CS559 – Lecture 24
B-Spline Basics

These are course notes (not used as slides)

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How do we make Smooth Curves?

- Will be approximating
- Want flexibility
  - Any number of points
  - Any degree of continuity
- Want good properties
  - Locality
  - Convex Hull
  - Variation Diminishing
  - Shift Invariant (sum of blending functions = 1)

B-Splines

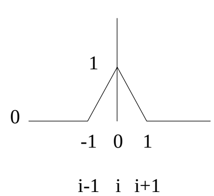
- Approximating splines
- Useful for making very smooth curves
- Very general
  - Any degree, lots of other generalities
- Lots of good properties
  - C(d-1), local, …
- Need to think about in terms of blending functions
  - Not useful to think segment by segment

Blending functions

- Re-write functions in terms of control points
  - Sum(i) Bi(u) Pi
- See this for 1 curve segment
  - Different "roles" for each point
- See this for the whole curve
  - Each point plays a different role at a different time
  - Blending function over all time is a piecewise polynomial
- Each blending function is same, but shifted

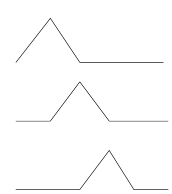
Blending functions for a line

- Each point active in 2 segments
- Blending function is linear in each segment
- Has a "bump shape"
- Zero outside of a range
  - Locality
- This is a B-Spline of degree 1!



An example of blending functions

- Lines
- Blending functions are "bumps"
- Each piece is a spline
  - Two polynomials, degree 1
- Locality
  - Non-Zero over small range
  - Between 3 knots (d+2)
- Smoothness
  - D-1 continuity
- Shift invariant
- Convex Hull Property
- Shiftable (periodic)



B-Spline properties

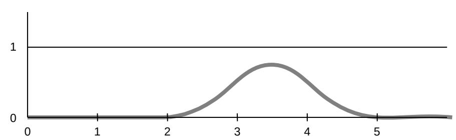
- Local = non-zero only for (d+1) segments
- C(d-1) = blending functions have this property, then linear combinations (blends) of them do too
- Shift invariant (functions are the same, just shifted)
  - symmetric
- Convex Hull property
- Variation Diminishing
- Can encode them in matrix form (just can't derive them that way)

Quadratic (d=2) B-Splines

- Needs to start (end) at zero
- Needs to have f'(0)=0
  - So can't be a line segment – wouldn't get off the ground
- Know what beginning and end pieces must be
  - How many pieces to turn around?
- Remember – blending function
  - At any time, 3 points are "active"
  - One in each phase

Quadratic, Uniform B-Splines

$$b_{i,3}(t) = \begin{cases} \frac{1}{2}u^2 & \text{if } i \le t < i+1 \quad u = t - i \\ -u^2 + u + .5 & \text{if } i+1 \le t < i+2 \quad u = t - (i+1) \\ \frac{1}{2}(1-u)^2 & \text{if } i+2 \le t < i+3 \quad u = t - (i+2) \\ 0 & \text{otherwise.} \end{cases}$$



Cubic B-Splines

- C(2)
- 4 phases – each a cubic polynomial