CS 559: Computer Graphics

Homework 4

This homework must be done individually. Submission date is Tuesday, October 26, 2004, in class. It may not be graded in time for the midterm, so we recommend making a copy before you hand this hoemwork in. You can then compare your copy to the solutions.

Question 1:

A pair of transformations is said to commute if the order in which you apply them does not matter. In terms of transformation matrices, that means that $\mathbf{AB} = \mathbf{BA}$. Consider three rotation matrices and a translation matrix:

$$\mathbf{R}_1 = \begin{bmatrix} c_1 & -s_1 & 0 & 0 \\ s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_2 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_2 & -s_2 & 0 \\ 0 & s_2 & c_2 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{R}_3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & c_3 & -s_3 & 0 \\ 0 & s_3 & c_3 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} 1 & 0 & 0 & t_x \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

a. What axis is $\mathbf{R}_1$ rotating about?

b. What axis are $\mathbf{R}_2$ and $\mathbf{R}_3$ rotating about?

c. Do $\mathbf{R}_1$ and $\mathbf{R}_2$ commute?

d. Do $\mathbf{R}_2$ and $\mathbf{R}_3$ commute?

e. Under what circumstances do two rotation matrices commute?

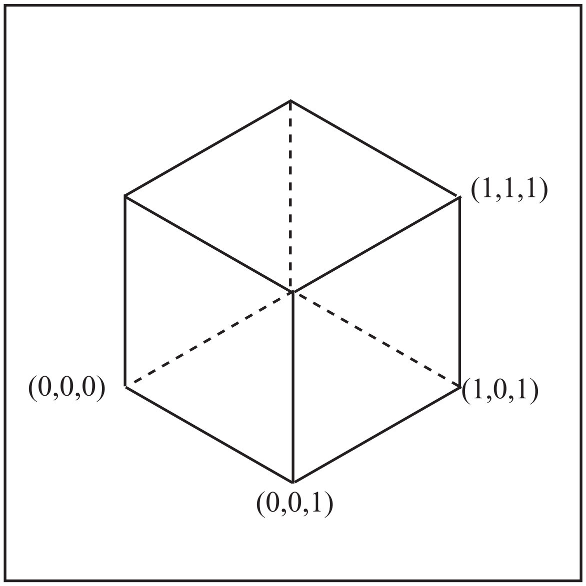
f. Do $\mathbf{R}_1$ and $\mathbf{T}$ commute?

g. Do $\mathbf{R}_2$ and $\mathbf{T}$ commute?

h. Under what circumstances can you say that a rotation and a translation do commute?

Question 2:

The image below is an orthographic projection of a cube. The cube has a side length of 1. Some of the corners are marked – enough to determine the others. Hidden edges are marked as dashed lines.



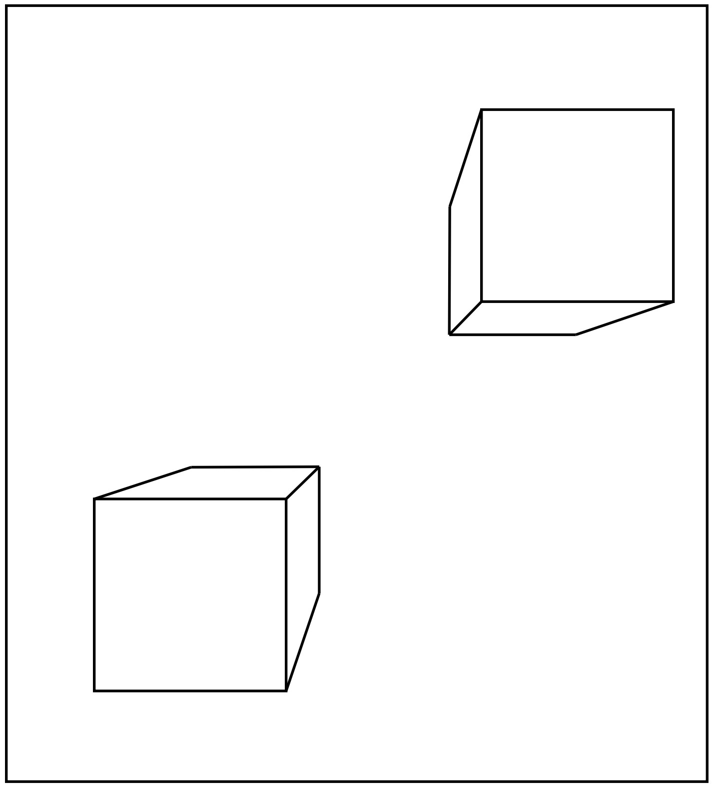
a. What is the gaze direction, g?

b. What is a suitable *up* vector?

c. Do you have enough information from the image alone to identify an *eye* location? If so, what is it? If not, what else do you need to know to determine an eye?

Question 3:

The image below shows two cubes drawn with perspective projection.



Aside from the fact that the objects are cubes, you have no information other than the contents of the image. This question is concerned with the conclusions you can draw about the scene based on the image.

a. Can you determine whether the cubes are aligned the same way? That is, can you decide if it is possible to scale and translate one cube to turn it into the other, without any rotation? You don't have to actually give the transformation, nor even say whether they are aligned.

