IDENTIFYING THE SEMANTIC AND TEXTUAL DIFFERENCES BETWEEN TWO VERSIONS OF A PROGRAM

by

Susan Horwitz

Computer Sciences Technical Report #895

November 1989

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SUSAN HORWITZ
University of Wisconsin – Madison

Text-based file comparators (*e.g.*, the Unix utility *diff*) are very general tools that can be applied to arbitrary files. However, using such tools to compare *programs* can be unsatisfactory because their *only* notion of change is based on program *text* rather than program *behavior*. This paper describes a technique for comparing two versions of a program, determining which program components represent changes, and classifying each changed component as representing either a *semantic* or a *textual* change.

Key words and phrases: file comparison, file difference, language-based tools, program maintenance, semantic difference.

1. INTRODUCTION

A tool that detects and reports differences between versions of programs is of obvious utility in a software-development environment. Text-based tools, such as the Unix utility *diff*, have the advantage of being applicable to arbitrary files; however, using such tools to compare *programs* can be unsatisfactory because no distinction can be made between textual and semantic changes.

This paper describes a technique for comparing two programs, *Old* and *New*, determining which components of *New* represent changes from *Old*, and classifying each changed component as representing either a *textual* or a *semantic* change. It is, in general, undecidable to determine precisely the set of semantically changed components of *New*; thus, the technique described here computes a safe approximation to (*i.e.*, possibly a superset of) this set. This computation is performed using a graph representation for programs and a partitioning operation on these graphs first introduced in [Yang89], and summarized in Section 2. The partitioning algorithm is currently limited to a language with scalar variables, assignment statements, conditional statements, while loops, and output statements. Because the partitioning algorithm is fundamental to the program-comparison algorithm described here, the program-comparison algorithm is also currently limited to the language described above. However, research is under way to expand the language; in particular, we are studying extensions for procedures and procedure calls, pointers, and arrays.

A precise definition of semantic change is given in Section 2; informally, a component c of *New* represents a semantic change either if there is no corresponding component of *Old* (because component c was added to *Old* to create *New*), or if a different sequence of values might be produced at c than at the corresponding component of *Old*. By "the sequence of values produced at c" we mean: if c is an assignment statement, the sequence of values assigned to the left-hand-side variable when the program is executed; if c is a predicate, the sequence of true-false values to which c evaluates when the program is executed; if c is an output statement, the sequence of values output when the program is executed.

This work was supported in part by the Defense Advanced Research Projects Agency, monitored by the Office of Naval Research under contract N00014-88-K, by the National Science Foundation under grant CCR-8958530, and by grants from Xerox, Eastman Kodak, and the Cray Research Foundation.

Author's address: Computer Sciences Department, Univ. of Wisconsin, 1210 W. Dayton St., Madison, WI 53706.

Example. Figure 1 shows a program *Old* and three different *New* programs; each *New* program is annotated to show its changes with respect to *Old*.

It is worthwhile to consider whether other approaches to program comparison could be used to detect the kinds of changes illustrated in Figure 1. In program New_1, the assignment "x := 2" is flagged as a semantic change because the value 2 is assigned to variable x whereas the corresponding component of *Old* assigns the value 1 to x. A text-based program comparator would also have flagged this as a changed component; however, the other changes flagged in New_1 would not have been detected by a text-based program comparator. These components represent semantic changes because they may use (directly or indirectly) the new value assigned to x.

The second and third semantic changes of program New_1 *could* have been detected by following def-use chains [Aho86] from the modified definition of x; however, program New_2 illustrates a situation in which following def-use chains leads to an erroneous detection of semantic change. In New_2, component "x := 0" is flagged as a semantic change because the sequence of values produced there is empty if variable P is true,[1] while the sequence of values produced at the corresponding component in *Old* is never empty (since the assignment is unconditional). Although "x := 0" represents a semantic change, the sequence of values produced at component "y := x" in New_2 is identical to the sequence of values produced at the corresponding component of *Old*; thus, "y := x" is *not* flagged as a change. Following def-use chains from "x := 0" would (incorrectly) identify both "y := x" and "output(y)" as semantic changes.

Finally, New_3 illustrates purely textual changes; again, following def-use chains from the changed component "y := a" would incorrectly identify "output(y)" as a semantic change.

In discussing the examples of Figure 1 we have talked about "corresponding components" in *Old* and the various *New* programs. How is this correspondence actually established? One possibility is to rely on the

Old	New_1	New_2	New_3
x := 0	x := 0	**if** P **then**	a := 0 ⟵ TEXTUAL
if P **then**	**if** P **then**	x := 1	**if** P **then**
x := 1	x := 2 ⟵ SEMANTIC	**else**	a := 1 ⟵ TEXTUAL
fi	**fi**	x := 0 ⟵ SEMANTIC	**fi**
y := x	y := x ⟵ SEMANTIC	**fi**	y := a ⟵ TEXTUAL
output(y)	output(y) ⟵ SEMANTIC	y := x	output(y)
		output(y)	

Figure 1. Program *Old* and three versions of *New*; each version of *New* is annotated to show its changes with respect to *Old*.

[1]The language under consideration does not include explicit input statements. However, variables can be used before being defined; these variables' values come from the initial state.

editing sequence used to create *New* from *Old*. For example, this correspondence could be established and maintained by the editor used to create *New* from *Old* as follows: Each component of *Old* has a unique tag; when a component is added, it is given a new tag, when a component is moved or modified it maintains its tag, when a component is deleted, its tag is never reused.

An algorithm for detecting the semantic and textual changes between *Old* and *New*, assuming editor-supplied tags, is given in Section 3.1; however, this approach has two important disadvantages:

(1) A special editor that maintains tags is required.

(2) The set of changes in *New* with respect to *Old* depends not only on the semantics of the two programs, but also on the particular editing sequence used to create *New* from *Old*. For example, it would be possible to use two different editing sequences to create programs *New* and *New′* from *Old*, such that the two new programs were *identical*, yet had different sets of changed components with respect to *Old*.

Section 3.2 considers how to determine semantic and textual changes between *Old* and *New* in the absence of editor-supplied tags; *i.e.*, the problem of finding the correspondence between the components of *Old* and *New* is included as part of the program-comparison algorithm. A reasonable criterion for determining the correspondence is that it should minimize the difference between *Old* and *New*; however, we show that it is *not* satisfactory to define "difference between *Old* and *New*" as simply the number of semantically or textually changed components of *New* with respect to *Old*. Instead, we propose defining "difference between *Old* and *New*" as the number of semantically or textually changed components of *New* plus the number of new *flow* or *control dependence edges* in the graph representation of *New* (flow and control dependence edges are defined in Section 2). Finding a correspondence that minimizes the difference between *Old* and *New* according to this definition is shown to be NP-hard in the general case; a study of real programs is needed to determine how difficult the problem will be in practice.

2. PARTITIONING PROGRAM COMPONENTS ACCORDING TO THEIR BEHAVIORS

The program-comparison algorithm described in this paper relies on an algorithm for partitioning program components (in one or more programs) so that two components are in the same partition only if they have equivalent behaviors [Yang89]. The Partitioning Algorithm uses a graph representation of programs called a *Program Representation Graph*. This section summarizes the definitions of Program Representation Graphs and partitioning given in [Yang89].

2.1. The Program Representation Graph

Program Representation Graphs (PRGs) are currently defined only for programs in a limited language that includes scalar variables, assignment statements, conditional statements, while loops, and output statements.[2]

PRGs combine features of program dependence graphs [Kuck81, Ferrante87, Horwitz88] and static single assignment forms [Shapiro70, Alpern88, Cytron89, Rosen88]. A program's PRG is defined in terms of an augmented version of the program's control-flow graph. The standard control-flow graph includes a special *Entry* vertex and one vertex for each *if* or *while* predicate, each assignment statement, and each

[2]The language used in [Yang89] is actually slightly more restrictive, including only a limited kind of output statement called an *end* statement, which can appear only at the end of a program; however, it is clear that no problems are introduced by allowing general output statements.

output statement in the program. As in static single assignment forms, the control-flow graph is augmented by adding special "ϕ vertices" so that each use of a variable in an assignment statement, an output statement, or a predicate is reached by exactly one definition.

(1) For each variable x that is defined within either (or both) branches of an *if* statement and is live at the end of the *if* statement, a "ϕ_{if}" vertex labeled "ϕ_{if}: $x := x$" is added to the control-flow graph immediately following the *if* statement. If there is more than one such vertex, their relative order is arbitrary.

(2) For each variable x that is defined within a *while* loop, and is live immediately after the loop predicate (*i.e.*, may be used before being redefined either inside the loop or after the loop), a "ϕ_{enter}" vertex labeled "ϕ_{enter}: $x := x$" is added to the control-flow graph inside the loop, before the loop predicate. If there is more than one such vertex, their relative order is arbitrary.

