CS 559: Computer Graphics

Homework 3

This homework must be done individually. Submission date is Tuesday, Oct 14 in class. It will be graded immediately and made available to you before the midterm. It is a long homework, but it should form an essential part of your study for the midterm.

Question 1:

The 2D edge-detect (high-pass) filter that we looked at in class has the following form if you ignore the constant:

```
-1   -2   -1
-2   12   -2
-1   -2   -1
```

a. What is the output if you filter the image below? Only apply the filter at places where all the underlying pixels exist, so you end up with a 4×1 result.

```
1   1   1   0   0   0
1   1   1   0   0   0
1   1   1   0   0   0
```

b. What is the output if you filter the image below (giving a 6×1 output image)?

```
1   1   1   1   1   0   0   0
1   1   1   1   0   0   0   0
1   1   1   0   0   0   0   0
```

c. Does the filter respond more to a diagonal or vertical edge?

d. An alternate edge-detect filter is given below. What is its output on each of the above images (you should have a 4×1 and a 6×1 answer)?

```
-1   2   -1
-1   2   -1
-1   2   -1
```

e. Does this filter respond more to vertical or diagonal edges?

f. Design a 3×3 filter that responds to diagonal edges like that of part (b), but gives no response to a vertical edge.

Question 2:

Say that we point sample a band-limited function into a 1D "image", by taking the value of the function at the points $x = \{0, 1, \ldots, 9, 10\}$. In class, we saw that this corresponds to convolution with a sequence of spikes in the frequency domain. We also saw that the function can be reconstructed perfectly by multiplying with a box filter in the frequency domain.

    a. What sort of function do we need to convolve with in the spatial domain to perform perfect reconstruction?

    b. Say that a function plotter works by taking the 1D image and drawing a narrow box for each sample - each box as wide as the original sample spacing. Draw the output from the plotter for the function samples below.
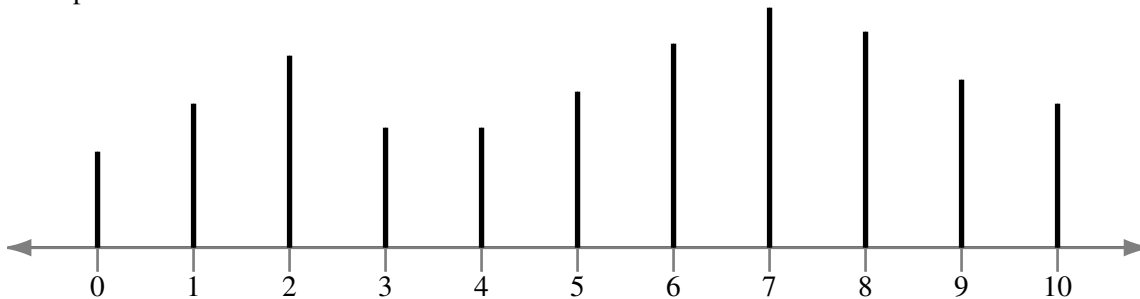
    c. What type of function is the plotter convolving our samples with in the spatial domain?

    d. What filter function is the plotter multiplying by in the frequency domain?

    e. What additional frequencies (high or low) will appear in the plotted image, and how do they visually manifest themselves?

Question 3:

Say you begin with a background image, b, and a foreground image, f. Consider corresponding pixels of b and f, with pre-multiplied colors and alphas (c_b, α_b) and (c_f, α_f) respectively. In this question, assume the $\alpha_f > 0$, and consider the foreground to be a brush that we repeatedly dab onto the background, as might be done in a paint program.

    a. What is the resulting pre-multiplied color and α if you compose f *over* b (one dab of the brush)?

    b. What is the resulting pre-multiplied color and α if you dab the brush a second time? In other words, what color and α result from f *over* (f *over* b)?

    c. What is the limit color and α if we dab the brush an infinite number of times? It helps to know that, for $0 < r \leq 1$:

$$\lim_{n \to \infty} a + a(1 - r) + a(1 - r)^2 + a(1 - r)^3 \ldots = \frac{a}{r}$$

Question 4

A pair of transformations is said to commute if the order in which you apply them does not matter. In terms of transformation matrices, that means that $\mathbf{AB} = \mathbf{BA}$. Consider three rotation matrices and a translation matrix:

$$\mathbf{R}_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_3 & -s_3 & 0 \\ 0 & s_3 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a. What axis is $\mathbf{R}_1$ rotating about?

b. What axis are $\mathbf{R}_2$ and $\mathbf{R}_3$ rotating about?

c. Do $\mathbf{R}_1$ and $\mathbf{R}_2$ commute?

d. Do $\mathbf{R}_2$ and $\mathbf{R}_3$ commute?

e. Under what circumstances do two rotation matrices commute?

f. Do $\mathbf{R}_1$ and $\mathbf{T}$ commute?

g. Do $\mathbf{R}_2$ and $\mathbf{T}$ commute?

h. Under what circumstances can you say that a rotation and a translation commute?