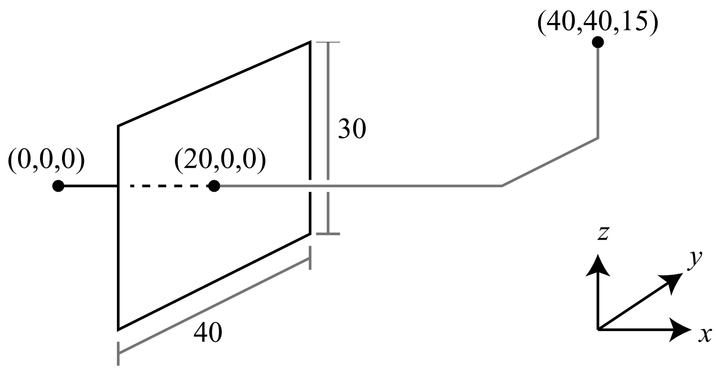
b. Can you determine which cube is bigger in the world, or if they are the same size?

Question 4:

Conventional graphics assumes that the person looking at the screen is seated directly in front of the screen and is focused at the center of the screen. However, for virtual reality applications you want the world to really appear to be behind the screen. This requires tracking the user's head and presenting an image that matches the arrangement of their eyes and the screen. In particular, the near clipping plane should coincide with the window, and the viewer's virtual eyes should coincide with their real location.

To make all this work, you first create a global coordinate system that is the same as a real world coordinate system. In the set-up below, the origin of the virtual world (and real world) is behind the screen. The center of the screen is at (20, 0, 0), the "right" on the screen is in the direction (0, 1, 0) and "up" on the screen is in the direction (0, 0, 1). The screen has width 40 and height 30. The viewer's eye is at (40,40,15).

Use the notation from class (and in the textbook).



a. What is the vector e, the origin of view space in world space?

b. What is the vector w, the image plane normal vector in world space?

c. What is the vector u, the world space direction that will become x in view space?

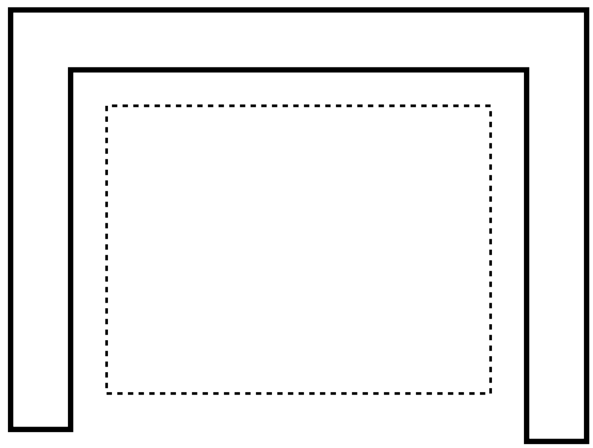
d. What is the vector v, the world space direction that will become y in view space?

e. What is the near clip plane distance?

f. What are l, r, b, t that define the left, right, top and bottom clip planes?

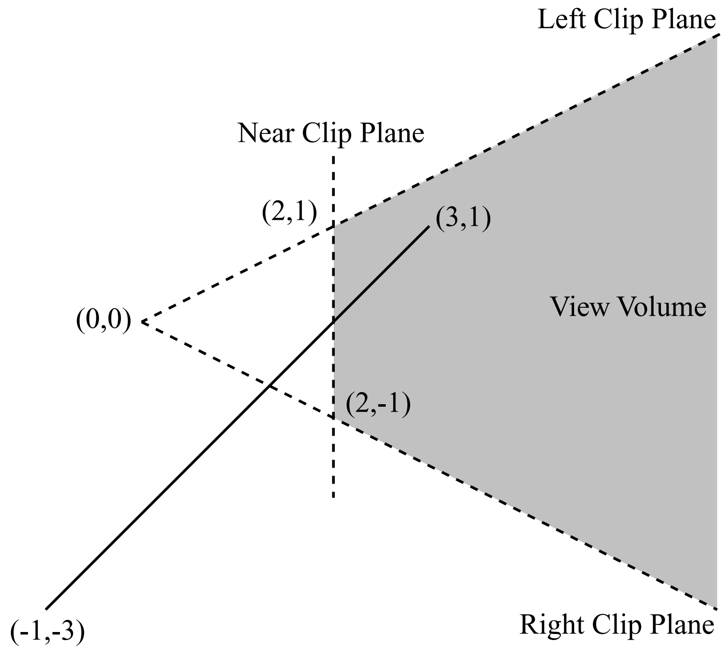
Question 5:

Perform Sutherland Hodgman clipping on the figure below to the rectangular clip region shown dashed. Show the intermediate results after clipping with the top edge, the results after clipping with the top and right edges, the results after the top, right and bottom edges, and the final results. (You should show 4 figures in all.)



Question 6:

This question explores Liang-Barsky clipping. Consider the line segment and 2D perspective clip region shown below.



a. What is the parametric equation of the line? Write it in the form:

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} a \\ b \end{bmatrix} + t \begin{bmatrix} c \\ d \end{bmatrix}$$

b. What is the parametric equation for the near clip plane?

c. What is the parametric equation for the left clip plane?

d. What is the parametric equation for the right clip plane?

e. What are the parametric coordinates (the t values) for the intersections of the line with each clip edge? Label them as entering or leaving intersections.

f. What are the parametric coordinates of the endpoints of the visible segment?

g. What are the (x,y) coordinates of the endpoints of the visible segment?