CS 559: Computer Graphics

Homework 3

This homework must be done individually. Submission date is Tuesday, March 2, 2004, in class.

Question 1:

In class we discussed the Gaussian filter:

$$\frac{1}{\sigma\sqrt{2\pi}}e^{-\frac{x^2}{2\sigma^2}}$$

a. Sketch or plot the Gaussian filter on the domain $[-5, 5]$ for the case when $\sigma = 1$.

b. Another filter commonly used in computer vision is the derivative of Gaussian filter, formed by taking the derivative of the Gaussian filter (with respect to x). Write down the formula for the derivative of Gaussian filter and sketch or plot it (again using $\sigma = 1$).

c. Build a discrete 1D filter mask for the derivative of Gaussian filter, using a filter width of 5 and $\sigma = 1$.

d. What is the output if you filter the 1D image below? Only apply the filter at places where all the underlying pixels exist, so you end up with six pixels as your output.

$$1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1$$

e. What is the output if you filter the 1D image below?

$$1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 0 \quad 0 \quad 0 \quad 0 \quad 0$$

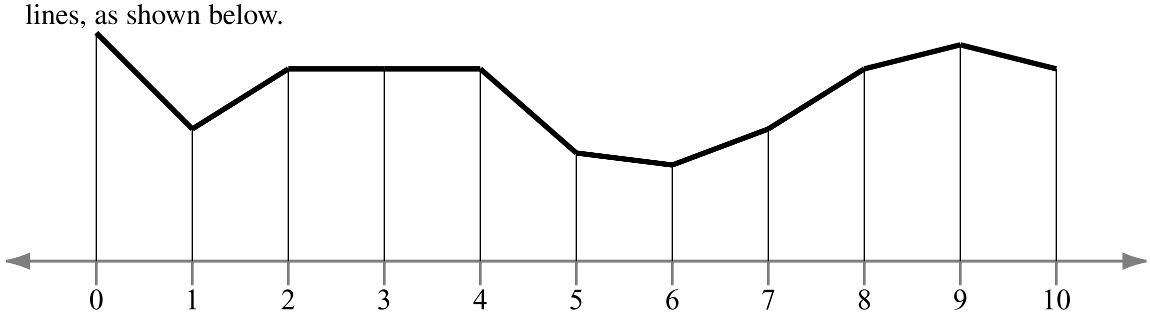
f. What kind of image feature – constant brightness or edge – do you expect a 2D derivative of Gaussian filter to respond to?

Question 2:

Say that we point sample a band-limited function into a 1D ”image”, by taking the value of the function at the points $x = \{0, 1, \ldots, 9, 10\}$. In class, we saw that this corresponds to convolution with a sequence of spikes in the frequency domain. We also saw that the function can be reconstructed perfectly by multiplying with a box filter in the frequency domain.

a. What sort of function do we need to convolve with in the spatial domain to perform perfect reconstruction?

b. Say that a function plotter works by taking the 1D image and joining the samples together with straight lines, as shown below.



What type of function is the plotter convolving our samples with in the spatial domain?

c. What filter function is the plotter multiplying by in the frequency domain?

d. What additional frequencies (high or low) will appear in the plotted image, and how do they visually manifest themselves?

Question 3:

Compositing is used in real time 3D graphics to mimic transparent or partially transparent surfaces. The *over* operator is most frequently used, although it is also possible to use the other operators. This question explores the importance of the order in which transparent objects are drawn. We wish to create an image of a room seen through a stained glass window.

a. Consider an image of a white wall, with pixel values $b = (1, 1, 1, 1)$ (fully opaque). We now draw a piece of green furniture in front of the wall by composing $f = (0, 1, 0, 1)$ over the wall: $f\ over\ b$. What pre-multiplied color results?

b. To simulate the effect of viewing this through an stained glass window, the window, with pre-multiplied color $w = (0.5, 0.0, 0.0, 0.5)$ is drawn over the furniture and wall using the operation $w\ over\ (f\ over\ b)$. What is the resulting pre-multiplied color?

c. Now consider the situation in which you draw the wall, then the window, and then the furniture. To give the correct appearance, you need some compositing operation, *op*, such that $f\ op(w\ over\ b) = w\ over\ (f\ over\ b)$. Do any of the compositing operations discussed in class give the right result?

d. What do you think the drawing order must be enable correct transparency through compositing?

