Load balancing while solving large linear integer problems for enumeration purposes

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introduction

disciplines:

- experimental design (statistics)
- integer programming (mathematical programming)
- high-throughput computing

goal:

- complete enumeration of MARS designs

what is a MARS design?

top of one of the buttes in Murray Buttes. Image processing by Paul Hammond.
Photo Credit: NASA/JPL-Caltech/MSSS/Paul Hammond

what is a MARS design?

$$\begin{pmatrix}
1 & 0 & 0 & -1 & 1 & -1 & 0 \\
0 & 1 & 0 & 0 & -1 & -1 & -1 \\
1 & 0 & 1 & 0 & 0 & 1 & -1 \\
1 & 1 & 0 & 1 & 0 & 0 & 1 \\
-1 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & -1 & -1 & 0 & 1 & 0 \\
0 & 0 & 1 & -1 & -1 & 0 & 1 \\
-1 & 0 & 0 & 1 & -1 & 1 & 0 \\
0 & -1 & 0 & 0 & 1 & 1 & 1 \\
-1 & 0 & -1 & 0 & 0 & -1 & 1 \\
-1 & -1 & 0 & -1 & 0 & 0 & -1 \\
1 & -1 & -1 & 0 & -1 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 & -1 & 0 \\
0 & 0 & -1 & 1 & 1 & 0 & -1
\end{pmatrix}$$

minimally
aliased
response
surface
designs

what is a MARS design?

$$m \times n \quad \{-1, 0, 1\} \quad matrix$$

n runs

$$\begin{pmatrix}
1 & 0 & 0 & -1 & 1 & -1 & 0 \\
0 & 1 & 0 & 0 & -1 & -1 & -1 \\
1 & 0 & 1 & 0 & 0 & 1 & -1 \\
1 & 1 & 0 & 1 & 0 & 0 & 1 \\
-1 & 1 & 1 & 0 & 1 & 0 & 0 \\
0 & 1 & -1 & -1 & 0 & 1 & 0 \\
0 & 0 & 1 & -1 & -1 & 0 & 1 \\
-1 & 0 & 0 & 1 & -1 & 1 & 0 \\
0 & -1 & 0 & 0 & 1 & 1 & 1 \\
-1 & 0 & -1 & 0 & 0 & -1 & 1 \\
-1 & -1 & 0 & -1 & 0 & 0 & -1 \\
1 & -1 & -1 & 0 & -1 & 0 & 0 \\
0 & -1 & 1 & 1 & 0 & -1 & 0 \\
0 & 0 & -1 & 1 & 1 & 0 & -1
\end{pmatrix}$$

m factors

$i)$ columns sum up to zero

$ii)$ columns are orthogonal

$iii)$ component-wise multiplication of any 3 columns produce a column that sums up to zero

desirable statistical properties

why is this important?

there is a small set of MARS designs and they have became standard in response surface methodology

designs with less runs which give the same amount of information of bigger ones

designs which perform well under conflicting criteria

how do we find them?

$$\sum_{p\in\Omega} y^p = n$$

$$\sum_{p\in\Omega_{0i}} y^p = n_0^{ME} \qquad 1 \le i \le m$$

$$\sum_{p\in\Omega_{0ij}} y^p = n_0^{IE} \qquad 1 \le i < j \le m$$

$$\sum_{p\in\Omega} \alpha_{ij}^p y^p = 0 \qquad 1 \le i < j \le m$$

$$\sum_{p\in\Omega} \alpha_{ijk}^p y^p = 0 \qquad 1 \le i \le j \le k \le m$$

$$y^p \in \{0,1\} \qquad p \in \Omega$$

$|\Omega| = 3^m - 1$

$S \subset \Omega :=$ basic design

$G :=$ group of permutations of levels and factors

$|G| = 2^m m!$

iteratively add isomorphism inequalities:

$$\sum_{p\in g(s)} y^p \le n-1, \forall g \in G$$

enumeration tree exploration

what are the problems?

Andy Warhol's Marilyn Monroe Series, 1967

MARS designs have
huge isomorphic groups

mathematical programming
techniques help with this

what are the problems?

$$\sum_{p \in \Omega} y^p = n$$

$$\sum_{p \in \Omega_{0i}} y^p = n_0^{ME} \qquad 1 \leq i \leq m$$

$$\sum_{p \in \Omega_{0ij}} y^p = n_0^{IE} \qquad 1 \leq i < j \leq m$$

$$\sum_{p \in \Omega} \alpha_{ij}^p y^p = 0 \qquad 1 \leq i < j \leq m$$

$$\sum_{p \in \Omega} \alpha_{ijk}^p y^p = 0 \qquad 1 \leq i \leq j \leq k \leq m$$