(3) For each variable x that is defined within a *while* loop and is live after the loop, a "ϕ_{exit}" vertex labeled "ϕ_{exit}: $x := x$" is added to the control-flow graph immediately after the loop. If there is more than one such vertex, their relative order is arbitrary.

In addition, for each variable x that may be used before being defined (*i.e.*, there is a x-definition clear path in the standard control-flow graph from the *Entry* vertex to a vertex that uses x), a vertex labeled "x := *Initial*(x)" is added to the control-flow graph after the *Entry* vertex. This vertex represents the assignment to x of a value from the initial state. If there is more than one such vertex, their relative order is arbitrary; however, they must appear sequentially, following the *Entry* vertex and preceding all other vertices in the control-flow graph.

Example. Figures 2(a) and 2(b) show a program and its augmented control-flow graph.

The vertices of a program's Program Representation Graph (PRG) are the same as the vertices in the augmented control-flow graph (an *Entry* vertex, one vertex for each predicate, each assignment statement, and each output statement, and for each *Initial*, ϕ_{if}, ϕ_{enter}, and ϕ_{exit} vertex). The edges of the PRG represent *control* and *flow* dependences.

The source of a control dependence edge is always either the *Entry* vertex or a predicate vertex; control dependence edges are labeled either **true** or **false**. The intuitive meaning of a control dependence edge from vertex v to vertex w is that if the program component represented by vertex v is evaluated during program execution and its value matches the label on the edge, then, (assuming termination of all loops) the component represented by w will eventually execute. (By definition, the *Entry* vertex always evaluates to **true**.)

Algorithms for computing control dependences in languages with unrestricted control flow are given in [Ferrante87, Cytron89]. For the restricted language under consideration here, control dependence edges reflect the nesting structure of the program (*i.e.*, there is an edge labeled **true** from the vertex that represents a *while* predicate to all vertices that represent statements inside the loop; there is an edge labeled **true** from the vertex that represents an *if* predicate to all vertices that represent statements in the true branch of the *if*, and an edge labeled **false** to all vertices that represent statements in the false branch; there is an edge labeled **true** from the *Entry* vertex to all vertices that represent statements or predicates that are not inside any *while* loop or *if* statement). In addition, there is a control dependence edge labeled **true** from every vertex that represents a *while* predicate to itself.

Flow dependence edges represent possible flow of values, *i.e.* there is a flow dependence edge from vertex v to vertex w if vertex v represents a program component that assigns a value to some variable x, vertex w represents a component that uses the value of variable x, and there is an x-definition clear path from v to w in the augmented control-flow graph.

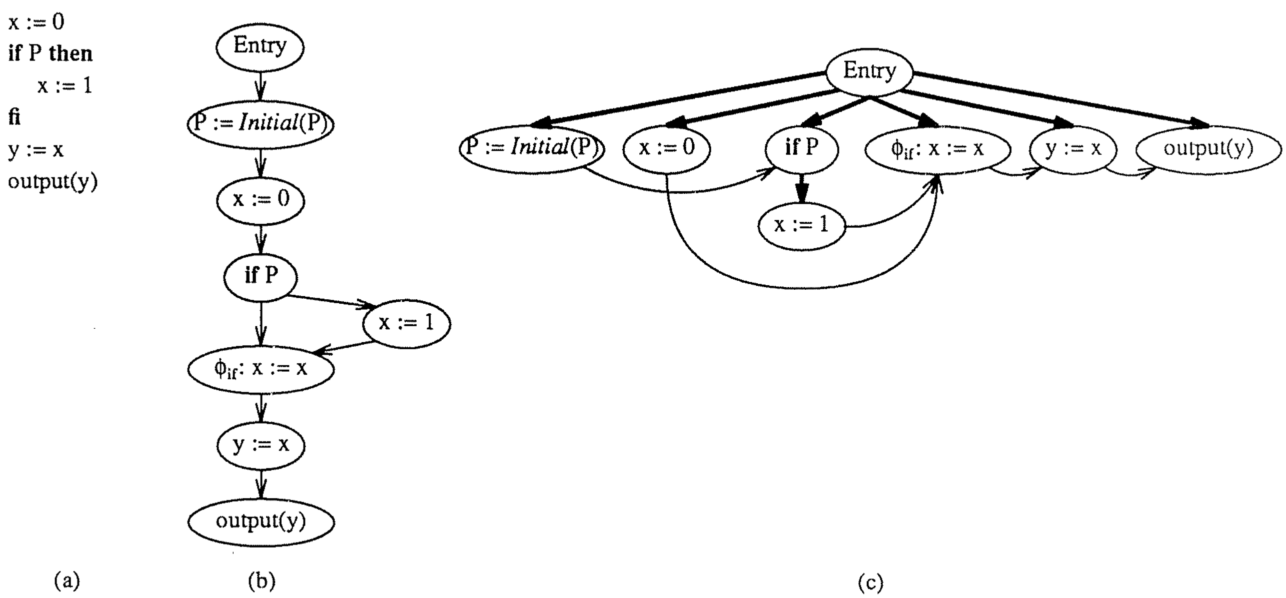


Figure 2. (a) A program; (b) its augmented control-flow graph; (c) its Program Representation Graph. In the Program Representation Graph, control dependence edges are shown using bold arrows and are unlabeled (in this example, all control dependence edges would be labeled **true**); data dependence edges are shown using arcs.

Example. Figure 2(c) shows the Program Representation Graph of the program of Figure 2(a). Control dependence edges are shown using bold arrows and are unlabeled (in this example, all control dependence edges would be labeled **true**); data dependence edges are shown using arcs.

2.2. The Partitioning Algorithm

The Partitioning Algorithm of [Yang89] can be applied to the Program Representation Graphs of one or more programs. The algorithm partitions the vertices of the graph(s) so that two vertices are in the same partition only if the program components that they represent have equivalent behaviors in the following sense:

Definition (equivalent behavior of program components). Two components c_1 and c_2 of (not necessarily distinct) programs P_1 and P_2 respectively, have *equivalent behaviors* iff all four of the following hold:

(1) For all initial states σ such that both P_1 and P_2 halt when executed on σ, the sequence of values produced at component c_1 when P_1 is executed on σ is identical to the sequence of values produced at component c_2 when P_2 is executed on σ.

(2) For all initial states σ such that neither P_1 nor P_2 halts when executed on σ, either the sequence of values produced at component c_1 is an initial sub-sequence of the sequence of values produced at c_2 or *vice versa*.

(3) For all initial states σ such that P_1 halts on σ but P_2 fails to halt on σ, the sequence of values produced at c_2 is an initial sub-sequence of the sequence of values produced at c_1.

(4) For all initial states σ such that P_2 halts on σ but P_1 fails to halt on σ, the sequence of values produced at c_1 is an initial sub-sequence of the sequence of values produced at c_2.

By "the sequence of values produced at a component" we mean: for an assignment statement (including *Initial* statements and ϕ statements), the sequence of values assigned to the left-hand-side variable; for an output statement, the sequence of values output; and for a predicate, the sequence of boolean values to which the predicate evaluates.

The Partitioning Algorithm uses a technique (which we will call the Basic Partitioning Algorithm) adapted from [Alpern88, Aho74] that is based on an algorithm of [Hopcroft71] for minimizing a finite state machine. This technique finds the coarsest partition of a graph that is consistent with a given initial partition of the graph's vertices. The algorithm guarantees that two vertices v and v' are in the same class after partitioning if and only if they are in the same initial partition, and, for every predecessor u of v, there is a corresponding predecessor u' of v' such that u and u' are in the same class after partitioning.

The Partitioning Algorithm operates in two passes. Both passes use the Basic Partitioning Algorithm, but apply it to different initial partitions, and make use of different sets of edges. The first pass creates an initial partition based on the operators that are used in the vertices;[3] flow dependence edges are used by the Basic Partitioning Algorithm to refine this partition. The second pass starts with the final partition produced by the first pass; control dependence edges are used by the Basic Partitioning Algorithm to further refine this partition.

The time required by the Partitioning Algorithm is $O(N \log N)$, where N is the size of the Program Representation Graph(s) (*i.e.*, number of vertices + number of edges).

Example. Figure 3 illustrates partitioning using the programs from Figure 1. Figure 3 shows the partitions created by the Partitioning Algorithm: the initial partition, the refinement created by Pass 1, and the final partition. Note that the components labeled "y := x" from *Old* and New_2 are in the same final partition (and thus have the same execution behaviors) even though they are transitively flow dependent on components that are not in the same final partition (namely, the components labeled "x := 0" from *Old* and New_2).

3. COMPUTING SEMANTIC AND TEXTUAL DIFFERENCES

This section presents three different algorithms to compute the semantic and textual differences between two versions of a program. All three algorithms operate on the programs' Program Representation Graphs; thus, in what follows, *New* and *Old* are Program Representation Graphs, and "program component" and "Program Representation Graph vertex" are used interchangeably.

