

TRANSFORMATIONS

Went through Line Drawing too fast

- it's in the book (The F+D chapter in reader)
- it's in Shirley
- i'll write up my notes

We'll come back to triangle drawing later

Why rush ahead -

- WANT TO GET TO OpenGL so you can do project
- WANT TO GET TO 3D so you can do project
- WANT TO ASSIGN A PROJECT USING 3D, TRANSFORMATIONS AND CURVES FOR PROJECT 2

GAME PLAN:

- RUSH THROUGH TRANSFORMATIONS : 3D quickly
- Get TO CURVES FOR PROJECT
- Go BACK TO IMPORTANT STUFF GLOSSED OVER

TODAY: Co-ORDINATE SYSTEMS (in 20) } Shirley Ch 4
 Transformations (in 20) }
 GL BASICS (in 20) } OGL

MONDAY: "EXTRA SESSION PLANNING"

TUESDAY: PROJECT SIGNUPS
 Co-ORDINATE SYSTEMS (in 30) } Shirley 4, OGL
 Projection } Shirley 5
 HOMOGENEOUS TRANSFORMS }

WEDS: SPIKE
THURS: 3D SURVIVAL

LAST TIME :

DRAW A POINT @ X, Y

LINE @ $X_1, Y_1 \quad X_2, Y_2$

....

What do these things mean?

NOT MUCH WITHOUT A CO-ORDINATE SYSTEM

CO-ORDINATE SYSTEM TELLS US HOW TO MAP sets of numbers to locations in our space

TODAY "Space" is 2D - easier to draw

What do we need for a Co-ordinate System?

- ① origin + where is 0
 - ② directions and scales to measure in
 - ↑ what happens as we change values
- point + 2 vectors

How TO INTERPRET A POINT X, Y

Start at Origin

add in x of vector 1, y of vector 2

Properties :

Redundancy - 2 vectors are enough if not parallel

Span the space

Relationships Between Co-ordinate Systems (3)

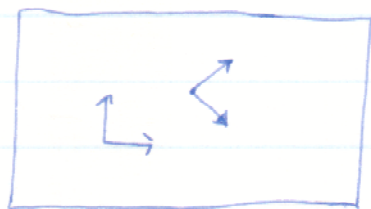
How TO SPECIFY WHERE ORIGIN IS?

ABSOLUTE POSITION

EXTERNAL REFERENCES

"WORLD" OR CANONICAL SYSTEMS

ONCE WE HAVE 1 CO-ORDINATE SYSTEM
WE CAN DEFINE OTHERS!



POINT IN SEVERAL CO-ORDINATE SYSTEMS

CAN MOVE POINTS BY MOVING CO-ORDINATE SYSTEM
PIECE OF GLASS / RUBBER SHEET

SWITCHING CO-ORDINATE SYSTEMS CHANGES POINTS

TALK ABOUT TRANSFORMING POINTS

→ MOVE C-SYSTEM TO CHANGE OBJECTS IN THEM

- different control knobs

- grouping

TRANSFORMATION

something that changes points

$$x', y' = f(x, y) \quad f \in \mathbb{R}^2 \rightarrow \mathbb{R}^2$$

EXAMPLES

$$f(x, y) = x + 2, y + 3$$

$$f(x, y) = -y, +x$$

$$f(x, y) = x^2, y$$

Interpretation as a new (wired) co-ordinate system
What happens to "graph paper"

Composition

$$f(g(h(x_j)))$$

Switch to Vector NOTATION



note old

LINEAR TRANSFORMATIONS

CRITICAL SPECIAL CASE!

$$Mx \quad \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{matrix} x \\ y \end{matrix}$$

↑ post-multiply convention
like function application convention

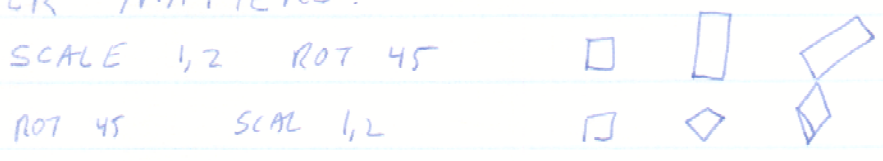
What can you do?

scale $\begin{bmatrix} s & 0 \\ 0 & s \end{bmatrix} \quad \begin{bmatrix} s & 0 \\ 0 & t \end{bmatrix}$

rotate $\begin{bmatrix} c & -s \\ s & c \end{bmatrix}$

skew $\begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix}$

ORDER MATTERS!



Composition by MATRIX MULTIPLY

ANALYSIS w/ LINEAR ALGEBRA

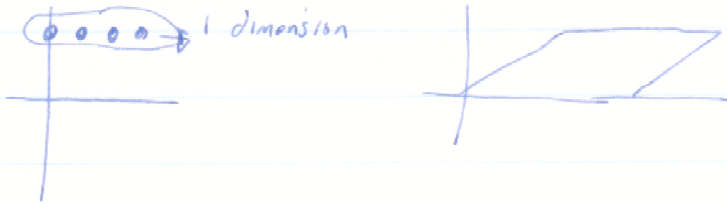
What ABOUT TRANSLATE?

LINEAR OPERATORS PRESERVE 0

6

BIG IDEA: HOMOGENEOUS CO-ORDINATES

TRANSLATION IN N -dimensions = Shear in $N+1$



Idea - take your points, move them away from origin
 distance from origin matters - for now just use unit values

Homogeneous Co-Ord's are useful for lots of things!

$$x, y \Rightarrow \begin{pmatrix} x \\ y \\ 1 \end{pmatrix}$$

$$\text{Translation} = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix}$$

we were careful to preserve 1

$$\text{Scale} = \begin{bmatrix} s & & \\ & s & \\ & & 1 \end{bmatrix}$$

$$\text{ROTATE} = \begin{bmatrix} c & -s & 0 \\ s & c & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

TRANSFORMATION MATRICES AS CO-ORD SYSTEMS

- ① where does origin go $\begin{matrix} 0 & 0 & 1 \end{matrix}$
- ② where does axis go $\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{matrix}$

Do we change the co-ord system, or the obj?
It's the same!

Composition: Multiply Matrices

Standard tricks-

- ~~mult~~ rotate around arbitrary point
- move center to origin
- rotate around origin
- move center back

$$T_c R T_c^{-1}$$

Scale along an axis

- move point to origin
- align axis w/ co-ordinate axis
- scale
- rotate back
- translate back

Inverses

EASY

GENERAL

What does this do in software?

OpenGL

Stated drawing system

current color / style

current co-ordinate system

↳ matrix from co-ord system (object) \Rightarrow screen
(simplification)

stack

multipln. load. clear