$$y^p \in \{0,1\} \qquad p \in \Omega$$

tree of exponential size

$|\Omega| = 3^m - 1$

$S \subset \Omega :=$ basic design

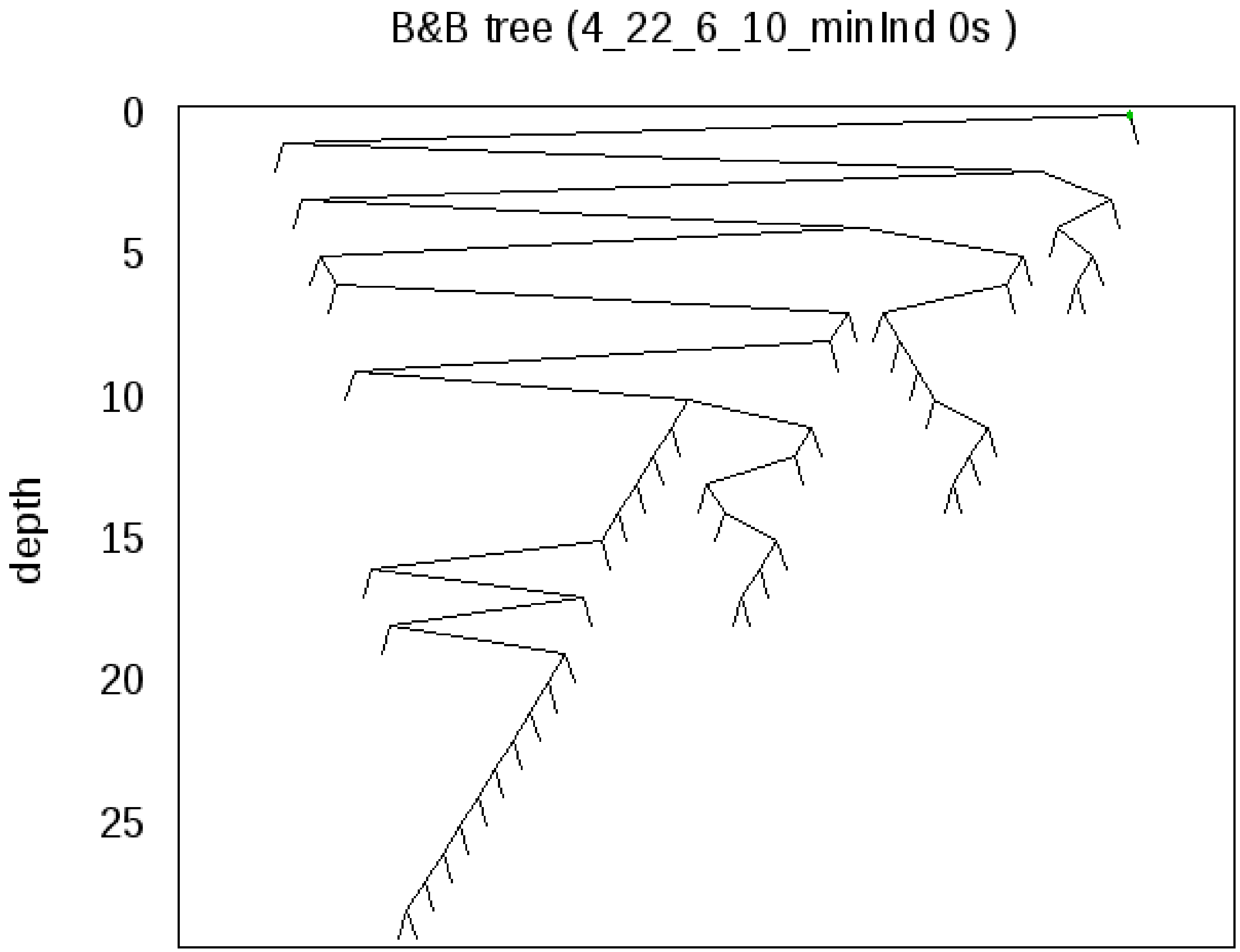
$G :=$ group of permutations of levels and factors

$|G| = 2^m m!$

iteratively add isomorphism inequalities:

$$\sum_{p \in g(s)} y^p \leq n - 1, \forall g \in G$$

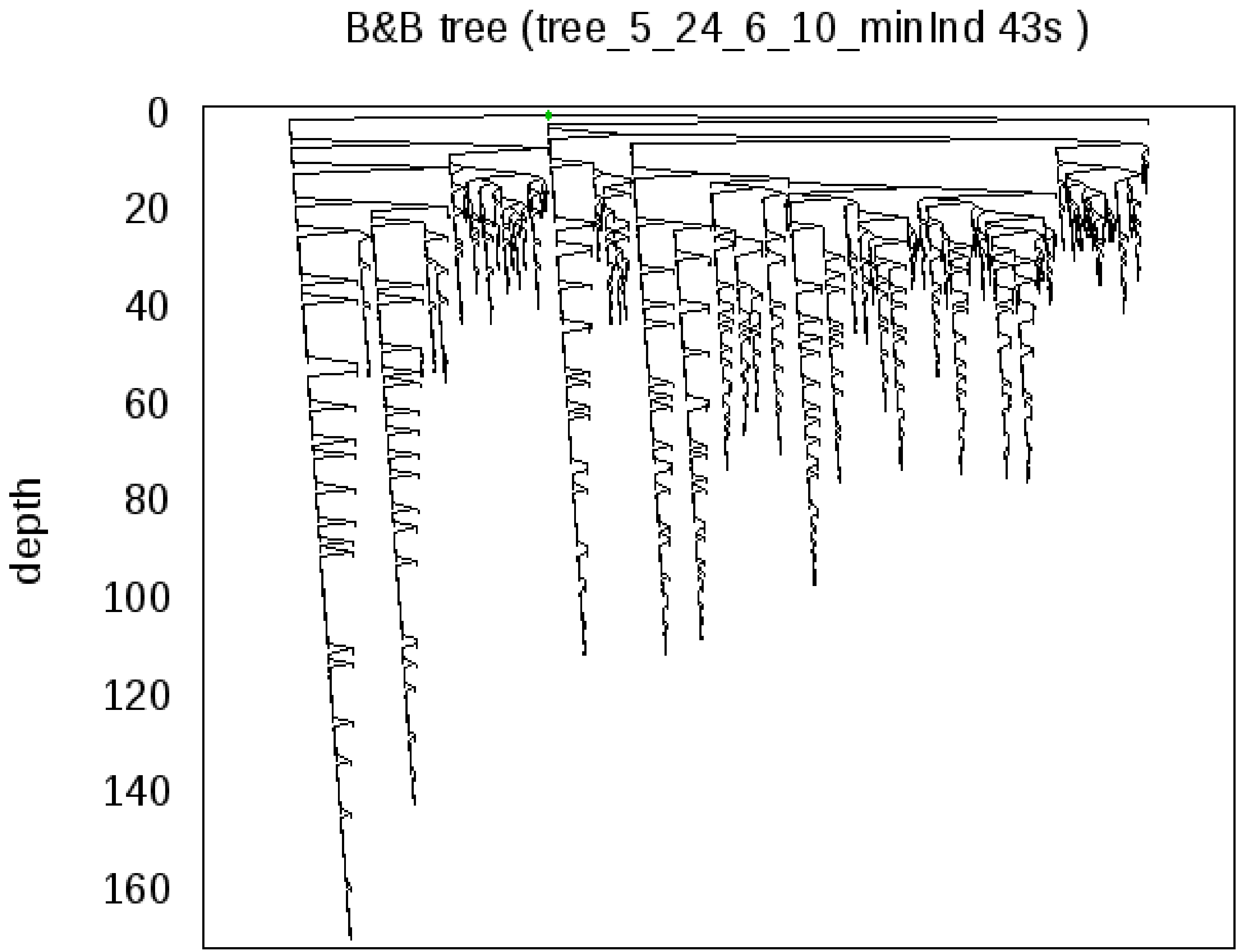
what does the enumeration tree look like?