Section 3.1 assumes that a special tag-maintaining editor is used to create program *New* from program *Old*. Section 3.2 assumes that the correspondence between the components of *New* and *Old* must be computed; Sections 3.2.1 and 3.2.2 use different criteria for determining the best correspondence. In both cases the goal is to find a correspondence that minimizes the size of the change between *New* and *Old*. However,

[3] The initial partition required for the purposes of this paper may be a refinement of that defined in [Yang89]. For the purposes of this paper, assignment statements, predicates, and output statements are put into different classes in the initial partition even when they use the same operator. For example, the statements "x := a **or** b", "output(c **or** d)", and "**if** x **or** y" all use the same operator (the logical or operator), but these statements would each be in a different class in the initial partition.

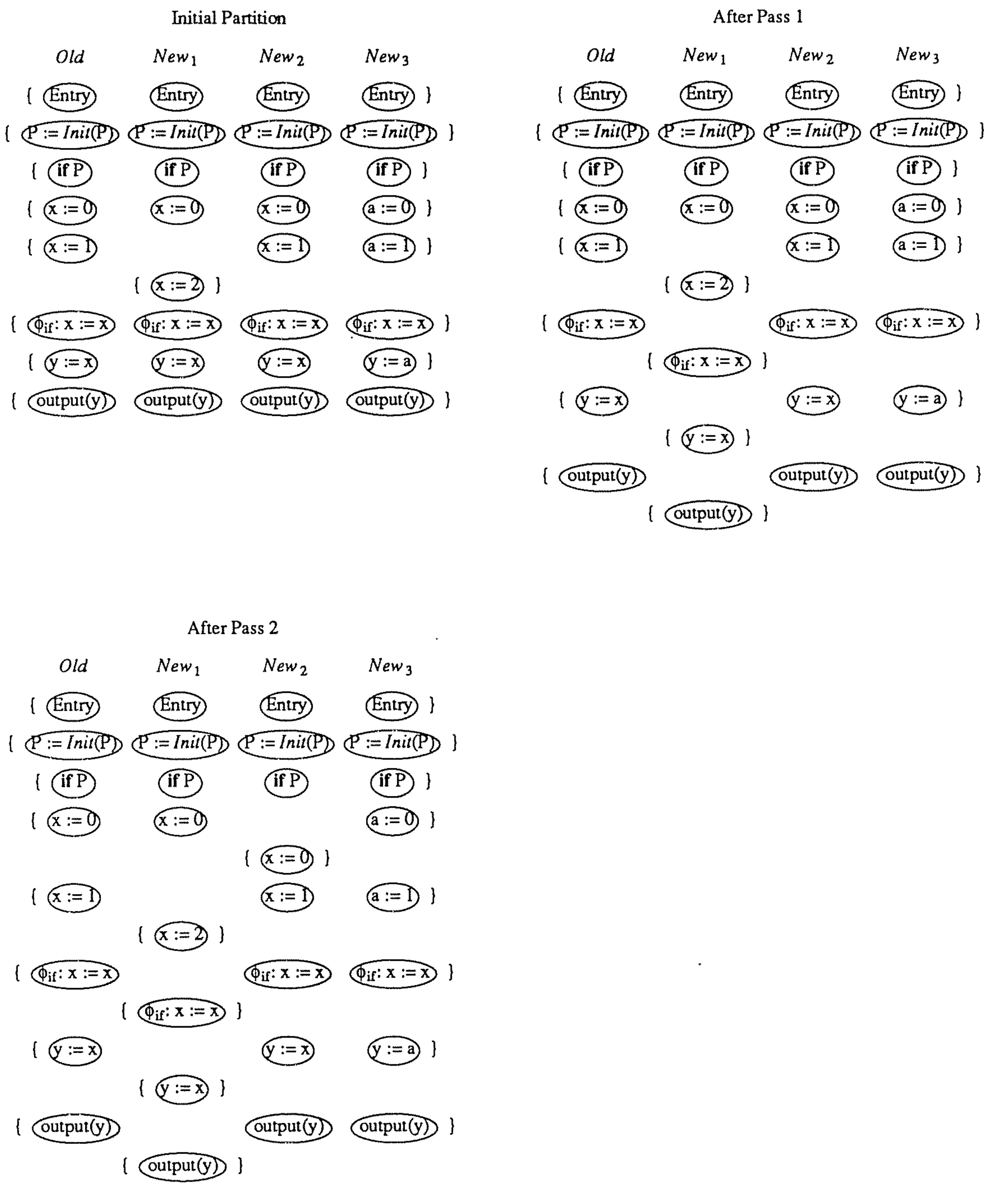


Figure 3. Partitioning Example. The partitions created by the Partitioning Algorithm for the programs of Figure 1.

in Section 3.2.1 "size of the change" is defined to be the number of semantically or textually changed components of *New*, while in Section 3.2.2 "size of the change" is defined to be the number of semantically or textually changed components, *plus* the number of new flow or control dependence edges in *New*.

3.1. Component Correspondence is Maintained by the Editor

If program *New* is created from program *Old* using an editor that maintains tags on program components, then determining which components of *New* represent changes from *Old* and classifying each changed component as either a textual or semantic change is quite straightforward. A procedure called ComputeChanges that classifies the components of *New* is given below. The procedure first partitions programs *Old* and *New* and then considers each component c of *New*. If there is no component of *Old* with the same tag, then c was added to *Old* to create *New*, and thus represents a semantic change. Similarly, if there is a component of *Old* with the same tag, but the component is not in the same partition as c, then c represents a semantic change. If there is a component of *Old* with the same tag and in the same partition but with different text, then c represents a textual change.

```
procedure ComputeChanges( Old, New: Program Representation Graphs )
returns two sets of components of New, representing semantic and textual changes, respectively
  declare semanticChange, textualChange: sets of program components
begin
  apply the Partitioning Algorithm to Old and New
  semanticChange := ∅
  textualChange := ∅
  for each component c of New do
    if (there is no component of Old with the same tag as c) or
      (the component of Old with the same tag as c is not in the same partition as c)
    then insert c into semanticChange
    else if the text of the component of Old that has the same tag as c ≠ the text of c
        then insert c into textualChange
        fi
    fi
  od
  return( semanticChange, textualChange )
end
```

Procedure ComputeChanges can be illustrated by considering programs *Old* and New_2 of Figure 1. Assume that program New_2 was created from *Old* by moving the statement "x := 0" into the *else* branch of the *if* statement. In this case, for every component of New_2 there is a component of *Old* with the same tag, and (as illustrated in Figure 3) for every component of New_2 other than component "x := 0", the component of *Old* with the same tag is in the same final partition. Thus, the only component of New_2 identified by procedure ComputeChanges as representing a change from *Old* is component "x := 0", which is identified as a semantic change.

3.2. Component Correspondence Must be Computed

In this section we consider how to compare programs *Old* and *New* assuming that program components are *not* tagged by the editor. Instead, the correspondence between the components of *Old* and *New* must be computed as part of the program-comparison algorithm. Our goal is to find a correspondence that minimizes the size of the change between *Old* and *New*. Sections 3.2.1 and 3.2.2 consider two different definitions of "the size of the change."

3.2.1. Size of change = the number of semantically or textually changed components of *New*

If we define the size of the change between *Old* and *New* as the number of semantically or textually changed components of *New*, then it is possible to define an efficient algorithm to find a correspondence that minimizes this size. A procedure called MatchAndComputeChanges that computes such a correspondence and simultaneously classifies the components of *New* with respect to *Old* is given below. The procedure first tries to match every component of *New* with a component of *Old* that is both semantically and textually equivalent. Next, the procedure considers all unmatched components of *New*, attempting to match them with unmatched components of *Old* that are semantically equivalent but textually different. These components of *New* are classified as textual changes. Components of *New* that remain unmatched are classified as semantic changes.

Applying procedure MatchAndComputeChanges to programs *Old* and New_2 of Figure 1 will produce the result pictured in Figure 1 even if the components of the two programs are not tagged by the editor. All components of New_2 other than "x := 0" will be matched with a component of *Old* that is both semantically and textually equivalent; component "x := 0" will be unmatched, and so will be classified as a semantic change.

