ICS59 – Lecture 6
Resampling, Color

These are course notes (not used as slides)
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With some slides adapted from the notes of Stephen Chenney

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Sampling Theory

- Sampling is multiply by spike chain in time domain
  - Fourier transform of spike chain is spike chain
  - Fourier transform of multiply is convolution
- Sampling is convolution by spike chain in frequency
- Makes infinite copies of signal
- Reconstruction low-pass filters to remove all but one
- Non-band limited, things “spill”

Sampling / Reconstruction

- Both sampling and reconstruction require Low Pass Filtering
- Sampling:
  - Low pass filter signal to make sure is band-limited
- Reconstruction:
  - Low pass filter spike chain to figure out what happens between samples
- Resampling:
  - Reconstruction followed by sampling

Resizing = Resampling

- Same image – different number of samples
- Issues:
  - New samples are in between old samples
  - Too few new samples to capture all the frequency
- Basic idea (in theory)
  - Reconstruct original signal (LPF the samples)
  - Low-pass filter (so sampling works)
  - Sample at new sampling rate

Resampling – Little Square Model

- Region of source = Region of Dst
- Pixel is a region
  - Dest region might be bigger than pixel in source
  - Average over the region (convolution gives us the weights)
- In-between pixels is piecewise constant
  - Chunky look is what the model says is right

Pre-Filtering

- If SRC is bigger than DST it may have HF
  - if its close, might need it anyway because of imperfect reconstruction
- Need to LPF
- LPF before sampling?
  - Requires you to do a complete reconstruction
  - Only really need to do it at points you will sample
- Pre-Filtering
  - Do LPF before reconstruction / as part of reconstruction
  - Order is OK (convolutions commute)
Reconstruction in Practice

- Sample a sample – no problem!
- Issue is samples between samples

- Theory: LPF a spike chain
  - Convolve “reconstruction kernel” with samples
  - Only really need to evaluate at places where you’ll sample
- Another view: interpolation
  - Different interpolations are different filters

Some reconstruction kernels

- Constant
- Triangle (Bartlet)
- Cubic (Catmull-Rom)

Spacing (1 unit = sample distance)
Scaling issues
Interpolating (non-interpolating kernels exist as well)
Approx to Ideal LPF

Reconstruction Example

- Could do this as linear interpolation
  - Generalizes nicely this way
- Need to evaluate filter for various values
- Convolve reconstruction kernel with sampling kernel (LPF for frequency limit)
- Easier ways to implement nearest neighbor

Functional Form for Filters

- Consider the Bartlett in 1D:
  \[ H(x) = \frac{2}{w} \left( 1 - \frac{|x|}{w} \right) \]
  - To apply it at a point \( x \) and find the contribution from point \( x' \) where the image has value \( f(x') \)
  \[ f(x) = \frac{2}{w} \left( 1 - \frac{|x|}{w} \right) f(x') \]
  - Extends naturally to 2D:
  \[ f(x, y) = \frac{4}{w^2} \left( 1 - \frac{|x|}{w} \right) \left( 1 - \frac{|y|}{w} \right) f(x, y) \]

General Resampling

- Could be any transformation on \( x, y \)
  - \( X', y' = f(x, y) \)
- Scale, translate, rotate, something weird
- Kernel should get warped too
  - Little square -> some weird shape
  - Little circle/square (of kernel) -> some weird shape
  - In practice, stick with squares

Reverse Warping

- Note we generally need the INVERSE:
  - \( X', y' = f(x, y) \) (\( X' = \text{dst}, x = \text{src} \))
  - Know \( x' \), need to find \( x \) is inverse
- Reverse warping is easier (scan over each pixel in the dst, figure out where it comes from)
- Forward warping is trickier
  - Usually can invert function, but if you can’t
  - Need to worry about holes
- Lots of fun warps to do!
Shift Gears: Color

- Color
- Quality of Light
  - Has a wavelength – not just an amount
- Can measure the spectrum of light
  - Graph wavelength vs. amount at the measurement
- Different spectra give different “color impressions”

Colors

- One dominant wavelength = pure color
- No dominant wavelength = “white” (or black/gray)
- What do we perceive?
  - Luminence (amount of light)
  - Color (dominant)
  - Purity of Color
- Complications
  - Differences in perception
  - Artist notions vs. physics vs. psychology

Sensing Color

- Different sensors have different sensitivities
  - Spectrum of sensor
  - Convolution with spectrum gives response
- Ideal photo sensor / real photo sensor
- Cameras – wide range sensor
  - Put filters in front of each CCD element
  - Different parts of spectra (R,G,B)
  - Bayer Mosaic (need to interpolate)
  - Foveon

Color Vision in Animals

- Rods = all the same
  - No color vision
- Cones = have different kinds
  - 1-chromat (can’t see color) -> Dogs
  - bi-chromat (2 different types) -> large mamals
  - Tri-chromat -> humans ***
    - Color blindness = lack of 1 type
    - Rare genetics condition gives a 4th type
  - Some birds have 4 or 5 types of cones
    - Ducks&Pigeons have 5, European starlings have 4

Distinguishing colors

- 1 sensor
  - All colors look the same
  - Combination of colors looks like any color
- Metamers – perceptually indistinguishable
- 2 sensors
  - Non-overlap case (what differences?)
  - Overlap case
    - Middle vs. combination of sides

Faking Colors

- Metamers allow for faking
  - 2 cones = 2 frequencies
- Two different overlapping cones respond
  - Some of each color?
  - Some of the in-between color
- Can fake responses using N “point” colors
- Get either cone, or anything in overlap
  - Colors outside of overlap can’t be faked