skinny

deep

what does the enumeration tree look like?



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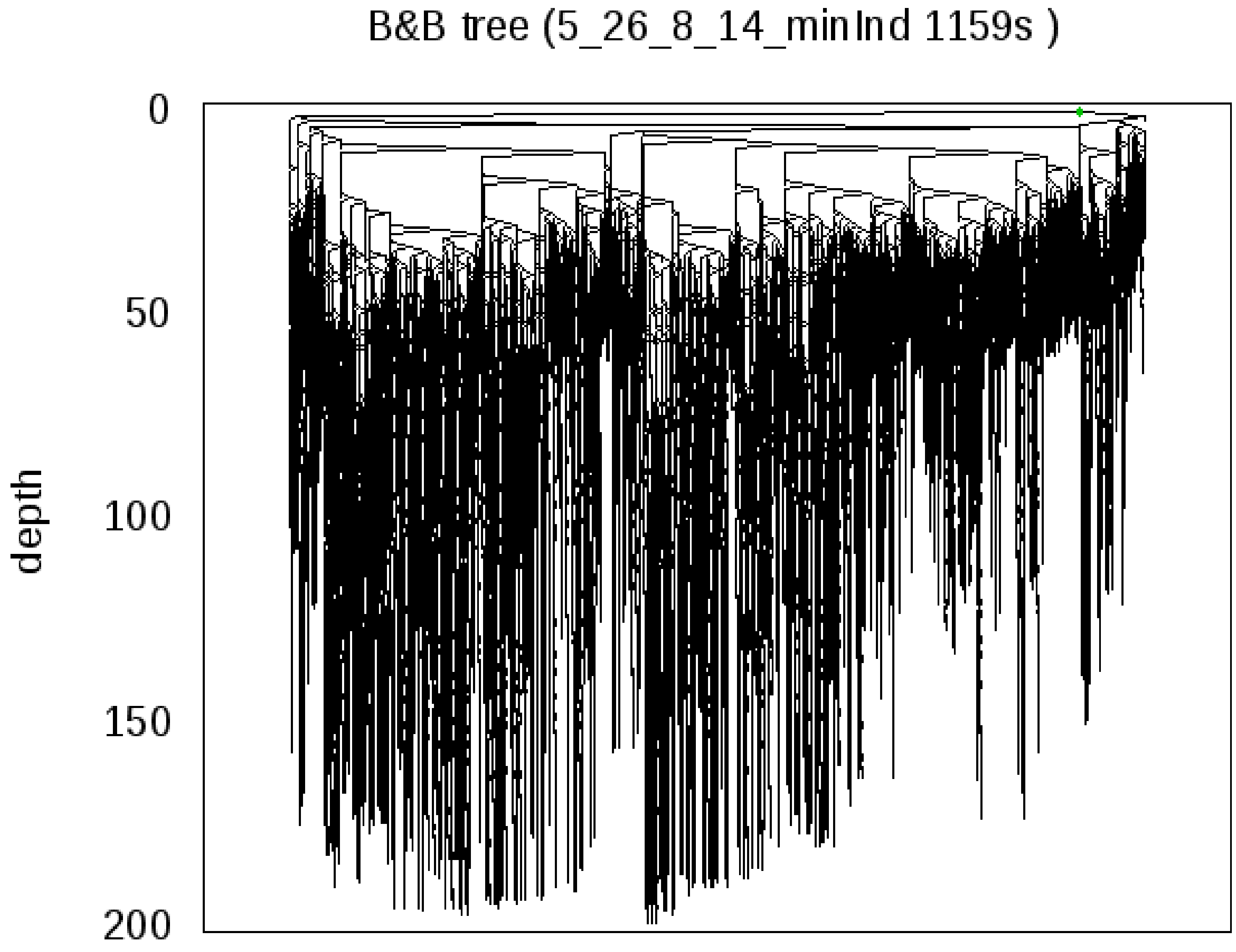