Procedure MatchAndComputeChanges first partitions *Old* and *New*, then makes two passes through *New* matching and classifying its components. Assuming that it is possible to determine in constant time whether there is an unmatched component of *Old* in the same partition and with the same text as a given

procedure MatchAndComputeChanges(*Old*, *New*: Program Dependence Graphs)
returns (1) a map from components of *New* to components of *Old*, and
        (2) two sets of components of *New*, representing semantic and textual changes, respectively
declare map: a set of program component pairs
        semanticChange, textualChange: sets of program components
begin
  apply the Partitioning Algorithm to *Old* and *New*
  map := ∅; semanticChange := ∅; textualChange := ∅
  for each component *c* of *New* **do**
    if there is an unmatched component *c′* of *Old* that is in the same partition as *c* and has the same text
    then insert the pair (*c*, *c′*) into map
        mark *c* "matched"
        mark *c′* "matched"
    fi
  od
  for each unmatched component *c* of *New* **do**
    if there exists an unmatched component *c′* of *Old* that is in the same partition as *c*
    then insert the pair (*c*, *c′*) into map
        insert *c* into textualChange
        mark *c′* "matched"
    else insert *c* into semanticChange **fi**
  od
  return(map, semanticChange, textualChange)
end

component of *New*, the time required for matching and classifying is linear in the size of *New*; thus, the time required for procedure MatchAndComputeChanges is dominated by the time required for partitioning. The total time for procedure MatchAndComputeChanges is $O(N \log N)$, where N is the sum of the sizes of *Old* and *New*.

3.2.2. Size of change includes the number of new edges in *New*

Simply minimizing the number of semantically and textually changed components does not always produce a satisfactory classification of the components of *New*; this is illustrated in Figure 4. Figure 4 shows programs *Old* and *New*, and four possible mappings from the components of *New* to the components of *Old*. All four mappings induce the same (minimum) number of changed components of *New* with respect to *Old*, yet there is something intuitively more satisfying about the first two mappings than the third and forth mappings. The problem with the third and forth mappings is that they "separate" a use of variable x from the corresponding definition of x.

We can avoid choosing mapping three or mapping four of Figure 4 by redefining the "size of the change between *Old* and *New*" to take into account PRG *edges* as well as vertices.

Definition (a correspondence between *New* and *Old*). A *correspondence* between *New* and *Old* is a 1-to-1 partial function f from vertices of *New* to vertices of *Old* such that (1) for all vertices v of *New*, $f(v)$ is either a vertex of *Old*, or is the special value $\perp$ ($f(v) = \perp$ means that there is no vertex of *Old* that corresponds to vertex v of *New*), and (2) if $f(v) = v'$, then vertices v and v' are in the same final partition.

Definition (unmatched vertex). A vertex v of *New* is unmatched under correspondence f iff $f(v) = \perp$.

Definition (unmatched edge). An edge $v_1 \to v_2$ of *New* is unmatched under correspondence f iff any of the following hold: (1) $f(v_1) = \perp$; (2) $f(v_2) = \perp$; (3) there is no edge $f(v_1) \to f(v_2)$ in *Old* of the same type[4] as the edge $v_1 \to v_2$.

Old	*New*	Mapping	Changed Components
[O1] x := 1	[N1] x := 1	{([N1]-[O1]), ([N2]-[O2])}	N3, N4
[O2] y := x	[N2] y := x	{([N3]-[O1]), ([N4]-[O2])}	N1, N2
	[N3] x := 1	{([N1]-[O1]), ([N4]-[O2])}	N2, N3
	[N4] y := x	{([N2]-[O2]), ([N3]-[O1])}	N1, N4

Figure 4. Programs *Old* and *New*, and four possible mappings from the components of *New* to the components of *Old*. Each mapping induces a set of changed components of size 2; however, the first two mappings each induce only one new data dependence, while the second two mappings each induce two new data dependences.

[4]A precise definition of edge type can be found in [Yang89]. Roughly, two edges are of the same type if (1) they are both control-dependence edges, or (2) they are both flow-dependence edges for the same operand of the target vertex. For example, vertices "$x := y + z$" and "$a := b + c$" both have two variable operands, so both have two incoming flow-dependence edges. The edge "carrying" the definition of variable y can only match the edge carrying the definition of variable b; it cannot match the edge carrying the definition of variable c.

Definition (size of change between *Old* and *New*). The size of the change between *Old* and *New* induced by correspondence f is: (the number of unmatched vertices v of *New*) + (the number of matched vertices v of *New* such that $f(v) = v'$ and the text of v is not identical to the text of v') + (the number of unmatched edges of *New*).

Figures 5(a), 5(b), and 5(c) give a procedure for computing a correspondence between *New* and *Old* that minimizes the size of the change between *Old* and *New* as defined above. However, since the problem of finding such a correspondence is NP-hard (as shown below) it is unlikely that an *efficient* procedure can be defined.

```
declare global bestSoFar: a correspondence between New and Old
declare global smallestChangeSoFar: integer


procedure Match(Old, New: Program Representation Graphs)
returns: a correspondence between New and Old that minimizes the size of the change between Old and New
  declare map: a correspondence between New and Old
  declare workingSet: a set of vertices of New
begin
  apply the Partitioning Algorithm to Old and New
  map := ∅
  /* match all "no-choice" vertices of New */
    for each partition that includes exactly one vertex v of New and one vertex v' of Old do
        insert (v, v') into map
        mark v "matched"
        mark v' "matched"
    od

  /* put all remaining matchable vertices of New into the working set */
     workingSet := ∅
     for all unmatched vertices v of New such that ∃ an unmatched vertex of Old in the same partition do
         insert v into workingSet
     od

  /* try all possible correspondences; keep track of the best one found */
     bestSoFar := ∅
     smallestChangeSoFar := ∞
     TryMatches(map, workingSet)

  /* the best correspondence has been saved in global variable bestSoFar */
     return( bestSoFar )
end
```

Figure 5(a). Procedure Match finds a correspondence between *New* and *Old* that minimizes the difference between *Old* and *New*. Procedure Match calls procedure TryMatches, which is shown in Figure 5(b).

```
procedure TryMatches( map: a correspondence between New and Old,
                      workingSet: a set of vertices of New )
begin
  if workingSet = ∅
  then /* no more matchable vertices of New
       * compute the size of the change induced by the current correspondence;
       * save the current correspondence if its change size is smaller than the best so far
       */

       if ChangeSize( map ) < smallestChangeSoFar
       then bestSoFar := map
            smallestChangeSoFar := ChangeSize( map )
       fi

  else /* try all remaining possible matches */

       select and remove an arbitrary vertex v from workingSet
       let P be v's partition in
         remove v from P
[L1]:    if (# of unmatched vertices of New in P) ≥ (# of unmatched vertices of Old in P)
         then /* must try correspondences in which v is unmatched, too */
              TryMatches( map, workingSet )
         fi
[L2]:    for each unmatched vertex v' of Old in partition P do
           insert (v, v') into map
           mark v' "matched"
           TryMatches( map, workingSet )
           remove (v, v') from map
           mark v' "unmatched"
         od
         /* put vertex v back into partition P and into workingSet so that it will be there next time TryMatches is called */
          add v to partition P
          insert v into workingSet
       ni
  fi
end
```

Figure 5(b). If there are no more matchable vertices of *New*, Procedure TryMatches computes the size of the change between *Old* and *New* induced by the current correspondence. Otherwise, it trys all correspondences consistent with the given (incomplete) correspondence.

```
procedure ChangeSize( map: a correspondence between New and Old )
returns the size of the change between Old and New induced by the given correspondence

/*
 * "size of change" = (# of unmatched vertices of New) +
 *                    (# of vertices of New matched with textually different vertices of Old) +
 *                    (# of unmatched edges of New)
 */

  declare size: integer
begin
  size := (# of vertices of New) + (# of edges of New)
  for each vertex v of New do
    if (v, v′) is in map
    then if text(v) = text(v′) then size := size − 1 fi
         for each edge v → w in New do
           if (w, w′) is in map
           then if (∃ edge v′ → w′ in Old) and (type(v → w) = type(v′ → w′))
                then size := size − 1
                fi
           fi
         od
    fi
  od
  return( size )
end
```

Figure 5(c). Procedure ChangeSize computes the size of the change between *Old* and *New* induced by the given correspondence.

Procedure Match of Figure 5(a) matches all "no-choice" vertices of *New*, *i.e.*, those vertices in partitions that include exactly one vertex of *Old* and one vertex of *New*. Match then puts all *matchable* vertices of *New* (those vertices of *New* that are unmatched and are in partitions with at least one unmatched vertex of *Old*) into a working set, and calls procedure TryMatches to try all correspondences that include the "no-choice" matchings performed so far.

To understand procedure TryMatches, consider what it does when the working set is empty, when the working set contains exactly one vertex, and when the working set contains more than one vertex.

The working set is empty.

> When the working set is empty there are no partitions that include both an unmatched vertex of *New* and an unmatched vertex of *Old*; *i.e.*, a complete correspondence has been defined. In this case, procedure TryMatches computes the size of the change induced by the current correspondence; the current correspondence and its change size are saved if it is the best correspondence found so far. (The size of the change induced by the current correspondence is computed by procedure ChangeSize, shown in Figure 5(c).)

The working set contains one vertex v.

In this case, v is removed from the working set and from its partition P. Now there are two subcases: (1) partition P contains no unmatched vertex of *Old*; (2) partition P contains one or more unmatched vertices of *Old*. In the first case, the correspondence is complete; the test at line [L1] will succeed (because both the number of unmatched vertices of *New* in P and the number of unmatched vertices of *Old* in P are zero), and a recursive call to TryMatches (with an empty working set) will be made. This recursive call will compute the cost of the current correspondence.