Question 5:

Conventional graphics assumes that the person looking at the screen is seated directly in front of the screen and is focused at the center of the screen. However, for virtual reality applications you want the world to really appear to be behind the screen. This requires tracking the user's head and presenting an image that matches the arrangement of their eyes and the screen. In particular, the near clipping plane should coincide with the window, and the viewer's virtual eyes should coincide with their real location.

To make all this work, you need a global coordinate system that is the same for the real world and the virtual world. In the set-up below, the origin of the virtual world (and real world) is behind the screen. The center of the screen is at (10, 0, 0), the "right" on the screen is in the direction (0, 1, 0) and "up" on the screen is in the direction (0, 0, 1). The screen has width 60 and height 40. The viewer's eye is at (50, 30, 20).

Use the notation from class (and in the textbook). Note that in this instance the gaze direction is **not** the direction the viewer is looking, rather it is the vector looking into the world perpendicular to the view plane. In this situation, in fact, it doesn't matter which direction the viewer is looking.

(50,30,20)

40

(0,0,0)   (10,0,0)

60
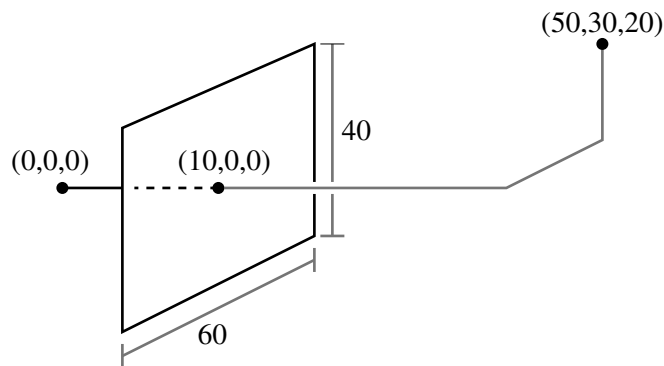
   a. What is the vector e?

   b. What is the vector w?

   c. What is the vector u?

   d. What is the vector v?

   e. What is the near clip plane distance?

   f. What are l, r, b, t that define the left, right, top and bottom clip planes?

Question 6:

Perform Sutherland Hodgman clipping on the figure below to the rectangular clip region shown dashed. Show the intermediate results after clipping with the top edge, the results after clipping with the top and right edges, the results after the top, right and bottom edges, and the final results. (You should show 4 figures in all.)
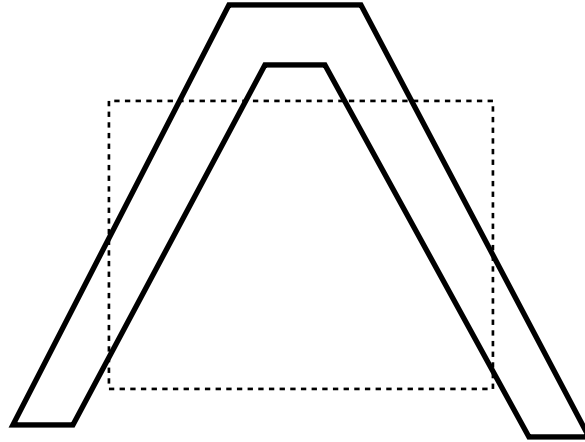
Question 7:

This question explores Liang-Barsky clipping. Consider the line segment and semi-infinite clip region shown below.
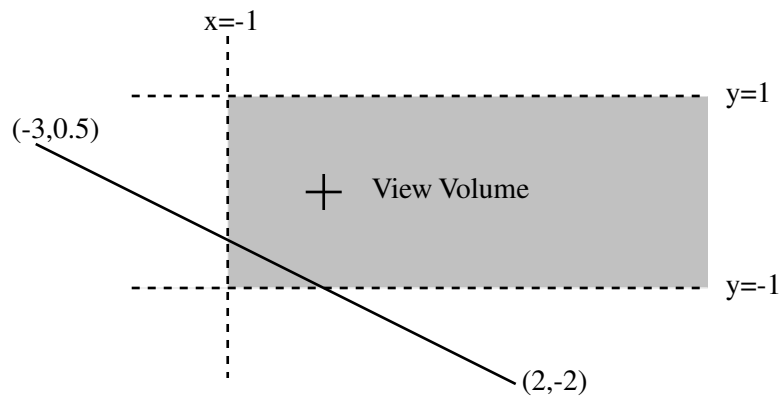
a. What is the parametric equation of the line? Write it in the form:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} + t \begin{bmatrix} c \\ d \end{bmatrix}$$

b. What are the parametric coordinates (the t values) for the intersections of the line with each clip edge? Label them as entering or leaving intersections.

c. What are the parametric coordinates of the endpoints of the visible segment?

d. What are the (x,y) coordinates of the endpoints of the visible segment?