Question 4:

It takes three points to define an affine transformation in 2D. Say that the point $(1, 1)$ goes to $(4, \sqrt{2})$, that $(1, -1)$ goes to $(4 + \sqrt{2}, 0)$, and that the point $(-1, 1)$ goes to $(4 - \sqrt{2}, 0)$. Assume that the affine transformation is described by the following homogeneous matrix equation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} & b_x \\ a_{yx} & a_{yy} & b_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

a. Write out six linear equations involving the unknowns in the matrix equation above and the coordinates of the given points. (Hint: Plug the values for each point in turn into the above matrix equation and expand the results.)

b. Solve the equations to find the unknowns and hence write out the transformation matrix.

c. Consider the points to be the corners of a triangle. Carefully draw a picture showing the initial and final positions of the triangle.

d. Determine from the picture the sequence of rotations and translations required for the transformation. Write out the rotation matrix (or matrices) required and the translation matrix (or matrices) required.

e. Show that the composition of the rotation and transformation matrices from the previous question is the same as the matrix you derived from the linear equations.

Question 5

A pair of transformations is said to commute if the order in which you apply them does not matter. In terms of transformation matrices, that means that $\mathbf{AB} = \mathbf{BA}$. Consider three rotation matrices and a translation matrix:

$$
\begin{aligned}
\mathbf{R}_1 &= \begin{bmatrix} c_1 & 0 & s_1 & 0 \\ 0 & 1 & 0 & 0 \\ -s_1 & 0 & c_1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\mathbf{R}_2 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\mathbf{R}_3 &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_3 & -s_3 & 0 \\ 0 & s_3 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \\
\mathbf{T} &= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & t_y \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

a. What axis is $\mathbf{R}_1$ rotating about?

b. What axis are $\mathbf{R}_2$ and $\mathbf{R}_3$ rotating about?

c. Do $\mathbf{R}_1$ and $\mathbf{R}_2$ commute?

d. Do $\mathbf{R}_2$ and $\mathbf{R}_3$ commute?

e. Under what circumstances do two rotation matrices commute?

f. Do $\mathbf{R}_1$ and $\mathbf{T}$ commute?

g. Do $\mathbf{R}_2$ and $\mathbf{T}$ commute?

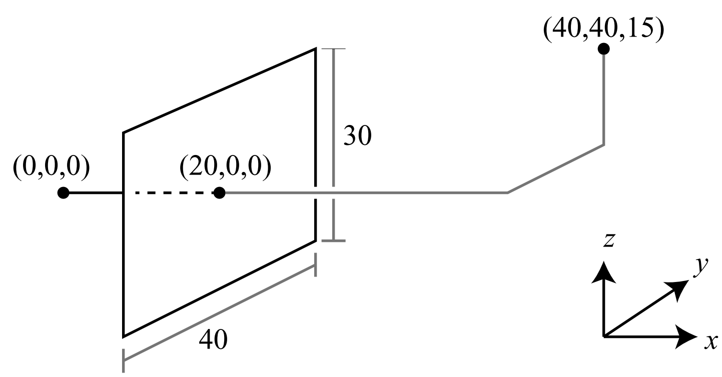
h. Under what circumstances can you say that a rotation and a translation commute?

Question 6:

Conventional graphics assumes that the person looking at the screen is seated directly in front of the screen and is focused at the center of the screen. However, for virtual reality applications you want the world to really appear to be behind the screen. This requires tracking the user's head and presenting an image that matches the arrangement of their eyes and the screen. In particular, the near clipping plane should coincide with the window, and the viewer's virtual eyes should coincide with their real location.

To make all this work, you first create a global coordinate system that is the same as a real world coordinate system. In the set-up below, the origin of the virtual world (and real world) is behind the screen. The center of the screen is at (20, 0, 0), the "right" on the screen is in the direction (0, 1, 0) and "up" on the screen is in the direction (0, 0, 1). The screen has width 40 and height 30. The viewer's eye is at (40,40,15).

Use the notation from class (and in the textbook). Note that in this instance the gaze direction is **not** the direction the viewer is looking, rather it is the vector looking into the world perpendicular to the view plane. In this situation, in fact, it only matters where the viewer's eyes are, not the direction they are looking.



a. What is the vector e?

b. What is the vector w?

c. What is the vector u?

d. What is the vector v?

e. What is the near clip plane distance?

f. What are l, r, b, t that define the left, right, top and bottom clip planes?