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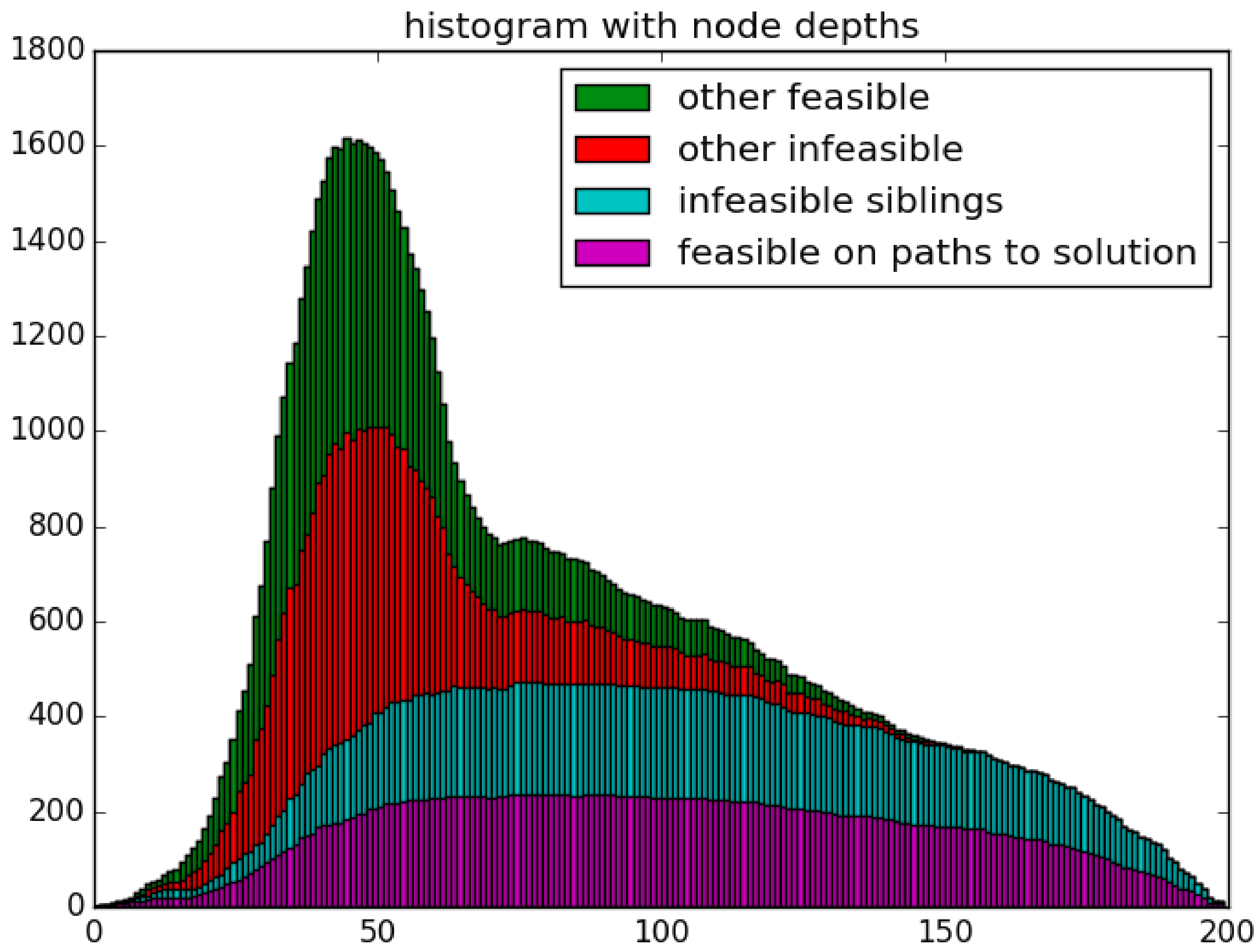


why htcondor?

unknown number of processed nodes (potentially huge)

long processing time

“pleasantly parallel”, little communication and synchronization needed

our load balancing scheme

element 1: Knuth estimation
done ntimes, if predicted size > threshold
then we do BFS, otherwise DFS

