPLUS1=NT+1
      NTMESH=NT-1
     DO 818 I=1,NTPLUS1
818   DT(I)=DDT
      EPS=.001
C    INITIALIZE VECTORS
      NZ=0
      KJ=0
      MP=5
      ISTOP=0
      R=500
      C1=.7
      F1=.3
      RW=1.0
104   CONTINUE
      KJ=0
      PRINT 2323,C1
2323  FORMAT(1H1,F8.2)
     DO 1 K=1,NTPLUS1
     DO 1 I=1,NPLUS1
     DO 1 J=1,MPLUS1
     SVOUT(I,J,K)=0
     SVPSI(I,J,K)=0
      PSI(I,J,K)=0
1     OMA(I,J,K)=0
      NM=0
      F2=1-F1
      C2=1-C1
      K=NT+1
C     READ IN STEADY STATE SOLUTION
      NE=0
      NN=7
      DO230 JP=1,NPLUS1,7
      NB=JP
      NE=NE+NN
      IF(NE .GT. NPLUS1)231,232
231   NE=NPLUS1
232   DO230 J=1,MPLUS1
```

```
      L=MPLUS1-J+1
      READ 233,(PSI(I,L,K),I=NB,NE)
230   CONTINUE
233   FORMAT(7F10.6)
      NE=0
      NN=7
      DO240 JP=1,NPLUS1,7
      NB=JP
      NE=NE+NN
      IF(NE .GT. NPLUS1)241,242
241   NE=NPLUS1
242   DO240 J=1,MPLUS1
      L=MPLUS1-J+1
      READ 233,(OMA(I,L,K),I=NB,NE)
240   CONTINUE
C     BEGIN LOOP FOR OUTER ITERATIONS
C   SAVE VORTICITY FUNCTION FROM PREVIOUS OUTER ITERATION
23    DO 40 K=2,NT
      DO 40 I=1,NPLUS1
      DO 40 J=1,MPLUS1
40   SVOUT(I,J,K)=OMA(I,J,K)
      NM=NM+1
C   BEGIN INNER ITERATION FOR STREAM FUNCTION
C     COMPUTE STREAM FUNCTION FOR INNER REGION
     DO 114 K=2,NT
      NCOUNT=0
11    DO 2 I=3,NMESH
      DO 2 J=3,MMESH
     SVPSI(I,J,K)=PSI(I,J,K)
2     PSI(I,J,K)=(-.8*PSI(I,J,K))+.45*(PSI(I,J-1,K)+PSI(I,J+1,K)+PSI(I-
    11,J,K) +PSI(I+1,J,K)+H2*OMA(I,J,K))
C     COMPUTE STREAM FUNCTION ON TOP AND BOTTOM INNER BOUNDARY LINES
      DO 3 I=2,N
     PSI(I,M,K)=.25*PSI(I,MMESH,K)+.5*H
3    PSI(I,2,K)=(.25*PSI(I,3,K))
C     COMPUTE STREAM FUNCTION ON LEFT AND RIGHT INNER BOUNDARY LINES
      DO 4 I=3,MMESH
     PSI(2,I,K)=.25*PSI(3,I,K)
4    PSI(N,I,K)=.25*PSI(N-1,I,K)
C    TEST STREAM FUNCTION FOR CONVERGENCE
      DO 5 I=3,NMESH
      DO 5 J=3,MMESH
     DIFF=ABSF(SVPSI(I,J,K)-PSI(I,J,K))
      IF(DIFF .GT. EPS) GO TO 6
5     CONTINUE
C     RECALCULATE STREAM  FUNCTION USING WEIGHTING
      DO 222 I=3,NMESH
      DO 222 J=3,MMESH
222  PSI(I,J,K)=F1*SVPSI(I,J,K)+F2*PSI(I,J,K)
114    CONTINUE
      GO TO 200
6    NCOUNT=NCOUNT+1
      IF(NCOUNT .GT. 100) GO TO 8
     GO TO 11
C    TEST STREAM FUNCTION FOR DIVERGENCE
```

```
8        IF(DIFF .GT. 10) GO TO 28
         PRINT 93
93       FORMAT(1H1,11H PSI VALUES)
         CALL PRNTLST(PSI)
10       FORMAT(10F11.6)
          NCOUNT=0
        GO TO 11
28        PRINT 81
81       FORMAT(13H PSI DIVERGED)
         CALL PRNTLST(PSI)
         CALL PRNTLST(OMA)
         GO TO 699
C     BEGIN INNER ITERATION FOR VORTICITY
200      NCOUNT=0
30        HCONST=C2*(-2./H2)
C        COMPUTE VORTICITY ON BOUNDARY LINES USING WEIGHTING
C        TOP AND BOTTOM BOUNDARY LINES
        DO 12 K=2,NT
         DO 12 I=1,NPLUS1
        OMA(I,M+1,K)=C1*OMA(I,M+1,K)+HCONST*(PSI(I,M,K)-H)
12      OMA(I,1,K)=C1*OMA(I,1,J)+HCONST*PSI(I,2,K)
C           LEFT AND RIGHT BOUNDARY LINES
         DO 13 K=2,NT
          DO 13 I=2,M
        OMA(1,I,K)=HCONST*PSI(2,I,K)+C1*OMA(1,I,K)
13      OMA(N+1,I,K)=HCONST*PSI(N,I,K)+C1*OMA(N+1,I,K)
90       CONTINUE
C        COMPUTE COEFFICIENTS FOR VORTICITY EQUATIONS
C        COMPLETE ONE SWEEP OF INTERIOR
         DO 14 K=2,NT
         DO 14 I=2,N
         DO 14 J=2,M
        A1=PSI(I+1,J,K)-PSI(I-1,J,K)
        B1=PSI(I,J+1,K)-PSI(I,J-1,K)
         A=ABSF(A1)
         B=ABSF(B1)
         W0=((4*R*H2+8*DT(K))/(2*DT(K)))+(A+B)*(R/2)
         HNEW=(R*H2)/(2*W0*DT(K))
         RNEW=1/W0
          IF(A1.GE. 0)15,16
15        W2=1+(R/2)*A
         W4=1
         GO TO 17
16       W2=1
         W4=1+A*(R/2)
17       IF(B1.GE. 0)18,19
18       W1=1
         W3=1+B*(R/2)
         GO TO 20
19       W1=1+B*(R/2)
         W3=1
20       SVPSI(I,J,K)=OMA(I,J,K)
         IF(ISTOP .EQ. 1)GO TO 305
        OMA(I,J,K)=      (RNEW*(W1*OMA(I+1,J,K)+W2*OMA(I,J+1,K)+W3*OMA(I-1,
       1J,K)+W4*OMA(I,J-1,K)) +HNEW*(3*OMA(I,J,K-1)+OMA(I,J,K+1)))*RW+(1-
```