In the second case, the test at line [L1] will fail, and the *for* loop at line [L2] will be executed. Each time around the loop the current correspondence is completed by matching vertex v with a different unmatched vertex of *Old* in P, and a recursive call to TryMatches (with an empty working set) is made.

The working set contains more than one vertex.

In this case, an arbitrary vertex v is selected and removed from the working set. The test at line [L1] serves two (similar) purposes. First, if there are *no* unmatched vertices of *Old* in v's partition P, the test will succeed, guaranteeing that the current correspondence will be completed with v unmatched (the *for* loop at line [L2] will not serve this purpose since it will execute zero times). Second, if, after removing v from P there are still at least as many unmatched vertices of *New* as unmatched vertices of *Old* left in P, the test will succeed, and the recursive call to TryMatches will complete the current correspondence in all possible ways with v unmatched. The *for* loop at line [L2] will take care of completions in which v is matched with an available vertex of *Old*.

The time requirements of procedure TryMatches can be analyzed as follows. Let M be 1 + the maximum number of unmatched vertices of *Old* in a partition with at least one unmatched vertex of *New*. Given a working set of size 1, TryMatches will make at most M recursive calls, each with an empty working set, so $T(1) \leq M$. Given a working set of size n, TryMatches will make at most M recursive calls, each with a working set of size $n-1$, so $T(n) \leq M * T(n-1)$. Solving this equation we find that the time required for a call to TryMatches with a working set of size n is $O(M^n)$.

The value of n for the original call to TryMatches made from procedure Match is the number of matchable vertices of *New* that remain after all no-choice matches are made. It remains to be seen how large this value, as well as the value of M, are in practice. An (unrealistic) upper bound for the time required by TryMatches is $O(O^N)$, where O is the number of vertices in *Old*, and N is the number of vertices in *New*.

Finding a minimum change correspondence is NP-hard.

In this section we prove that finding a correspondence between *New* and *Old* that minimizes the size of the change between *New* and *Old* is NP-hard (where "size of the change" is as defined above). We call this problem the "Minimum Correspondence" problem. We show that the Minimum Correspondence problem is NP-hard by showing that a related problem, the "k-Correspondence" problem, is NP-complete. An algorithm for k-Correspondence answers the question, "Is there a correspondence that induces a change between *Old* and *New* of size $\leq k$?". It is clear that a solution to the Minimum Correspondence problem provides a solution to the k-Correspondence problem; thus, if the k-Correspondence problem is NP-complete, the Minimum Correspondence problem is NP-hard.

To show that k-Correspondence is NP-complete, we must

(1) show that k-Correspondence is in NP, and

(2) show that a polynomial-time solution to k-Correspondence can be used to find a polynomial-time solution to a known NP-complete problem.

It is clear that k-Correspondence is in NP; a nondeterministic algorithm to solve the k-Correspondence problem partitions *Old* and *New*, then, for each vertex v of *New*'s PRG, matches v with a (nondeterministically chosen) unmatched vertex of *Old*'s PRG that is in the same partition. Finally, the size of the change induced by the resulting correspondence is computed; if this size is less than or equal to k, the algorithm returns *true*, otherwise it returns *false*.

Next, we show that a polynomial-time solution to k-Correspondence can be used to find a polynomial-time solution to 3-CNF-Satisfiability (a known NP-complete problem). We show that, given a 3-CNF formula, we can produce (in polynomial time) Program Representation Graphs *Old* and *New*, and an integer k, such that there is a correspondence between *New* and *Old* that induces a change of size $\leq k$ iff the given 3-CNF formula is satisfiable.

The following terminology is used: A 3-CNF formula uses a set of *variables* $x_1, x_2, \cdots x_n$. The formula consists of the conjunction of a set of *clauses* $c_1, c_2, \cdots c_m$. Each clause c_j is of the form $(t_{j1} \vee t_{j2} \vee t_{j3})$, where each *term* t_{jk} is a barred or unbarred variable. For example:

		$t_{11} = x_1$	$t_{21} = \overline{x_2}$
$(x_1 \vee \overline{x_1} \vee x_2) \wedge (\overline{x_2} \vee x_3 \vee x_4)$	$\{x_1, x_2, x_3, x_4\}$	$t_{12} = \overline{x_1}$	$t_{22} = x_3$
		$t_{13} = x_2$	$t_{23} = x_4$
3-CNF Formula	Set of Variables	Terms	

Figure 6 shows the general form of the Program Representation Graphs *Old* and *New* produced from a given 3-CNF formula; Figure 7 shows the PRGs for the example 3-CNF formula given above. In Figure 6, the notation "<ji>" (used in both *Old* and *New*) means the index of the variable that appears in the i^{th} term of the j^{th} clause; thus, "v<j2>" is the identifier whose first character is "v" and whose remaining characters are the index of the variable that appears in the 2^{nd} term of the j^{th} clause. The notation "t_{jk}" (used in *New*) means the *value* of the term t_{jk}; *i.e.*, t_{jk} is a barred or unbarred variable.

To understand how a solution to the given 3-CNF-Satisfiability problem provides a solution to the corresponding Minimum Matching problem and *vice versa*, consider the equivalence classes produced by applying the Partitioning Algorithm to the programs *Old* and *New* illustrated in Figure 6. These classes are shown in Figure 8. The equivalence classes that contain three vertices (one from *Old* and two from *New*) "force" each variable x_i to be assigned a unique value. (Think of the vertex from *Old*, "vi := i" as the value *true*; that vertex can be matched with at most one of the two vertices from *New*; either vertex "xi := i" or vertex "$\overline{\text{xi}}$:= i". The former corresponds to assigning variable x_i the value *true*; the latter corresponds to assigning variable x_i the value *false*.)

The equivalence classes that contain four vertices (one from *Old* and three from *New*) "choose" one term from each clause. Again, the *Old* vertex, "cj := v<j1> + v<j2> + v<j3> + j" can be matched with at most one of the three vertices from *New*, either "cj := t_{j1} + y<j2> + y<j3> + j" or "cj := y<j1> + t_{j2} + y<j3> + j" or "cj := y<j1> + y<j2> + t_{j3} + j". Matching the first *New* vertex with "cj := v<j1> + v<j2> + v<j3> + j" corresponds to choosing term t_{j1}; matching the second *New* vertex corresponds to choosing term t_{j2}; and matching the third *New* vertex corresponds to choosing term t_{j3}. The same number of *vertices* of *New* can be matched whether or not the given 3-CNF formula is satisfiable; however, if the formula is not satisfiable, then the number of matched *flow edges* of *New* will be insufficient to allow the size of the change to be $\leq k$.