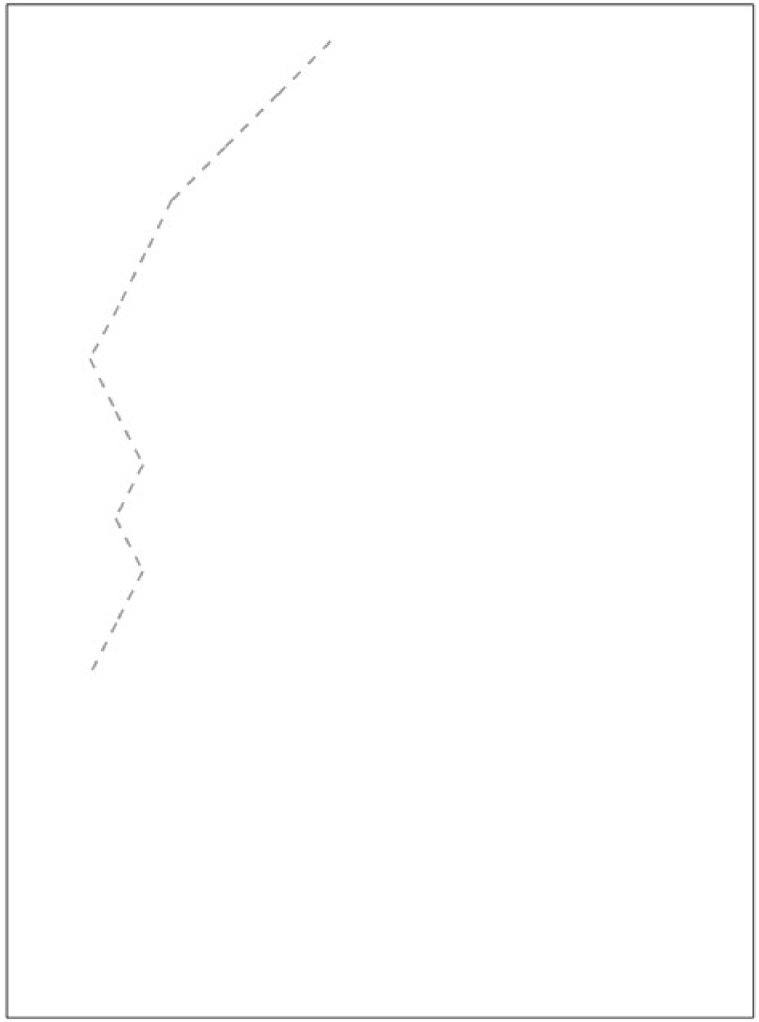
element 2: breadth-first-search (BFS)
until a certain depth determined
dynamically by a max processing time
OR a max number of open nodes

our load balancing scheme

element 3: depth-first-search (DFS)
faster and more memory efficient than BFS, creates less open nodes while evaluating more nodes, <u>max processing time</u>

element 4: trimming
after BFS and DFS (if not solved) we solve every open node if the solution time < <u>max processing time of a trivial node</u>, otherwise we store the open node data

our load balancing scheme

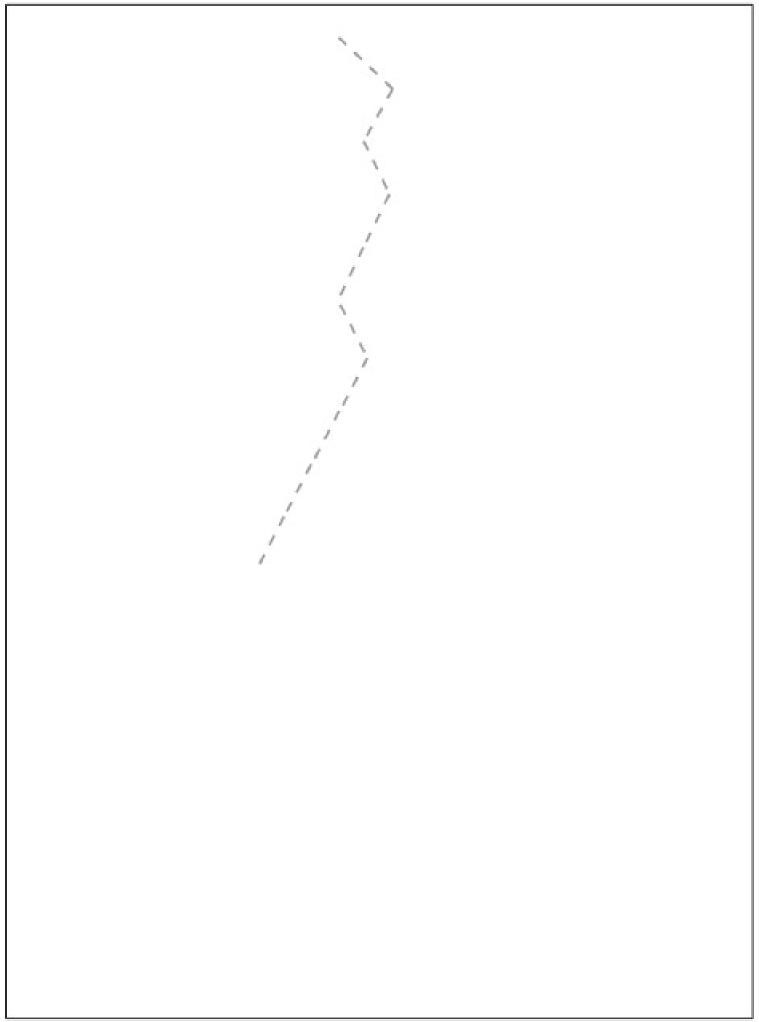


Knuth dives from root node

this is diving 1...

