Before the Discovery of Perspective

- di Bartolo, “The Nativity of the Virgin” (c. 1400)
- di Giovanni Fei, “The Presentation of the Virgin” (c. 1400)
- Ambrogio Lorenzetti (1342) The presentation in the temple. Panel, Uffizi, Florence
Euclid's Optics (300 BC)

- Visual ray: from point on object to eye
- Visual cone: from contour of object to eye
- Euclid's Law: diminution in visual angle with distance

Natural Perspective

Italian Renaissance

- Linear perspective
  - Illusionistic 3D space
  - Sculptural body
  - Natural pose, individual expression
  - Humanized suffering

"Perspective is nothing else than the seeing of an object through a sheet of glass, on the surface of which may be marked all the things that are behind the glass."

-- Leonardo

"First of all, on the surface on which I am going to paint, I draw a rectangle of whatever size I want, which I regard as an open window, through which the subject to be painted is seen."

-- Alberti (1435-6)
**Alberti’s Veil**

- Grid system

Alberti’s Veil

Draughtsman

drawing a reclining nude. Woodcut.

Reconstructed view through

**Point-Plotting Method**

- Use strings to embody Euclid’s visual rays

Point-Plotting Method

Two draughtsmen plotting points for the drawing of a lute in foreshortening. Woodcut.

**Alberti’s Method (1435): “Construzione Legittima”**

1. Draw “open window”, with a human figure 3 braccia high
2. Mark baseline in units of 1 braccio
3. Draw Centric Point at eye level (determines severity of convergence)
4. Draw orthogonals
5. Draw horizon line

Centric Point

horizon line
orthogonals

6. Draw “little space”, with a point at the height of the Centric Point (like elevation view, with eye point)
7. Draw baseline with units of 1 braccio (like ground plane)
8. Draw a vertical line (like picture plane)
9. Draw diagonals (like visual rays)
10. Draw transversals at intersections

“Little Space”

perpendicular
diagonals
transversals

c
d
Modified Alberti Method

- Slide the “little space” over so the right side of the rectangle becomes the picture plane
- DB is a “check line” for verifying correctness

Masaccio’s “Trinity” (c. 1425-8)

- The oldest existing example of linear perspective in Western art
- Use of “snapped” rope lines in plaster
- Vanishing point below orthogonals implies looking up at vaulted ceiling

Piero della Francesca, “Flagellation of Christ” (c. 1455)

- Carefully planned
- Strong sense of space
- Low eye level

Leonardo da Vinci, “Last Supper” (c. 1497)

- Use of perspective to direct viewer’s eye
- Strong perspective lines to corners of image
Properties of Perspective Projection

- Object size changes as it translates along $z$ axis (scale effect)

$\text{Magnification } m = \frac{|z|}{|z|}
\Rightarrow \text{distance btw points not preserved}$

- As $f$ gets smaller, more world points project onto finite image plane $\Rightarrow$ more wide angle image

- As $f$ gets larger, more telescopic

- Lines in 3D project to lines in 2D

Distant Objects are Smaller

Geometric Properties of Projection

- Points go to points
- Lines go to lines
- Planes go to whole image
- Polygons go to polygons
- Degenerate cases
  - line through focal point to point
  - plane through focal point to line

Parallel Lines Meet

Common to draw film plane in front of the focal point.

Moving the film plane merely scales the image.
Vanishing Points

- each set of parallel lines (= direction) meets at a different point
  - The vanishing point for this direction
- Sets of parallel lines on the same plane lead to collinear vanishing points
  - The line is called the horizon for that plane

- Good ways to spot faked images
  - scale and perspective don’t work
  - vanishing points behave badly
  - supermarket tabloids are a great source
Camera Matrix

- Turn previous expression into HC's
  - HC's for 3D point are (X,Y,Z,T)
  - HC's for point in image are (U,V,W)
Projection Matrix for Orthographic Projection

\[
\begin{pmatrix}
U
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 & 0
\end{pmatrix}
X
\]

\[
\begin{pmatrix}
V
\end{pmatrix} = \begin{pmatrix}
0 & 1 & 0 & 0
\end{pmatrix}
Y
\]

\[
\begin{pmatrix}
W
\end{pmatrix} = \begin{pmatrix}
0 & 0 & 1 & 0
\end{pmatrix}
Z
\]

Camera Parameters

- Issue
  - camera may not be at the origin, looking down the z-axis
    - extrinsic parameters
  - one unit in camera coordinates may not be the same as one unit in world coordinates
    - intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.
  
\[
\begin{pmatrix}
U
\end{pmatrix} = \begin{pmatrix}
\text{Transformation} & 1 & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\text{Transformation} & 0 & 1 & 0 & 0
\end{pmatrix}
\begin{pmatrix}
\text{Intrinsic parameters} & 0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
X
\end{pmatrix}
\]

- Note: Since image plane at \( e \neq f \), perspective projection equation can be written as:

\[
\begin{pmatrix}
X'
Y'
Z'
\end{pmatrix} = \begin{pmatrix}
f & 0 & 0 & 0
0 & f & 0 & 0
0 & 0 & 1 & 0
\end{pmatrix}
\begin{pmatrix}
x
y
z
\end{pmatrix}
\]

and \( x' = \frac{X'}{Z'} \)

\( y' = \frac{Y'}{Z'} \)

\( \Rightarrow \) Camera = linear projective transform from 3D projective space to 2D projective plane

- 3x4 matrix called camera perspective projection matrix