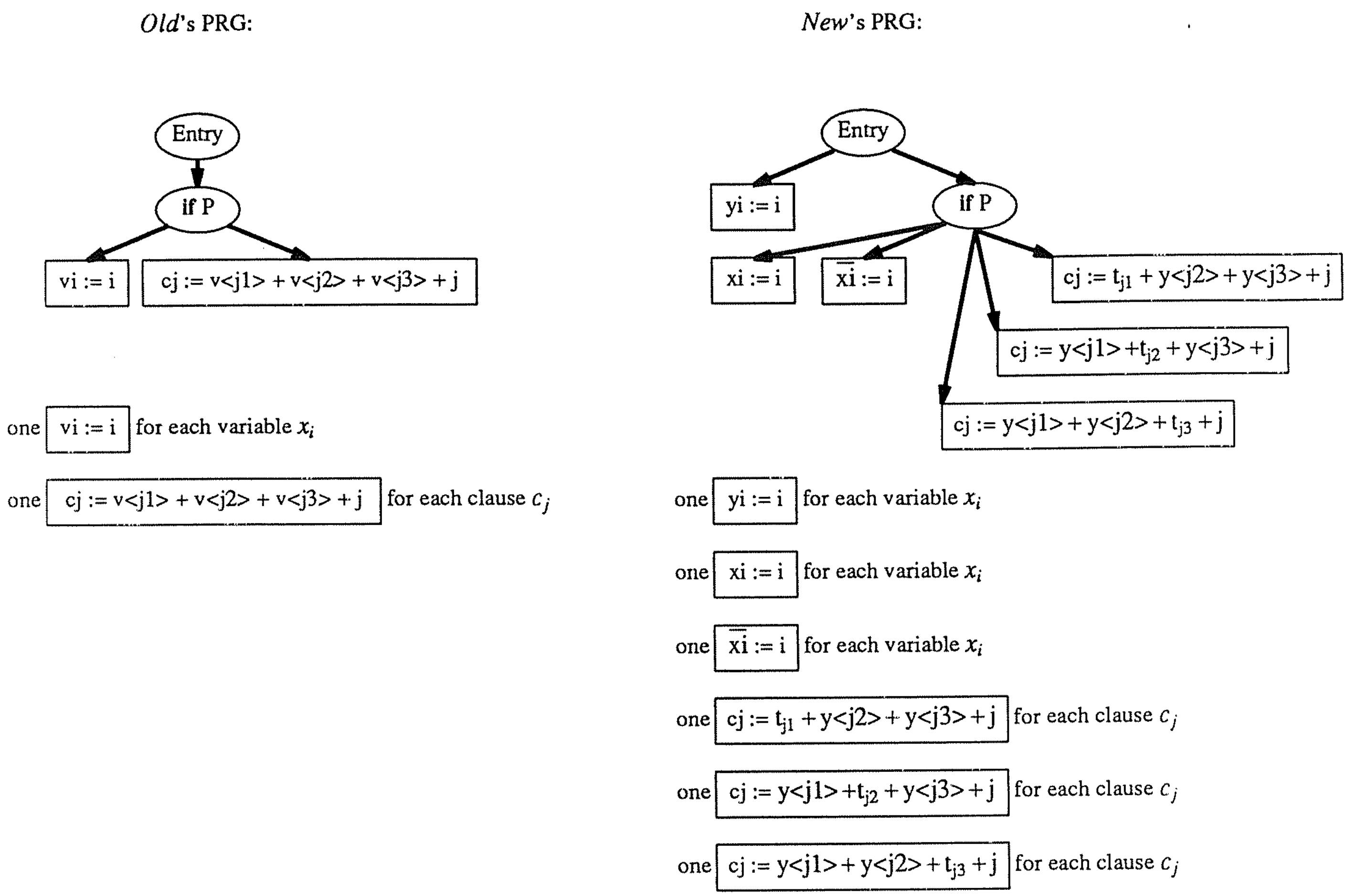


Figure 6. The general form of PRGs *Old* and *New* built from a given 3-CNF formula. Flow edges are omitted. Control edges are shown unlabeled (these edges would all be labeled **true**). See the text for an explanation of notation like "v<j1>" and t_{j1}.

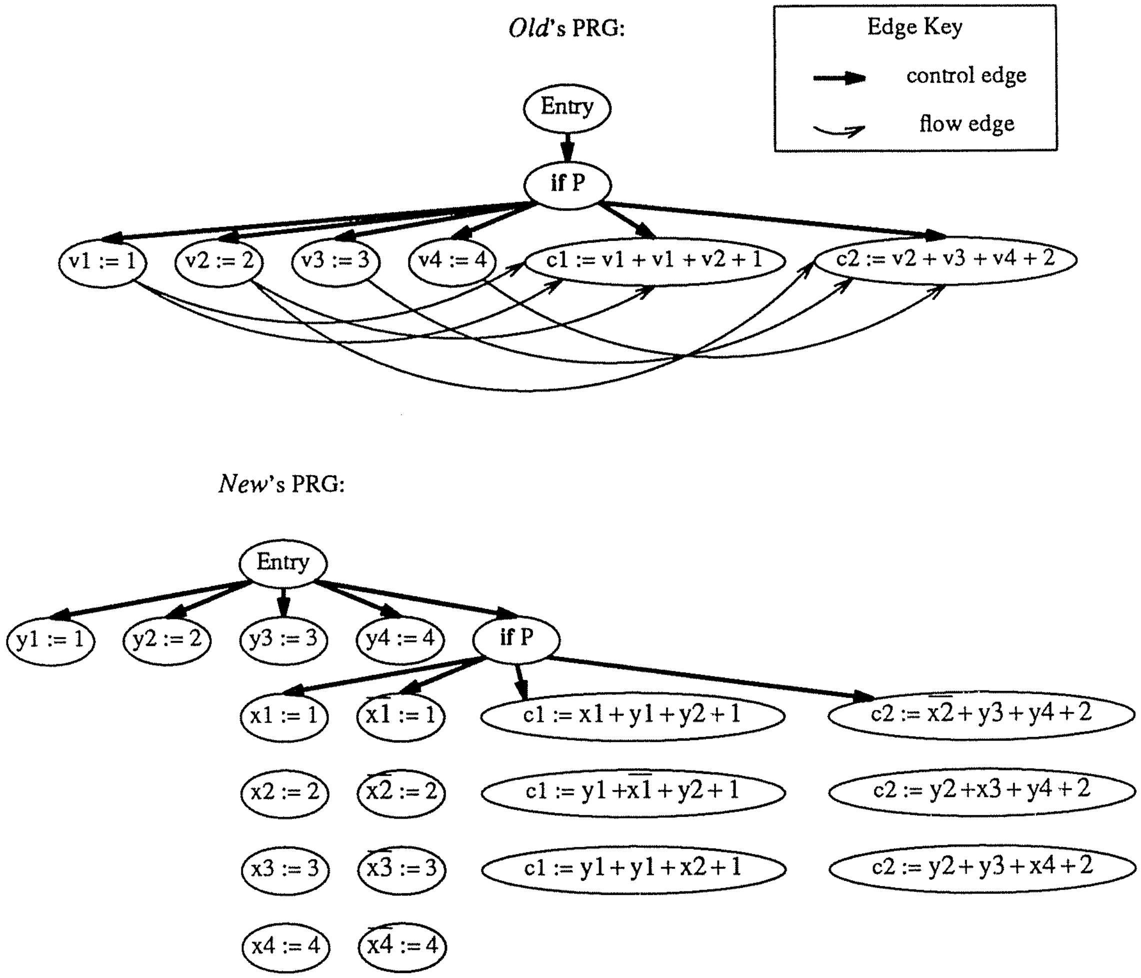


Figure 7. PRGs *Old* and *New* produced from 3-CNF formula $(x_1 \vee \overline{x_1} \vee x_2) \wedge (\overline{x_2} \vee x_3 \vee x_4)$. All flow edges and some control edges are omitted from *New*'s PRG. Control edges are shown unlabeled (these edges would all be labeled **true**).

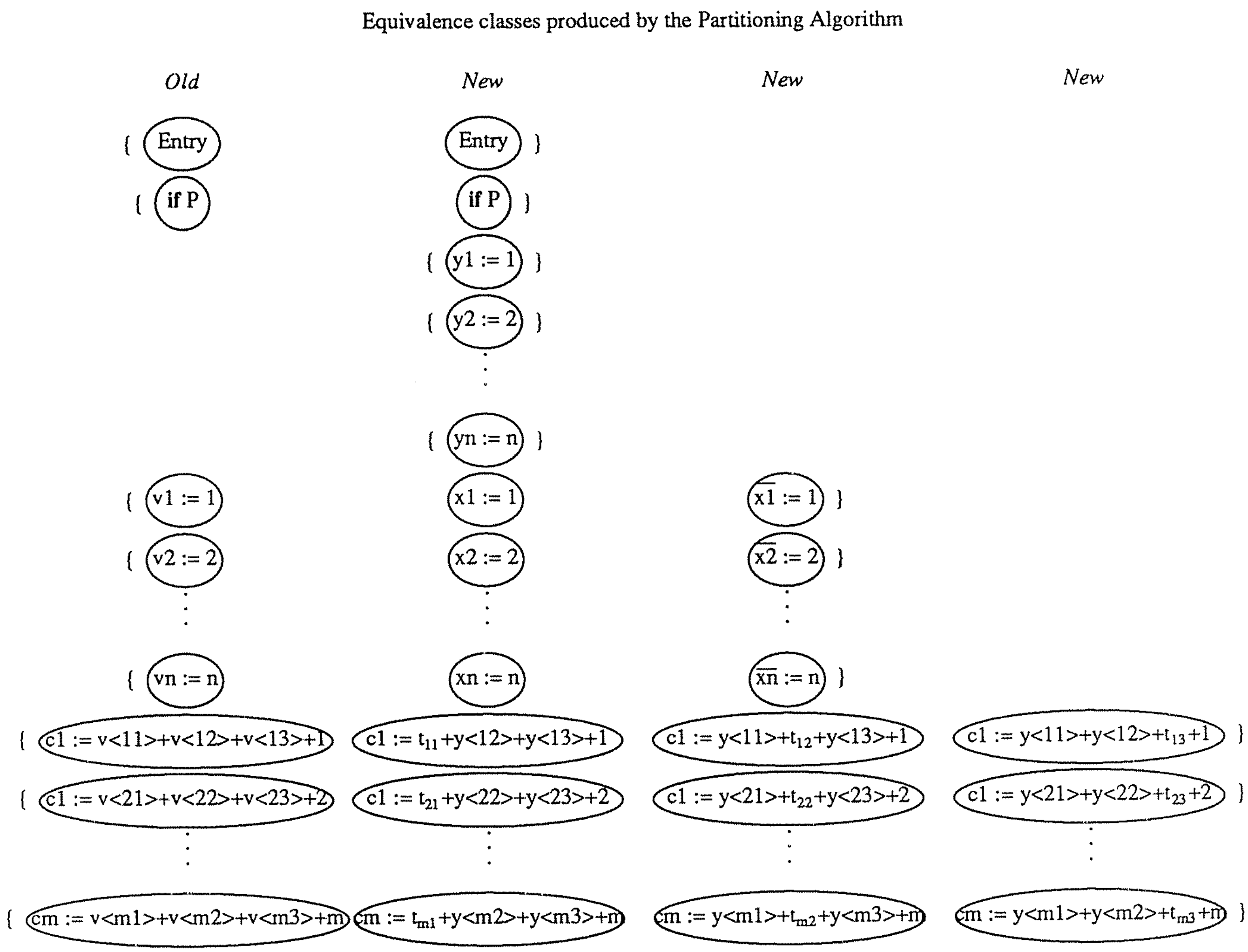


Figure 8. The equivalence classes produced by applying the Partitioning Algorithm to programs *Old* and *New* shown in Figure 6.

