CS 559: Computer Graphics

Homework 3

This homework must be done individually. Submission date is Tuesday, March 6, 2001, in class.

Question 1:

Say that we point sample a band-limited function into a 1D "image". In class, we saw that this corresponds to convolving with a sequence of spikes in the frequency domain. We also saw that the function can be reconstructed perfectly by multiplying with a box filter in the frequency domain.

1. What sort of function do we need to convolve with in the spatial domain to perform the reconstruction?
2. Say that a function plotter works by taking the 1D image and drawing a narrow box for each sample - each box as wide as the original sample spacing. What type of function is the plotter convolving our samples with in the spatial domain?
3. What type of filter is the plotter using in the frequency domain?
4. What additional frequencies will appear in the plotted image, and how do they visually manifest themselves?

Question 2:

In class we discussed the Gaussian filter:

$$\frac{1}{\sqrt{2\pi}}e^{-\frac{x^2}{2}}$$

1. Sketch or plot the Gaussian filter on the domain $[-5, 5]$.
2. Another filter commonly used in computer vision is the derivative of Gaussian filter, formed by taking the derivative of the Gaussian filter (with respect to x). Write down the formula for the derivative of Gaussian filter and sketch or plot it.
3. What type of image feature will a derivative of Gaussian filter give a high response to?
4. Another filter, that models certain early human visual processing, is the second derivative of Gaussian filter, formed by taking the second derivative of a Gaussian filter with respect to x. Write down its formula and sketch or plot the function.
5. What type of image feature does the second derivative of Gaussian filter give its highest response to?

Question 3:

Say you begin with a background image, b, and a foreground image, f. Consider corresponding pixels of b and f, with pre-multiplied colors and alphas (c_b, α_b) and (c_f, α_f) respectively. In this question, assume the $\alpha_f > 0$, and consider the foreground to be a brush that we repeatedly dab onto the background.

1. What is the resulting pre-multiplied color and α if you compose f *over* b (one dab of the brush)?

2. What is the resulting pre-multiplied color and α if you dab the brush a second time? In other words, what color and α result from f *over* (f *over* b)?

3. What is the limit color and α if we dab the brush an infinite number of times? It helps to know that, for $0 < r \leq 1$:

$$\lim_{n\to\infty} a + a(1-r) + a(1-r)^2 + a(1-r)^3 ... = \frac{a}{r}$$

Question 4:

It takes three points to define an affine transformation in 2D. Say that the point $(1, 1)$ goes to $(4, 4)$, that $(1, -1)$ goes to $(4 + \sqrt{2}, 4 - \sqrt{2})$, and that the point $(-1, 1)$ goes to $(4 - \sqrt{2}, 4 - \sqrt{2})$. Assume that the affine transformation is described by the following homogeneous matrix equation:

$$\begin{bmatrix} x' \\ y' \\ 1 \end{bmatrix} = \begin{bmatrix} a_{xx} & a_{xy} & b_x \\ a_{yx} & a_{yy} & b_y \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

1. Write out six linear equations involving the unknowns in the matrix equation above and the coordinates of the given points.

2. Solve the equations to find the unknowns and hence write out the transformation matrix.

3. Consider the points to be the corners of a triangle. Draw a picture showing the initial and final positions of the triangle.

4. Determine from the picture the sequence of rotations and translations required for the transformation.

5. Show that the composition of the rotation and transformation matrices from the previous question is the same as the matrix you derived from the linear equations.

Question 5

A pair of transformations is said to commute if the order in which you apply them does not matter. In terms of transformation matrices, that means that $\mathbf{AB} = \mathbf{BA}$. Which of the following transformation pairs commute **in 3D**?

1. translate - translate

2. scale - scale

3. rotate - translate

4. rotate - scale