CS 559: Computer Graphics

Homework 3

This homework must be done individually. Submission date is Tuesday, October 12, 2004, in class.

Question 1:

The 2D edge-detect (high-pass) filter that we looked at in class has the following form if you ignore the constant:

-1	-2	-1
-2	12	-2
-1	-2	-1

a. What is the output if you filter the image below? Only apply the filter at places where all the underlying pixels exist, so you end up with a 4×1 result.

1	1	1	0	0	0
1	1	1	0	0	0
1	1	1	0	0	0

b. What is the output if you filter the image below (giving a 6×1 output image)?

1	1	1	1	1	0	0	0
1	1	1	1	0	0	0	0
1	1	1	0	0	0	0	0

c. Which type of edge, diagonal or vertical, produces higher values with this filter?

d. An alternate edge-detect filter is given below. What is its output on each of the above images (you should have a 4×1 and a 6×1 answer)?

-1	2	-1
-1	2	-1
-1	2	-1

e. What type of edge gives higher values ofr this filter?

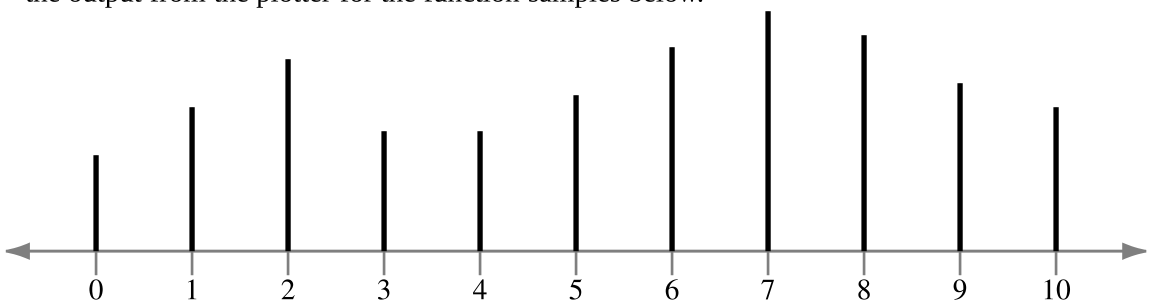
f. Design a 3×3 filter that responds to diagonal edges like that of part (b), but gives no response to a vertical edge.

Question 2:

Say that we point sample a band-limited function into a 1D "image", by taking the value of the function at the points $x = \{0, 1, \ldots, 9, 10\}$. In class, we saw that this corresponds to convolution with a sequence of spikes in the frequency domain. We also saw that the function can be reconstructed perfectly by multiplying with a box filter in the frequency domain.

a. What sort of function do we need to convolve with in the spatial domain to perform perfect reconstruction?

b. Say that a function plotter works by taking the 1D image and drawing the top edge of a narrow box for each sample - each box as wide as the original sample spacing and centered on the sample. Draw the output from the plotter for the function samples below.



c. What type of function is the plotter convolving our samples with in the spatial domain?

d. What filter function is the plotter multiplying by in the frequency domain?

e. What additional frequencies (high or low) will appear in the plotted image, and how do they visually manifest themselves?

Question 3:

You wish to use compositing operations to perform a stencil operation. You have a foreground image, f, that you wish to place into a background image, b, only at places where a stencil mask, s, has a particular α value. For example, of the foreground image is all white with $\alpha = 1$, the background is all red with $\alpha = 1$ and the stencil has holes for a word, inserting the foreground into the background would result in a white word on a red background.

a. Which α value would you use for the parts of the stencil that represent holes? Which value would you use for the rest? (There are two good answers to this question.)

b. You plan to use two compositing operations to combine the images, with the form $(f\ \mathbf{op}_1\ s)\ \mathbf{op}_2\ b$, where brackets indicate precedence. Which compositing operations would you use for $\mathbf{op}_1$ and $\mathbf{op}_2$?

Question 4:

It takes three points to define an affine transformation in 2D. Say that the point $(1, 1)$ goes to $(-\frac{4}{\sqrt{2}}, \frac{2}{\sqrt{2}})$, that $(1, 0)$ goes to $(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})$, and that the point $(0, 0)$ goes to $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. Assume that the affine transformation is described by the following homogeneous matrix equation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} & b_x \\ a_{yx} & a_{yy} & b_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

a. Draw one sketch showing the locations of the points before the transformation, and one showing their locations after. Join the points to form a triangle in each figure.

b. Write out six linear equations involving the unknowns in the matrix equation above and the coordinates of the given points.

c. Solve the equations to find the unknowns and hence write out the transformation matrix.

d. Determine from your sketch a sequence of rotations, scalings or translations required for the transformation.

e. Write out the transformation matrices for your sequence in part (d) and compose them to verify your answer to part (c).