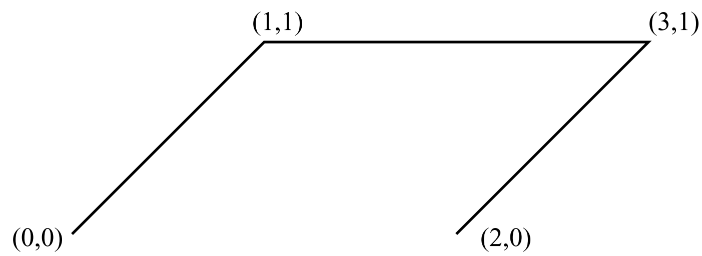
CS 559: Computer Graphics

Homework 7

This homework must be done individually. Submission date is Thursday, May 6, 2004, in class.

Question 1:

Use de-Casteljau's algorithm to find the point corresponding to $t = \frac{1}{3}$ on the Bezier curve defined by the given control points. Show your construction.



Question 2:

A Bezier curve will be used to represent a straight line of length 1. The first control point, $\mathbf{x}_0$, is at (0,0,0).

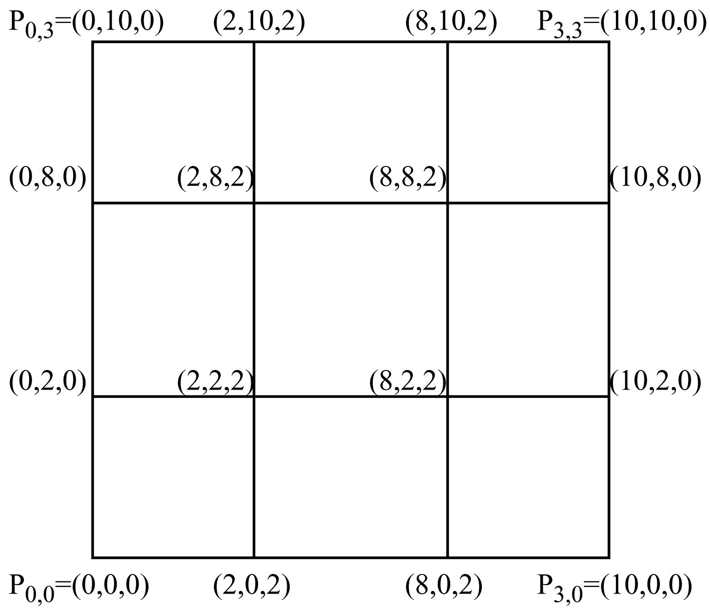
a. The line is to point along the y-axis. Where should the final control point, $\mathbf{x}_3$, be located?

b. Say we want the magnitude of the parametric derivative of the curve to equal 1 at both the start and end of the curve. Where should we place the other two control points, $\mathbf{x}_1$ and $\mathbf{x}_2$?

c. Show that the magnitude of the parametric derivative is always 1 for the curve you have created.

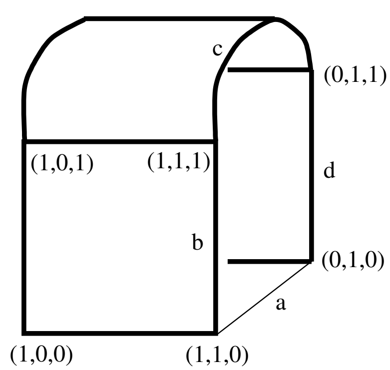
Question 3:

Compute the normal vector to the Bezier patch with control points given below, at the parameter values (0.25,0.25). Also give the tangent vectors you compute on the way to the normal vector. You might like to write a program to do it.



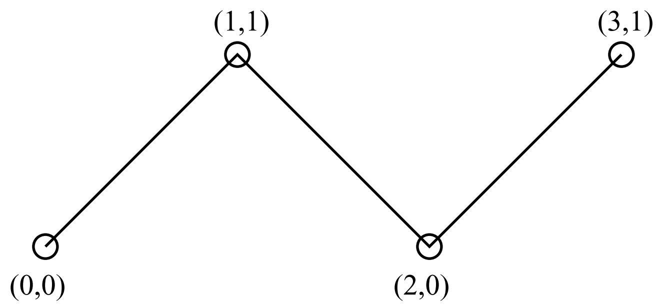
Question 4:

Give a set of 16 control points that will put a single Bezier patch on top of two parallel squares with G^1 continuity across the edges between the squares and the patch, as sketched below. It is best to draw the control mesh of the patch and label the vertices with their coordinates, as was done in Question 3.



Question 5:

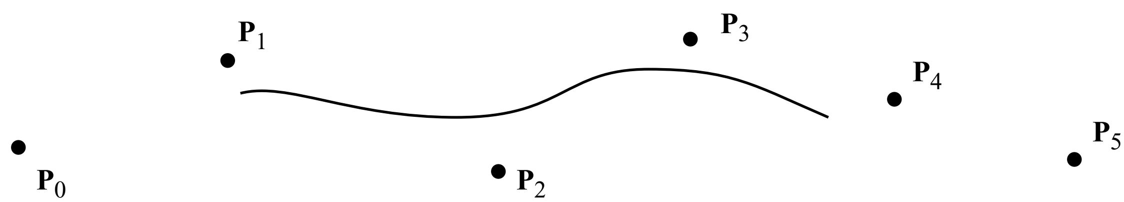
Refine the B-spline curve below by one level, creating 5 new control points. Give the locations of the new points.



Question 6:

B-spline curves have the property that at integral values of the parameter ($t = 0$, $t = 1$, $t = 2$ etc.) the curve is determined by only three control points. Similarly, the derivative is determined by only two control points.

a. Consider the piece of curve with control points labeled as below. At $t = 2$, which three control points define the curve? What is the equation for the point $\mathbf{x}(2)$ in terms of the control points?



b. Show that the curve interpolates the control points at $t = 2$ if $\mathbf{P}_2 = \mathbf{P}_3 = \mathbf{P}_4$ (a triple repeated control point).

c. Compute the parametric derivative of the curve at the parameter value $t = 1$, expressed in terms of the control points.

d. Show that the curve is tangent to the line joining $\mathbf{P}_1$ and $\mathbf{P}_2$ at parameter value $t = 1$ if $\mathbf{P}_2 = \mathbf{P}_3$ (a double control point). This requires showing something about both the point and the derivative at $t = 1$.