CS559 Final Exam

December 20, 2006

This exam is closed book. You may use 1 sheet of paper with notes, but you must turn this page in with the exam (write your name on it).

You will have the entire period (12:25-2:25) to complete the exam, although the exam is designed to take less time.

Please write your name and CS login on every page!

Write numerical answers in fractional form or use radicals (square root symbols) – we would prefer to see $\sqrt{3}$ than .866. You should not need a calculator for this exam.

Unless otherwise noted, assume that everything is a right-handed coordinate system and that angles are measured counter clockwise. E.g. to find the direction of rotation, point your thumb along the axis and curl your fingers.

If you need extra space, use the back of a page, but clearly mark what everything is. We may look at your work to determine partial credit.

The exam has 100 points.

Q1: __ / 8    A    >=  85 pts
Q2: __ / 12    AB  >= 75 pts
Q3: __ / 10    B    >=  65 pts
Q4: __ / 12    BC >=  55 pts
Q5: __ / 10    C    >=  45 pts
Q6: __ / 18
Q7: __ / 8    Mean = 69.5
Q8: __ / 8    Median = 71
Q9: __ / 6
Q10: __ / 8

Total: ___ / 100
**Question 1: (8 points)**

The following compound curve is made up of semi-circular arcs (some of radius 1, some of radius 2), and a line segment. Each curve segment is unit parameterized, so that any semi-circle or line segment takes the same amount of “time” to traverse. Each curve segment is drawn in a different shade of gray to make it easier to see where one begins and the next begins, and the points at these transitions are labels A-E.

![Diagram of the curve](image)

For each point, state the highest degree of C continuity and G continuity that the curve has at that point. We’ve answered the question for point A. For point A, the curve is C(0) (but not C(1)) and G(0) (but not G(1)).

A: C(0) G(0)

B: C(0) G(1) C: C(1) G(1)

D: C(0) G(1) E: C(0) G(\text{infinity})

Note: this one is tricky, partial credit
For anything > 1
Question 2: (12 points)

A Bezier curve segment (7th degree) has the following 8 control points:
(0,0) (4,0) (4,4) (4,8) (8,8) (8,4) (12,4) (12,0)

The curve is divided into two curves at (u=.5).
What are the first three control points for the first part of the curve?

*Hint: only determine the position of the first three control points – computing more points would be a lot of work!*

(0,0) (2,0) (3,1)

(use DeCastljam’s algorithm - partial credit for a picture)

Question 3: (10 points)

Some of the following curve segments could not possibly be a 4th degree Bezier curve segment (e.g. a Bezier curve segment with 5 control points) because they violate a property of Bezier curves. For each of the following curve segments, explain which property it violates, or say that it could be a 4th degree Bezier curve.

<table>
<thead>
<tr>
<th></th>
<th>3A: Yes</th>
<th>3B: No – has a discontinuity</th>
<th>3C: No – too many wiggles – violates variation diminishing</th>
<th>3D: Yes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td><img src="image1.png" alt="Curve" /></td>
<td><img src="image2.png" alt="Curve" /></td>
<td><img src="image3.png" alt="Curve" /></td>
<td><img src="image4.png" alt="Curve" /></td>
</tr>
</tbody>
</table>
Question 4: (12 points)

Bump Mapping is an approximation to Displacement Mapping.

4A: Describe two different situations where the difference between Bump Mapping and Displacement Mapping would be visible.

- **Edges of Objects (silhouettes)**
- **Self Shadowing**
- **Self Occlusion**
  - Viewing parallel to surface

4B: While it is easy to implement Bump-Mapping per-pixel using “standard” programmable graphics hardware (e.g. with separate Pixel (or Fragment) and Vertex shaders), it would be difficult (or maybe impossible) to do per-pixel displacement mapping. Explain why.

- **Pixel Shader cannot move a point to another pixel, so a displacement could only be towards the camera.**

Question 5: (10 points)

Environment mapping makes some assumptions that allow for the creation of shiny surfaces using standard graphics hardware.

5A: Explain what the assumptions are and describe a situation in which Environment mapping works well. (6pts)

- **Assume the object is very small relative to the distance to the reflected object.**
- **Alternatively, the reflected objects are far away.**
- **Or the mirror is approximated as a single point**

Other observations that are true are OK, but no points:
- e.g. No self-reflection

Partial credit for the consequence of assumptions (only normal direction matters).

- **A small, spherical object in a big room works well.**
- **Reflections of distant landscapes and sky on a car work well.**

5B: Describe a situation where the assumptions of Environment mapping are violated, and it would be a poor approximation of a shiny object. (4pts)

- **A planar mirror (or anything that isn’t “just a point” relative to the objects)**
- **If a mirror is flat, the position matters so the “just a point” fails.**
Question 6: (18 points)

A planar mirror shows the same image that would be seen through the mirror if the eye point was placed on the opposite side of the mirror. (equivalently, you could imagine moving the object to the other side of the mirror). We use the term “reflection point” to mean the location on the mirror’s surface where a point on an object appears to be.

Use the convention that the Y axis points upwards, so the Y=0 plane is the floor.