with predicted size < threshold

our load balancing scheme

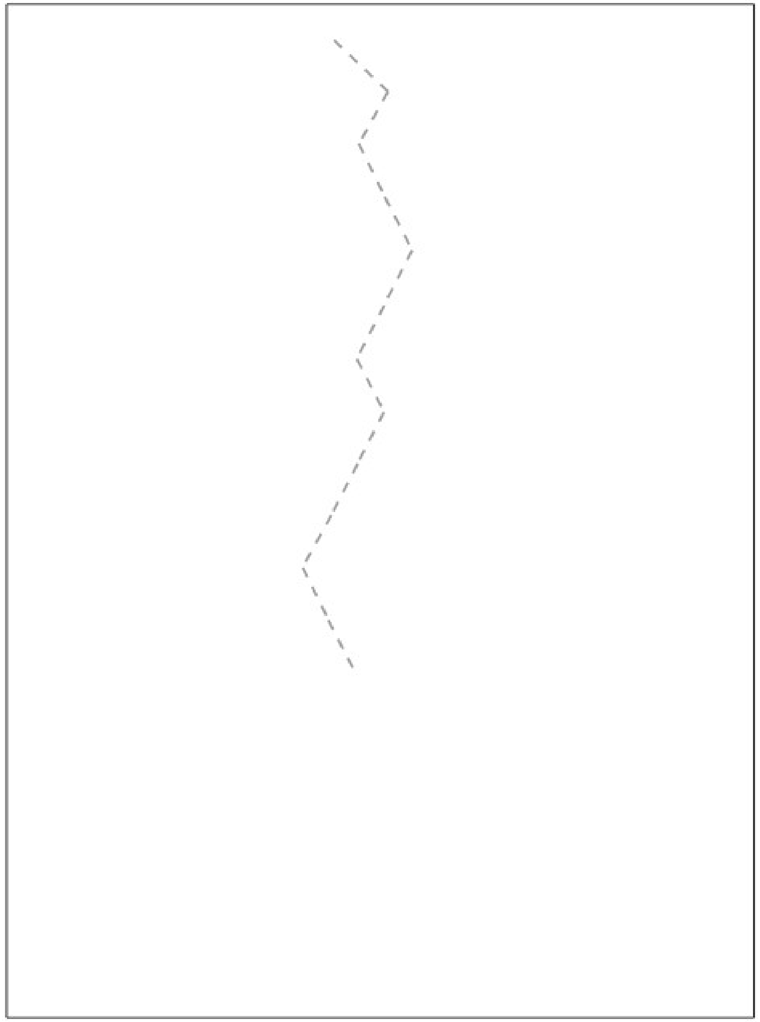


Knuth dives from root node

this is diving 2...

with predicted size < threshold

our load balancing scheme

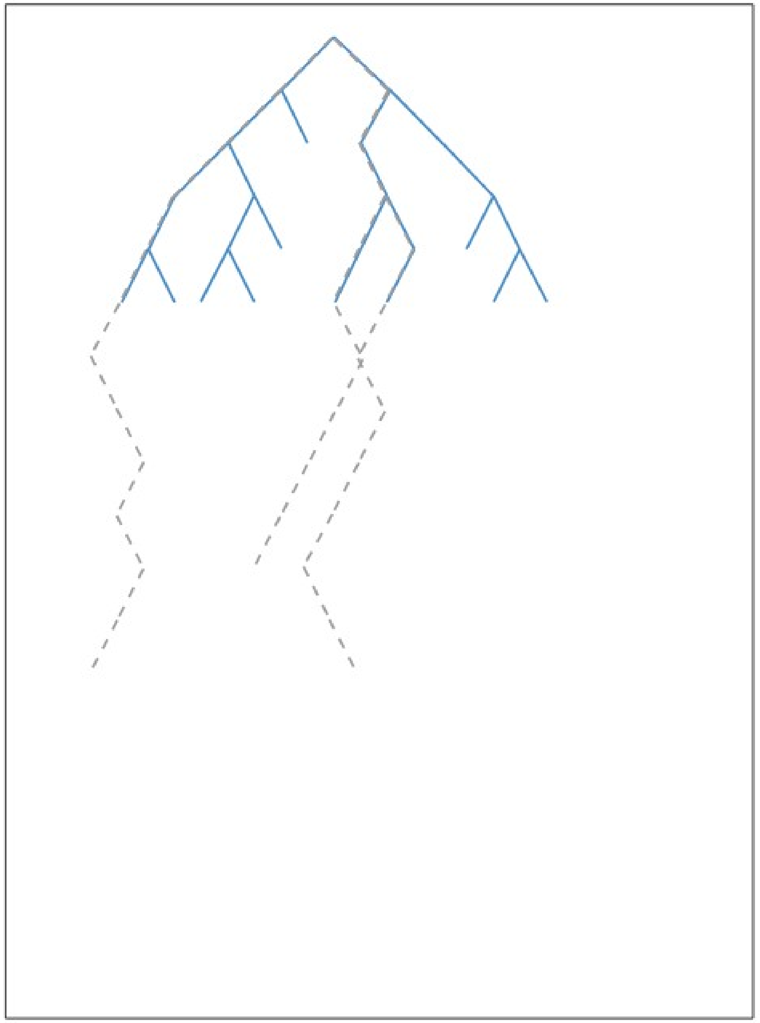


Knuth dives from root node

this is diving 3...