The value selected for k for a given 3-CNF formula is:

(# of vertices and edges in *New*) – (# of potentially unchanged vertices in *New* + # of matchable edges in *New*) =
((3 + (6 * #-of-variables) + (15 * #-of-clauses)) – (2 + (1 + #-of-variables + (2 * #-of-clauses))) =
(5 * #-of-variables) + (13 * #-of-clauses)

This number is explained below.

(1) Number of vertices and edges in *New*

The number of vertices in *New* is 2 + (3 * # vars) + (3 * # clauses): the *Entry* vertex, the "if P" vertex, one "yi := i" vertex for each variable, one "xi := i" vertex for each variable, and one "$\overline{\text{xi}}$:= i" vertex for each variable; and three "cj := ..." vertices for each clause.

The number of control dependence edges in *New* is one less than the number of vertices (because every vertex other than the *Entry* vertex has exactly one incoming control dependence edge); thus, there are 1 + (3 * # vars) + (3 * # clauses) control dependence edges.

The number of flow dependence edges in *New* is (9 * # clauses): the only vertices with incoming flow dependence edges are the "cj := ..." vertices; each such vertex uses three variables (so it has three incoming flow dependence edges), and there are three such vertices for each clause.

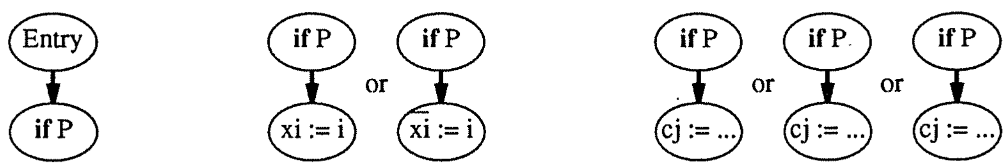
(2) Number of potentially unchanged vertices in *New*

The only vertices of *New* that can be matched with textually and semantically identical vertices of *Old* are the *Entry* vertex and the "if P" vertex. These vertices can be matched whether or not the 3-CNF formula is satisfiable.

(3) Number of matchable edges in *New*

Control dependence edges

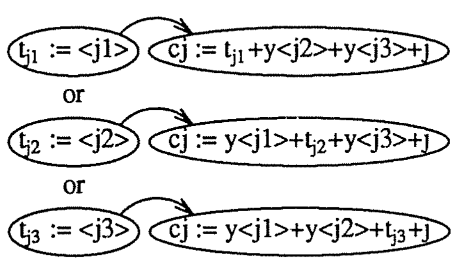
A control dependence edge in *New* can be matched only if both endpoints are matched (however, the endpoints need not be matched with textually identical vertices of *Old*). There are (1 + #-of-variables+#-of-clauses) matchable control dependence edges. These edges, enumerated in the figure below, can be matched whether or not the 3-CNF formula is satisfiable.



1   1 for each variable   1 for each clause

Flow dependence edges

There are (#-of-clauses) matchable flow dependence edges. These edges, enumerated in the figure below, can all be matched *only* if the 3-CNF formula is satisfiable.



1 for each clause

As stated in the discussion above on the value of k, it is always possible to find a correspondence that matches two vertices of *New* with textually and semantically equivalent vertices of *Old*, and matches (1 + #-of-variables + #-of-clauses) control-dependence edges of *New*. The crux of the proof that the 3-CNF formula is satisfiable iff there exists a correspondence between *New* and *Old* that induces a change of size $\leq k$, is showing that (#-of-clauses) flow-dependence edges can be matched iff the 3-CNF formula is satisfiable.

It is clear that (#-of-clauses) flow-dependence edges can be matched if the 3-CNF formula is satisfiable. In this case, for each variable x_i, with value *true*, the *Old* vertex "vi := i" is matched with the *New* vertex "xi := i"; for each variable x_i with value *false*, the *Old* vertex "vi := i" is matched with the *New* vertex "$\overline{\text{xi}}$:= i." For each clause c_j with *true* term t_{jk}, the *Old* vertex "cj := v<j1>+v<j2>+v<j3>+j" is matched with the *New* vertex that "chooses" term t_{jk}. The flow-dependence edge from the *New* vertex that "assigns" *true* to term t_{jk} to the *New* vertex for clause c_j that "chooses" term t_{jk} is matched because both of its endpoints are matched.

Example. Figure 9 uses the example 3-CNF formula $(x_1 \vee \overline{x_1} \vee x_2) \wedge (\overline{x_2} \vee x_3 \vee x_4)$ to illustrate how a solution to the 3-CNF Satisfiability problem can be used to find a solution to the k-Correspondence problem. In this example, the number of vertices in *New* is 20, the number of edges in *New* is 37, the number of potentially unchanged vertices in *New* is 2, and the number of matchable edges in *New* is 9; thus the value of k for this example is 46. The correspondence shown in Figure 9 matches two vertices of *New* with textually and semantically equivalent vertices of *Old* (namely, the *Entry* vertex and the "**if** P" vertex); nine edges of *New* (seven control-dependence edges and two flow-dependence edges) are matched. Therefore, this correspondence induces a change of size 46 (18 unmatched vertices, 6 matched but textually different vertices, 12 unmatched control-dependence edges, and 16 unmatched flow-dependence edges), which is the value of k.

It is also clear that the 3-CNF formula is satisfiable if (#-of-clauses) flow dependence edges can be matched. By construction, each subgraph that corresponds to a clause can contribute at *most* one matched flow-dependence edge; thus, if (#-of-clauses) flow-dependence edges can be matched, there must be exactly one such edge in each "clause" subgraph. Again by construction, the source of the matched flow-dependence edge is a vertex either of the form "xi := i", or of the form "$\overline{\text{xi}}$:= i". Since for each variable x_i at most one of these two vertices can be matched, this matching provides a satisfying truth value assignment for the variables of the formula.

4. RELATED WORK

Related work falls into two categories: techniques for computing *textual* differences, and techniques for computing *semantic* differences. The first category includes techniques for comparing strings [Sankoff72, Wagner74, Nakatsu82, Tichy84, Miller85] and techniques for comparing trees [Selkow77, Lu79, Tai79, Zhang89]. Although such work has a different goal than the technique described here, these textual differencing techniques might be useful in practice as a compromise between requiring editor-supplied tags and solving an NP-hard problem; *i.e.*, one of these algorithms might be used to compute tags for program components. Once tags are available, the procedure ComputeChanges of Section 3.1 can be used to classify the components of *New*. In this case, no special editor is required, and tags are not a function of the particular edit sequence used to create program *New* from program *Old*; however, there is no guarantee that the size of the change between *Old* and *New* will be minimal in the sense of Section 3.2.2.

