Rotations
- preserve 0
- preserve handedness
- preserve distance

Are linear transforms (N x N orthonormal matrices with positive determinant)

They compose
if $A$ is a rotation and $B$ is a rotation

$R = AB$ is a rotation
$R' = BA$ is a rotation
$R'' = R$ (they do not commute)

Rot 90 X, Rot 90 Z
vs.
Rot 90 Z, Rot 90 X

Local vs. Global axes
Can do either - but they are different
2-Axis

\[
\begin{bmatrix}
\cos \Theta & -\sin \Theta & 0 & 0 \\
\sin \Theta & \cos \Theta & 0 & 0 \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

Y-Axis

- remember the right hand rule
to get the right

\[
\begin{bmatrix}
\cos \Theta & 0 & \sin \Theta & 0 \\
0 & 1 & 0 & 0 \\
-\sin \Theta & 0 & \cos \Theta & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]

X-Axis

\[
\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & \cos \Theta & -\sin \Theta & 0 \\
0 & \sin \Theta & \cos \Theta & 0 \\
0 & 0 & 0 & 1
\end{bmatrix}
\]
(x, y, z) rotate about an arbitrary axis 
(x, y, z), θ

1. rotate so axis is aligned w/ x-axis
   rotate around z-axis (point is on xz plane)
   rotate around y-axis (point is on x axis)

Note 1: You can pick any 3 axes!

Note 2:
   We can make a rotation just using rotations around the fixed axes!
Euler's Theorems

Any rotation can be represented by:

1. A single rotation about an arbitrary axis
2. A sequence of 3 rotations around fixed axes

So you can represent a rotation (a 3x3 orthonormal, positive matrix) by:

9 numbers - if you're very careful
4 numbers - axis + angle
3 numbers $\text{Rot}_x \times \text{Rot}_y \times \text{Rot}_z$
   or $Z \times Y \times Z$
   or $Z \times Z$

Good things about Euler Angles

compact = 3 numbers
any 3 numbers $\rightarrow$ rotation
complete (any rotation can be represented)
efficient (relatively easy - make 3 1d rotations)
easy
GIMBALL LOCK

Assume XYZ (any order has this problem)

if \( Y = 90^\circ \), \( X \) and \( Z \) are the same!
arrow points down \( X \) axis

\[ \begin{array}{c}
\text{if } Y = 90^\circ \\
\text{how to rotate}
\end{array} \]

Can't rotate \( X \) (does nothing; arrow is axis)

Can't rotate \( Z \) (does nothing; arrow is axis - after \( Y \) rotation)

Why is this bad?

How to get from a small change = a big change in rotation in numbers

\[ 0, 90, 0 \rightarrow ? \]
I can't even figure it out!

or if you go right to left, imagine an airplane starting down the \( Z \) axis - how to make it point towards \( X \), but a little bit down
What do we do with rotation in 559

1. rotate around a single axis
   no problems

2. build a rotation matrix as needed
   use cross-product of first 2 vectors
   to get Z

3. use axis angle

4. Euler Angles are ok if:
   - you can figure out their values
   - you don't need to compose them
   - you don't need to interpolate them

5. you can always compose rotation matrices