CS 559: Computer Graphics

Homework 2

This homework must be done individually. Submission date is Tuesday, October 1, 2002, in class.

Question 1:

The $L^*u^*v^*$ space (defined below) is approximately perceptually uniform. Hence, one way to decide whether two pairs of colors in RGB space, say a, b and c, d, are separated by the same perceptual distance is to first convert all the colors into LUV space then compute their relative distances there using a standard distance metric. To get from RGB to XYZ, use the following matrix:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0.4124 & 0.3576 & 0.1805 \\ 0.2126 & 0.7151 & 0.0721 \\ 0.0193 & 0.1192 & 0.9505 \end{bmatrix} \begin{bmatrix} r \\ g \\ b \end{bmatrix} \tag{1}$$

LUV coordinates, (L^*, u^*, v^*) are computed in several steps. First compute (X_n, Y_n, Z_n) which are the XYZ coordinates of white. Then compute the following four values:

$$u' = \frac{4X}{X + 15Y + 3Z} \tag{2}$$

$$v' = \frac{9Y}{X + 15Y + 3Z} \tag{3}$$

$$u'_n = \frac{4X_n}{X_n + 15Y_n + 3Z_n} \tag{4}$$

$$v'_n = \frac{9Y_n}{X_n + 15Y_n + 3Z_n} \tag{5}$$

Finally, compute:

$$L^* = 116\left(\frac{Y}{Y_n}\right)^{\frac{1}{3}} - 16 \tag{6}$$

$$u^* = 13L^*(u' - u'_n) \tag{7}$$

$$v^* = 13L^*(v' - v'_n) \tag{8}$$

When $Y/Y_n < 0.01$, $L^* = 903.3Y/Y_n$, rather than the equation above. Note that when $r = g = b$, $u' = u'_n$ and $v' = v'_n$ and hence $u^* = v^* = 0$. In other words, the L^* component of $L^*u^*v^*$ encodes intensity. You might like to write a program to do the color conversions, and test it before trying to answer the questions (testing on white, black and grey is a good start.)

Continued ...

Colors are frequently linearly interpolated in graphics, as we will see later with *Gourand* shading. That is, to obtain colors between (r_1, g_1, b_1) and (r_2, g_2, b_2), we use the formula

$$\begin{aligned} r &= tr_1 + (1-t)r_2 \\ g &= tg_1 + (1-t)g_2 \\ b &= tb_1 + (1-t)b_2 \end{aligned}$$

where t varies from 0 to 1.

a. Linearly interpolate from (0.5, 0, 0) to (1, 1, 1) in 5 steps. That is, evaluate the formula above for $t \in \{0, 0.25, 0.5, 0.75, 1\}$.

b. Convert each of the colors you obtained above into (L^*, u^*, v^*).

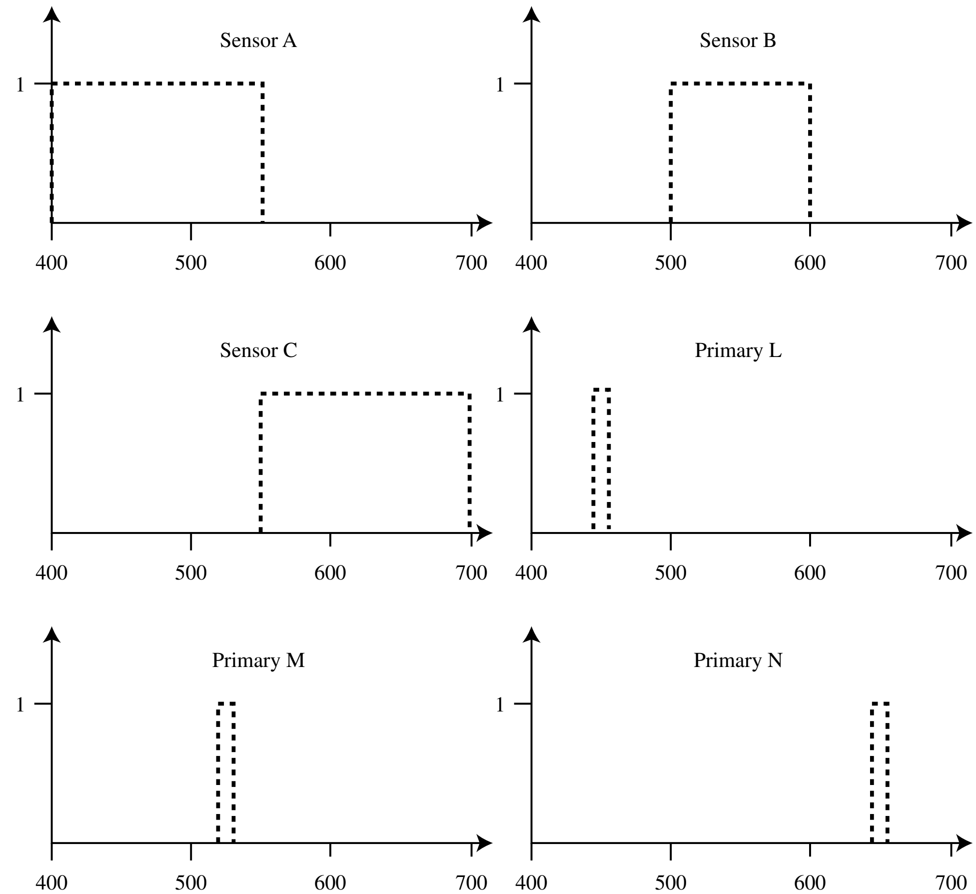
c. Plot three graphs: one showing L^* as a function of t, one showing u^* and one showing v^*.

d. Does linear interpolation in (r, g, b) give a linear interpolation when converted to (L^*, u^*, v^*)?

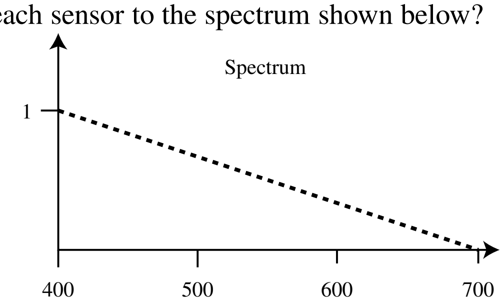
To interpolate in a perceptually linear manner, we would interpolate in (L^*, u^*, v^*) space and then convert back into RGB. This is not done for speed reasons.

Question 2:

Consider the three sensors, A, B and C, shown below, and the three primaries, L, M, and N. Sensor A has a response of 1 between 400nm and 550nm, Sensor B responds between 500nm and 600nm, and Sensor C responds between 550nm and 700nm. Primary L emits energy between 445nm and 455nm. Primary M emits between 520nm and 530nm. Primary L emits between 645nm and 655nm.



a. What is the response of each sensor to the spectrum shown below?



b. How much of each primary would be required to simulataneously generate a response from each sensor that is the same as their response to the spectrum above? (In other words, how much of each primary is required to match the spectrum response?)

Question 3:

Show the results of running Floyd-Steinberg, as presented in class, on a 4x2 image (4 columns, 2 rows) with a constant 0.5 grey intensity. Use the horizontal zig-zag pattern, and if there is no pixel to receive a part of the error, just ignore that term.