with predicted size > threshold

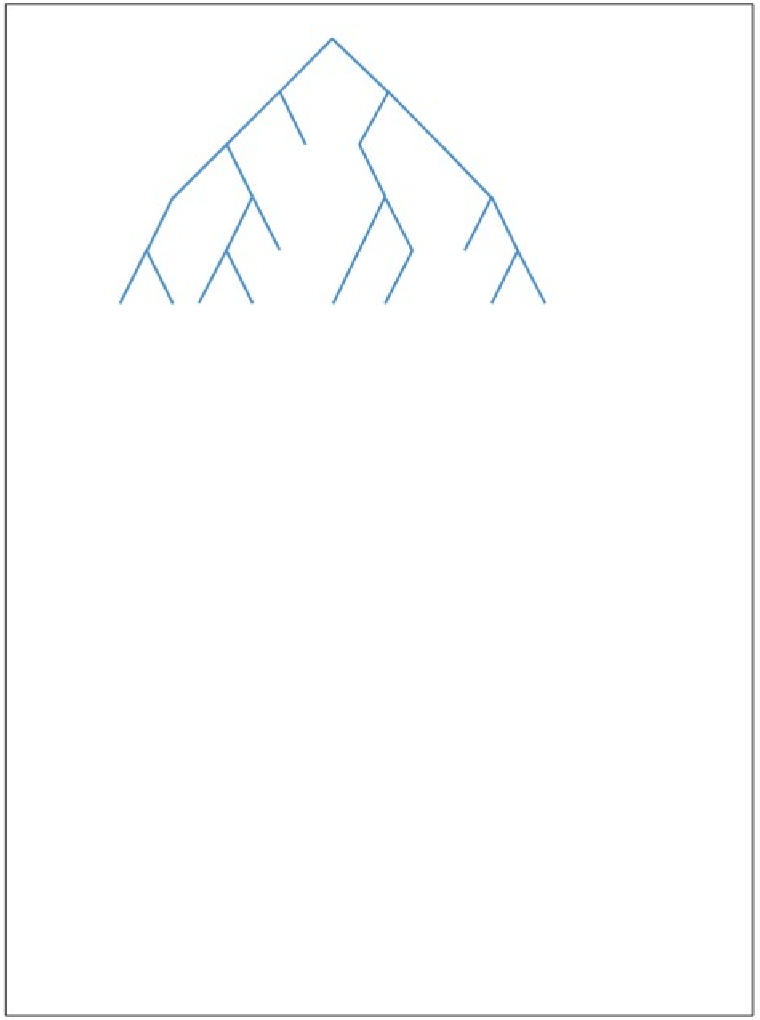
our load balancing scheme



we then do BFS from root node

dynamical depth, which depends on time/number of open nodes

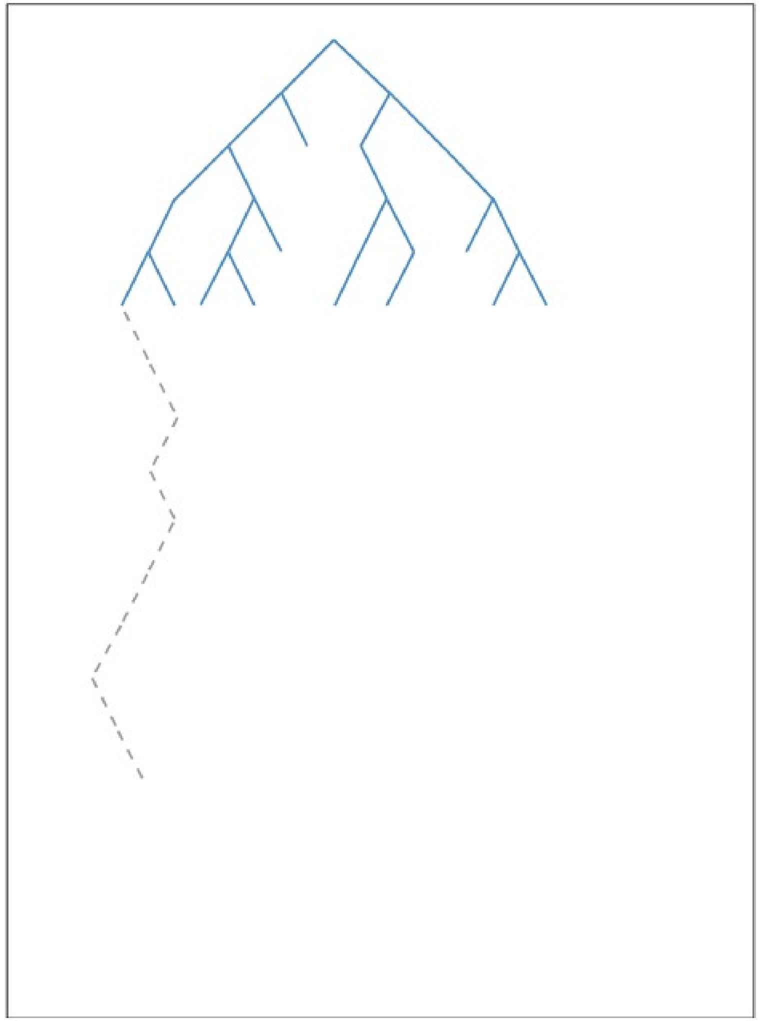
our load balancing scheme



now we repeat the process for each one of the open nodes …

let's do some Knuth dives ...