Program slicing [Weiser84, Ottenstein84, Horwitz88a] is a technique for identifying just those program components that might affect the values of a given set of variables at a given program point. Slicing is used by the program integration algorithm of [Horwitz89] to determine the semantic differences between two versions of a program. Program slicing could potentially be used in place of partitioning to identify sets of components of *Old* and *New* that have the same execution behavior. However, in the absence of tags, it is not clear whether this could be done efficiently since it would include determining isomorphism of slices as a subproblem. Furthermore, partitioning is superior to slicing in the sense that the equivalence classes determined by partitioning are supersets of the equivalence classes determined using slicing

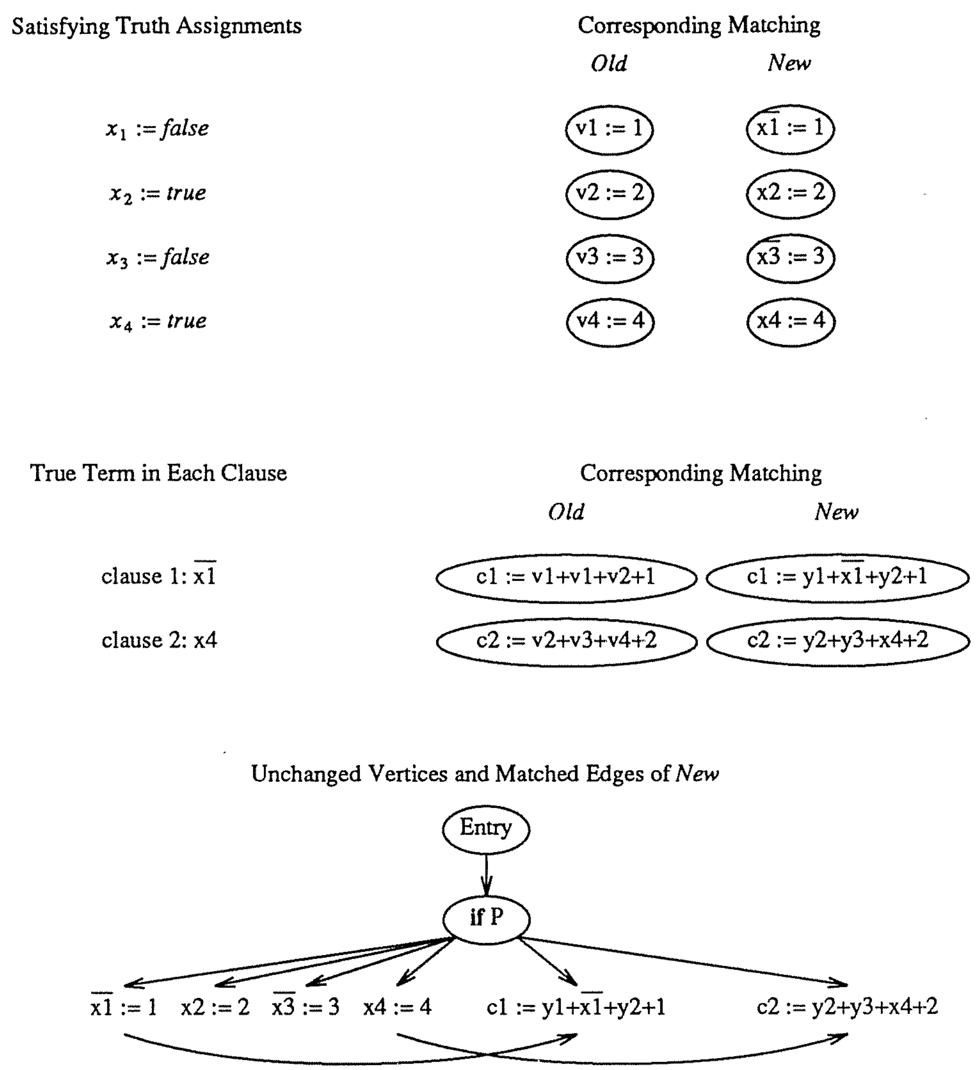


Figure 9. The matching of the vertices and edges of *New* and *Old* that corresponds to a solution for the 3-CNF formula $(x_1 \vee \overline{x_1} \vee x_2) \wedge (\overline{x_2} \vee x_3 \vee x_4)$. In the illustration of the unchanged vertices and matched edges of *New*, only the vertices shown inside circles are matched with textually identical vertices of *Old*.

[Yang89]; for example, the components "y := x" and "output(y)" of program New_2 of Figure 1, and the components "y := a" and "output(y)" of program New_3 of Figure 1 would be classified as semantic changes if slicing were used in place of partitioning.

5. CONCLUSIONS

We have discussed three algorithms for comparing two versions of a program and identifying their semantic and textual differences. All three algorithms use the technique for partitioning programs introduced in [Yang89]. Although the partitioning technique is currently applicable only to a limited language, we believe that it can be extended to include many standard programming language constructs. Extensions to the partitioning algorithm translate directly into extensions to the program-comparison algorithms; thus, we believe that the algorithms described here will soon be applicable to a reasonable language, for example, Pascal without procedure parameters.

After extending the partitioning algorithm, we will be able to implement the three program-comparison algorithms to determine how well they work in practice. We will determine whether the third algorithm, which in theory should provide a better classification of changes than the second algorithm, does so in practice, and whether or not the NP-hard matching problem that it incorporates makes it unusable on real programs.

References

Aho74.

Aho, A., Hopcroft, J.E., and Ullman, J., *The Design and Analysis of Computer Algorithms,* Addison-Wesley, Reading, MA (1974).

Aho86.

Aho, A., Sethi, R., and Ullman, J., *Compilers: Principles, Techniques and Tools,* Addison-Wesley, Reading, MA (1986).

Alpern88.

Alpern, B., Wegman, M.N., and Zadeck, F.K., "Detecting equality of variables in programs," pp. 1-11 in *Conference Record of the Fifteenth ACM Symposium on Principles of Programming Languages,* (San Diego, CA, January 13-15, 1988), ACM, New York (1988).

Cytron89.

Cytron, R., Ferrante, J., Rosen, B.K., Wegman, M.N., and Zadeck, K., "An efficient method of computing static single assignment form," pp. 25-35 in *Conference Record of the Sixteenth ACM Symposium on Principles of Programming Languages,* (Austin, TX, Jan. 11-13, 1989), ACM, New York, NY (1989).

Ferrante87.

Ferrante, J., Ottenstein, K., and Warren, J., "The program dependence graph and its use in optimization," *ACM Transactions on Programming Languages and Systems,* (1987).

Hopcroft71.

Hopcroft, J.E., "An *n* log *n* algorithm for minimizing the states of a finite automaton," *The Theory of Machines and Computations,* pp. 189-196 (1971).

Horwitz88.

Horwitz, S., Prins, J., and Reps, T., "Integrating non-interfering versions of programs," pp. 133-145 in *Conference Record of the 15th ACM Symposium on Principles of Programming Languages,* (San Diego, CA, January 13-15, 1988), ACM, New York (1988).

Horwitz88a.

Horwitz, S., Reps, T., and Binkley, D., "Interprocedural slicing using dependence graphs," *Proceedings of the SIGPLAN 88 Conference on Programming Language Design and Implementation,* (Atlanta, GA, June 22-24, 1988), *ACM SIGPLAN Notices* **23**(7) pp. 35-46 (July, 1988).

Horwitz89.

Horwitz, S., Prins, J., and Reps, T., "Integrating non-interfering versions of programs," *ACM Transactions on Programming Languages and Systems* **11**(3) pp. 345-387 (July, 1989).

Kuck81.

Kuck, D.J., Kuhn, R.H., Leasure, B., Padua, D.A., and Wolfe, M., "Dependence graphs and compiler optimizations," pp. 207-218 in *Conference Record of the Eighth ACM Symposium on Principles of Programming Languages,* (Williamsburg, VA, January 26-28, 1981), ACM, New York (1981).

Lu79.

Lu, S.Y., "A tree-to-tree distance and its application to cluster analysis," *IEEE Transactions on Pattern Analysis and Machine Intelligence* **PAMI-1**(2) pp. 219-224 (April, 1979).

Miller85.

Miller, W. and Myers, E.W., "A file comparison program," *Software – Practice and Experience* **15**(11) pp. 1025-1040 (November, 1985).

Nakatsu82.

Nakatsu, N., Kambayashi, Y., and Yajima, S., "A longest common subsequence algorithm suitable for similar text strings," *Acta Informatica* **18** pp. 171-179 (1982). (as cited in [Miller85])

Ottenstein84.

Ottenstein, K.J. and Ottenstein, L.M., "The program dependence graph in a software development environment," *Proceedings of the ACM SIGSOFT/SIGPLAN Software Engineering Symposium on Practical Software Development Environments, (Pittsburgh, PA, April 23-25, 1984), ACM SIGPLAN Notices* **19**(5) pp. 177-184 (May, 1984).

Rosen88.

Rosen, B., Wegman, M.N., and Zadeck, F.K., "Global value numbers and redundant computations," pp. 12-27 in *Conference Record of the Fifteenth ACM Symposium on Principles of Programming Languages,* (San Diego, CA, January 13-15, 1988), ACM, New York (1988).

Sankoff72.

Sankoff, D., "Matching sequences under deletion/insertion constraints," *Proc. Nat. Acad. Sci.* **69**(1) pp. 4-6 (January, 1972).

Selkow77.

Selkow, S.M., "The tree-to-tree editing problem," *Information Processing Letters* **6**(6) pp. 184-186 (December, 1977).

Shapiro70.

Shapiro, R. M. and Saint, H., "The representation of algorithms," Technical Reprot CA-7002-1432, Massachusetts Computer Associates (February, 1970). (as cited in [Alpern88, Rosen88])

Tai79.

Tai, K.C., "The tree-to-tree correction problem," *JACM* **26**(3) pp. 422-433 (July, 1979).

Tichy84.

Tichy, W., "The string-to-string correction problem with block moves," *ACM Transactions on Computer Systems* **2**(4) pp. 309-321 (November, 1984).