

CS 559: Computer Graphics

Homework 3

This homework must be done individually. Submission date is Tuesday, March 5 in class.

Question 1:

The 2D edge-detect (high-pass) filter that we looked at in class has the following form if you ignore the constant:

$$\begin{array}{ccc} -1 & -2 & -1 \\ -2 & 12 & -2 \\ -1 & -2 & -1 \end{array}$$

- a. What is the output if you filter the image below? Only apply the filter at places where all the underlying pixels exist, so you end up with a 4×1 result.

$$\begin{array}{cccccc} 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \end{array}$$

- b. What is the output if you filter the image below (giving a 6×1 output image)?

$$\begin{array}{ccccccc} 1 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \end{array}$$

- c. Does the filter respond more to a diagonal or vertical edge?
- d. An alternate edge-detect filter is given below. What is its output on each of the above images (you should have a 4×1 and a 6×1 answer)?

$$\begin{array}{ccc} -1 & 2 & -1 \\ -1 & 2 & -1 \\ -1 & 2 & -1 \end{array}$$

- e. Does this filter respond more to vertical or diagonal edges?
- f. Design a 3×3 filter that responds to diagonal edges like that of part (b), but gives no response to a vertical edge.

Question 2:

You wish to use compositing operations to perform a stencil operation. You have a foreground image, f , that you wish to place into a background image, b , only at places where a stencil mask, s , has a particular α value. For example, if the foreground image is all white with $\alpha = 1$, the background is all black with $\alpha = 1$ and the stencil has holes for a word, inserting the foreground into the background would result in a white word on a black background.

- Which α value would you use for the parts of the stencil that represent holes? Which value would you use for the rest? (There are two good answers to this question.)
- You plan to use two compositing operations to combine the images, with the form $(f \text{ op}_1 s) \text{ op}_2 b$, where brackets indicate precedence. Which compositing operations would you use for op_1 and op_2 ?

Question 3:

It takes three points to define an affine transformation in 2D. Say that the point $(1, 1)$ goes to $(-\frac{4}{\sqrt{2}}, \frac{2}{\sqrt{2}})$, that $(1, 0)$ goes to $(-\frac{3}{\sqrt{2}}, \frac{3}{\sqrt{2}})$, and that the point $(0, 0)$ goes to $(-\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}})$. Assume that the affine transformation is described by the following homogeneous matrix equation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} & b_x \\ a_{yx} & a_{yy} & b_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

- Draw one sketch showing the locations of the points before the transformation, and one showing their locations after. Join the points to form a triangle in each figure.
- Write out six linear equations involving the unknowns in the matrix equation above and the coordinates of the given points.
- Solve the equations to find the unknowns and hence write out the transformation matrix.
- Determine from your sketch a sequence of rotations, scalings or translations required for the transformation.
- Write out the transformation matrices for your sequence in part (d) and compose them to verify your answer to part (c).

Question 4:

You wish to construct a view that has the origin of the image plane at $(4, 0, 4)$, has the point $(0, 0, 2)$ appearing in the center of the image, and has the direction $(0, 0, 1)$ appearing to be up in the image. The following questions use the notation from lecture 10 for constructing the world to view matrix.

- a. What is the vector c ?
- b. What is the vector n ?
- c. What is the vector u ?
- d. What is the vector v ?
- e. What is the 4×4 world to view matrix?