```
      2RW)*OMA(I,J,K)
        GO TO 14
C       CHECK TO SEE IF DIFFERENCE EQUATIONS ARE SATISFIED TO EPS1
 305         DIFF=      (RNEW*(W1*OMA(I+1,J,K)+W2*OMA(I,J+1,K)+W3*OMA(I-1,
      1J,K)+W4*OMA(I,J-1,K)) +HNEW*(3*OMA(I,J,K-1)+OMA(I,J,K+1)))-OMA(I,J
      2,K)
        DIF=ABSF(DIFF)
        IF(DIF .GT. EPS1)282,14
 282    PRINT 183,I,J,K
        GO TO 700
 14     CONTINUE
        IF (ISTOP .EQ. 1) GO TO 700
C     TEST   VORTICITY     FOR CONVERGENCE
       DO 21 K=2,NT
        DO 21 I=2,N
       DO 21 J=2,M
       DIFF=ABSF(SVPSI(I,J,K)-OMA(I,J,K))
        IF(DIFF .GE. EPS) GO TO 22
 21    CONTINUE
C       RECALCULATE VORTICITY        USING WEIGHTING
       DO 144 K=2,NT
        DO 144 I=2,N
        DO 144 J=2,M
 144   OMA(I,J,K)=C1*SVPSI(I,J,K)+C2*OMA(I,J,K)
        JM=JM+1
C       PRINT OUT EVERY 2 OUTER ITERATES
        IF(JM .EQ. 2)89,59
 89     JM=0
        PRINT 79,NM
 79     FORMAT(1H1,I2,17H OUTER ITERATIONS)
        PRINT 91
        CALL PRNTLST(PSI)
        PRINT 92
        CALL PRNTLST(OMA)
C     TEST OUTER ITERATIONS FOR CONVERGENCE
 59     CONTINUE
       DO 45 K=2,NT
        DO 45 I=1,NPLUS1
        DO 45 J=1,MPLUS1
       DIFF=ABSF(SVOUT(I,J,K)-OMA(I,J,K))
        IF(DIFF .GT. EPS) GO TO 7
 45      CONTINUE
        NZ=1
         PRINT 99,NM
 99     FORMAT(1H1,22H PROBLEM CONVERGED IN ,I4)
        PRINT 91
 91     FORMAT(1X,11H PSI VALUES)
        CALL PRNTLST(PSI)
        PRINT 92
 92     FORMAT(1H1,14H OMEGA  VALUES)
        CALL PRNTLST(OMA)
        EPS1=.001
        RMAX=0
        ISTOP=1
C       CHECK TO SEE IF DIFFERENCE EQUATIONS FOR STREAM FUNCTION ARE SATISFIED TO
```

```
C       A TOLERANCE OF EPS1
        DO 181 KK=2,NT
         DO 181 II=3,NMESH
         DO 181 JJ=3,MMESH
        A=-4*PSI(II,JJ,KK)+PSI(II+1,JJ,KK)+PSI(II,JJ+1,KK)+PSI(II-1,JJ,KK)++
      1+PSI(II,JJ-1,KK)
        B=-H*H*OMA(II,JJ,KK)
         D=A-B
         IF(D .GT. EPS1) GO TO 182
181       CONTINUE
         GO TO 90
182     PRINT 183,II,JJ,KK
183      FORMAT(1H1,41H DIFFERENCE EQU. NOT SATISFIED AT POINT (,I2,1H,,I2
      1,1H,,I2,1H))
          GO TO 699
C      TEST OUTER ITERATIONS FOR DIVERGENCE
7        IF(DIFF .GT. 100)199,23
22       NCOUNT=NCOUNT+1
         IF(NCOUNT .GT. 300) GO TO 24
         GO TO 90
C      TEST   VORTICITY     FOR DIVERGENCE
24       IF(DIFF .GT. 10) GO TO 29
         PRINT 94
94       FORMAT(1H1,14H OMAEGA VALUES)
         CALL PRNTLST(OMA)
         PRINT 91
         CALL PRNTLST(PSI)
32       FORMAT(10F11.6)
         NCOUNT=0
         GO TO 90
29       PRINT 82
82        FORMAT(13H OMA DIVERGED)
         CALL PRNTLST(PSI)
         CALL PRNTLST(OMA)
         GO TO 699
199      PRINT 189
189      FORMAT(26H OUTER ITERATIONS DIVERGED)
699      CONTINUE
700       CONTINUE
         PRINT 303,RMAX
303      FORMAT(1H1,17H PSI CONVERGED TO,E12.4)
         END
```