our load balancing scheme

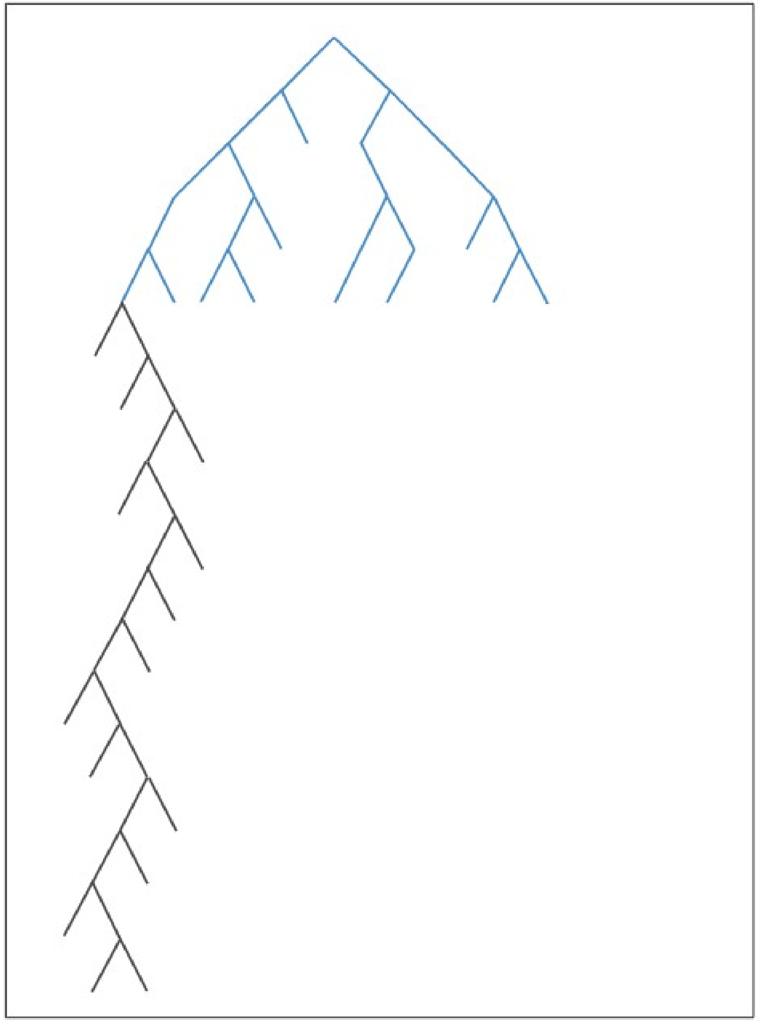


this time

predicted size < threshold

do DFS from this node

our load balancing scheme

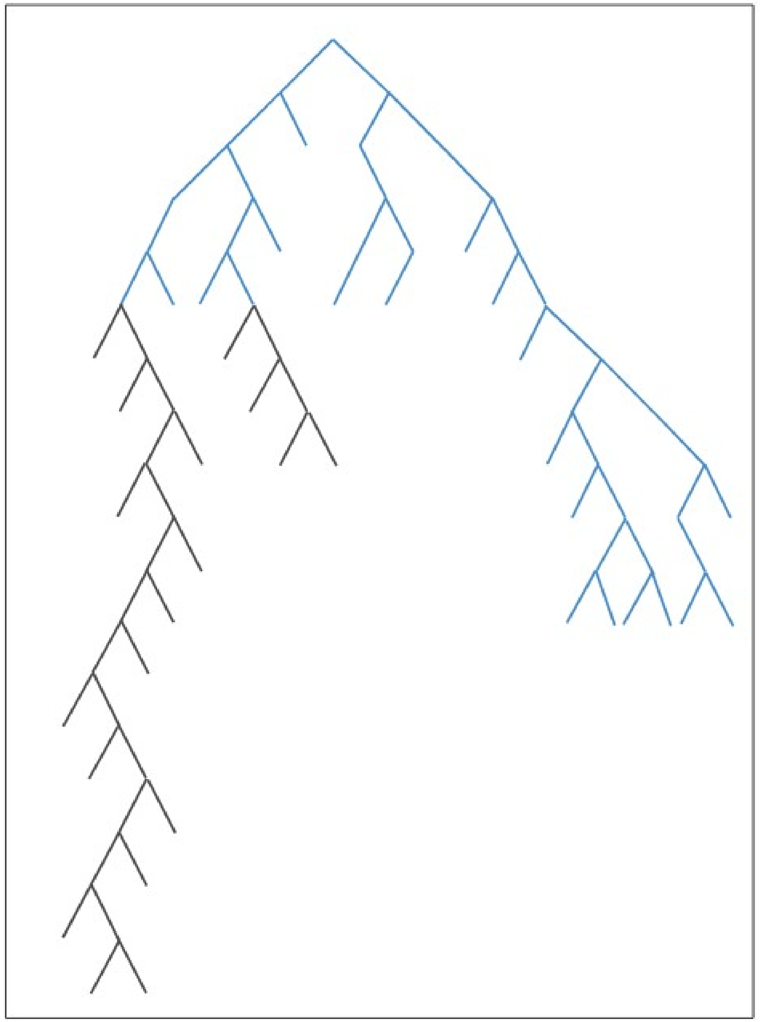


this time

predicted size < threshold

do DFS from this node

our load balancing scheme



we repeat the process on the open nodes

HTCondor DAGMan files

marsd.dag

```
SUBDAG EXTERNAL workers workers.dag
SCRIPT POST workers marsdOneIter.sh 7
RETRY workers 1000
```

workers.dag

```
JOB main submit-solve.cmd
```

input data: 30-36MB

output data < 1MB

executables

marsdOneIter.sh

identifies the open nodes
writes the htcondor submit file
dinamically tune the parameters

marsd

does the load balance

achievements

size	6	7	8
#nodes	$2{,}276 \times 10^6$	704×10^6	166×10^6
CPU time (years)	3.32	5.57	20.52
#solutions	296,193	20,184	521

acknowledgement

Peter Goos, my promotor at KU Leuven

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thank you!

any questions?