Imagine that we have a mirrored floor (the Y=0 plane), and the eye point is at the position (10,5,0).

6A: Where would the reflection point of 0,2,2 appear be in the mirror? (your answer should be a point with Y=0)

\[ \frac{20}{7}, 0, \frac{10}{7} \]

See attached derivation. I formed the matrix and plugged in the numbers. You could have done it geometrically.

6B: Give a 4x4 homogeneous transformation matrix that would transform any point with Y>0 to the place where its reflection would appear. Your answer should be a 4x4 matrix that contains only numbers (no variables) in it.

See attached derivation
Mirror:

Let the position of the camera be: $C_x$, $C_y$, $C_z$
So the "virtual Eye Point" is: $C_x$, $-C_y$, $C_z$
Let the position of the point we'll looking at be $X, Y, Z$

Let the position of the apparent reflection be: $X', Y', Z'$
(we known that $X'$ is 0)

The $X$ and $Z$ directions are done the same way, but since for the problem we only need to consider $C_z=0$, its a little easier so we'll derive that case first.

The picture (1) shows the similar triangles.

\[
\begin{align*}
\frac{Z'}{Z} &= \frac{Cy}{Cy + Y} \\
\end{align*}
\]

So:

\[
Z' = \frac{Z Cy}{Cy + Y}
\]

For the $X$ direction, we have to account for $C_x$.
We have a very similar picture.

To make things a little easier, we define the distance $L'$ that is the unknown in the similar triangles.

\[
\begin{align*}
\frac{L'}{C_x - X} &= \frac{Cy}{Cy + Y} \\
\end{align*}
\]

Which is quite similar to the $Z$ case, so we can solve:

\[
L' = \frac{Cy (C_x - X)}{Cy + Y}
\]

Since $X' = C_x - L'$

\[
X' = C_x - \left(\frac{Cy (C_x - X)}{Cy + Y}\right)
\]

Put everything (including that first $C_x$ over the common denominator $(Cy+Y)$):

\[
X' = \frac{C_x (Cy + Y) - Cy (C_x - X)}{Cy + Y}
\]

Collect up common terms

\[
X' = \frac{(C_x Y + Cy X)}{Cy + Y}
\]

(Notice that if $C_x$ is 0, we get the same answer as we got for $Z$).

Plugging in $XYZ=(0,2,2)$ and $CxCyCz = (10,5,0)$

We get $X' = \frac{(20+0)}{(5+2)} = \frac{20}{7}$
$Y' = 0$
and $Z' = \frac{10}{7}$

To write this as a matrix, we note that in projective coordinates, the
$W$ component is $(Cy + Y)$

So

\[
\begin{align*}
0 & Cy & 0 & 0 & 0 & 5 & 10 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & Cy & 0 & 0 & 0 & 5 & 0 & 0 \\
0 & 1 & 0 & Cy & 0 & 1 & 0 & 5 & 0 \\
\end{align*}
\]

Confirm that when this matrix is multiplied by the column vector $[X Y Z 1]$, we get:

$X_h = (Cy X + Cx Y)$, $Z_h = (Cy Z)$, and $W_h = Y+Cy$. 

Question 7: (8 points)

Lit from above using the Phong lighting model, a shiny sphere looks (approximately) like: The light source is straight above the sphere, and the camera is viewing the sphere horizontally.

Sketch how the sphere would look with:

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>A) No Specular Lighting</td>
<td>B) No Diffuse Lighting</td>
<td>C) With the shininess increased to a large value</td>
<td>D) more ambient lighting</td>
</tr>
</tbody>
</table>

Question 8: (8 points)

Give two situations where the order that objects are drawn matters when using a Z-Buffer.

When Z-values are the same or very close (Z-Fighting)
When objects are transparent
When there are aliasing artifacts
Intersecting objects is right IF you explained they are Z equal
Question 9: (6 points)

A vertex is drawn at the origin (0,0,0). It is viewed through a camera that has its viewing matrix:

$$\begin{bmatrix}
\frac{1}{2} & -\frac{1}{2} & 0 & -2 \\
\frac{1}{2} & \frac{1}{2} & 0 & -2 \\
0 & 0 & 1 & 2 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

The modeling transformation that is applied to the point is:

$$\begin{bmatrix}
0 & -1 & 0 & 2 \\
1 & 0 & 0 & 4 \\
0 & 0 & 1 & 6 \\
0 & 0 & 0 & 1
\end{bmatrix}$$

If this matrix is used for the perspective projection:

$$\begin{bmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1 \\
0 & 0 & -1 & 0
\end{bmatrix}$$

Where does the point appear in the image? (the x,y position)

$$\frac{3}{8}, -\frac{1}{8}$$

Question 10: (8 points)

For each of these properties, say whether they occur for Perspective projection, Orthographic projection, Both, or Neither (mark each P,O,B or N).

A) Far away objects are smaller  P

B) The far clipping plane's position influences how much z-fighting will occur  B

   The range from near to far sets how much z-resolution there is.

C) Can be implemented using homogeneous transformations  B

D) Can sight down any axis  B