This homework must be done individually. Submission date is Tuesday, May 8, 2001, in class.

Question 1: (8 points)

Subdivide each of the Bezier curves below into two segments. You should give the coordinates of the new vertices and indicate somehow which vertices go with which piece of new curve. Fractions are fine as part of the vertex coordinates.
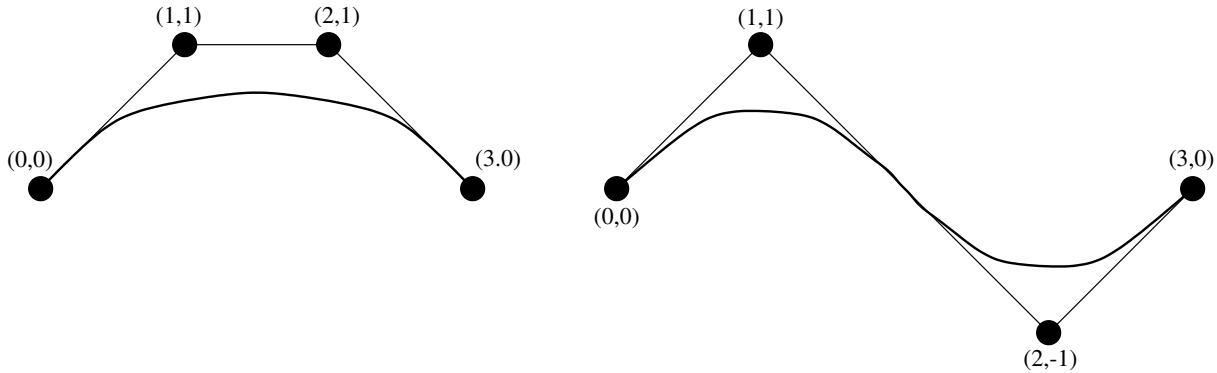
Question 2: (6 points)

Find the matrix, $M_{B \to H}$ that converts control points for a Bezier curve into control points a Hermite curve. Recall from lecture how this is done:

a. Write a point on the curve as it is evaluated using the matrix equation for Hermite curves: $x(t) = P_H^T M_H T$. Write down the appropriate matrix M_H.

b. Write a point on the curve as it is evaluated using the matrix equation for Bezier curves: $x(t) = P_B^T M_B T$. Write down the appropriate matrix M_B.

c. Now equate the two equations and rearrange and simplify them to find an equation of the form $P_H = M_{B \to H} P_B$. Show your working and write down the matrix $M_{B \to H}$.

Question 3:

Prove that uniform cubic B-spline curves are C^2 continuous everywhere. It suffices to show that, for a curve with four control points:

a. The value of the point, the first derivative and the second derivative at the parameter value $t = 0$ does not depend on the last control point.

b. The value of the point, the first derivative and the second derivative at the parameter value $t = 1$ does not depend on the first control point.

If this is true, then the point where two curve segments meet must be C^2, because the values for both segments at the join depend only on the same set of three control vertices. Everywhere else is C^2 because the blending functions are C^2.

Question 4:

Give a set of 16 control points that will put a single Bezier patch on top of two parallel squares with G^1 continuity, as sketched below. It is best to draw the control mesh of the patch and label the vertices with their coordinates. Also give a set of control points for a patch to cover one end of the open space. The second patch should have C^0 continuity with each of the edges marked a, b, c and d.