CS 559: Computer Graphics

Homework 1

This homework must be done individually. Submission date is Tuesday, September 14, 2004, in class.

Encouragement: Some of these questions may look a little difficult; if they do, it would be worth reviewing the math associated with vectors and matrices. An excellent reference for this material is the textbook or the appendices of "Real Time Rendering" by Moller and Haines. Several of the other graphics books on reserve in the library also have good sections on this material.

Question 1: Vectors are extremely important to computer graphics. They are used to represent both locations in space (points) and directions. Assume you have three points in space, represented by $\mathbf{a} = (a_x, a_y, a_z)$, $\mathbf{b} = (b_x, b_y, b_z)$ and $\mathbf{c} = (c_x, c_y, c_z)$.

a. How do you find the direction vector $\mathbf{v}$ that points *from* $\mathbf{b}$ *toward* $\mathbf{a}$?

b. How is the length, $||\mathbf{v}||$, of $\mathbf{v}$ computed?

c. A *unit vector*, $\hat{\mathbf{v}}$, in the direction $\mathbf{v}$ is a vector in the same direction as $\mathbf{v}$ but with length 1. How do you compute $\hat{\mathbf{v}}$? Computing $\hat{\mathbf{v}}$ is also referred to as *normalizing* $\mathbf{v}$.

d. If you wanted to *rapidly* compute which point, $\mathbf{b}$ or $\mathbf{c}$, was closer to $\mathbf{a}$, which quantities would you compare? Why? (Hint: We are not concerned about the actual distances, only about which point is closer.)

Question 2: Consider two vectors in 3D, $\mathbf{a}$ and $\mathbf{b}$.

a. How is the dot product $\mathbf{a} \cdot \mathbf{b}$ computed? The dot product is more generally called the inner product.

b. What is the relationship between $\mathbf{a} \cdot \mathbf{b}$ and the angle, θ, between $\mathbf{a}$ and $\mathbf{b}$?

c. For this part of the question, assume that $\mathbf{a}$ and $\mathbf{b}$ are *unit vectors* – that is, their length is 1. What is the value of $\mathbf{a} \cdot \mathbf{b}$ if:

   (i) $\mathbf{a}$ and $\mathbf{b}$ point in the same direction?

   (ii) $\mathbf{a}$ and $\mathbf{b}$ point in opposite directions?

   (iii) $\mathbf{a}$ and $\mathbf{b}$ are *orthogonal* (at right angles)?

d. How can you write $||\mathbf{a}||$ in terms of a dot product?

Question 3: Consider two vectors in 3D, $\mathbf{a}$ and $\mathbf{b}$.

a. How are the elements of the cross product vector $\mathbf{c} = \mathbf{a} \times \mathbf{b}$ computed?

b. What is the geometric relationship between $\mathbf{a}$, $\mathbf{b}$ and $\mathbf{c}$?

c. What is the geometric relationship between $\mathbf{a} \times \mathbf{b}$ and $\mathbf{b} \times \mathbf{a}$?

d. What is the relationship between $\mathbf{c}$ and the angle, θ, between $\mathbf{a}$ and $\mathbf{b}$?

e. For this part of the question, assume that $\mathbf{a}$ and $\mathbf{b}$ are unit vectors. What is the length of $\mathbf{c}$ if:

   (i) $\mathbf{a}$ and $\mathbf{b}$ point in the same direction?

   (ii) $\mathbf{a}$ and $\mathbf{b}$ are orthogonal?

Question 4: This question concerns the definition of a plane in three dimensions.

a. What is the minimum number of points needed to define a unique plane in 3D that passes through all the points? What other conditions must the points satisfy for the plane to be unique?

b. Given more than the minimum number of points, is it in general possible to find one plane that passes through all of them?

c. Label your points $\mathbf{p}_1$, $\mathbf{p}_2$, etc. Using these points it is possible to define several vectors that all lie in the plane. Give two such vectors.

d. How would you compute the *normal vector* for the plane given your points? (Hint: Use the results from Part *c* and Question 3.)

e. Typically, the implicit equation of a plane is written as either $\mathbf{n} \cdot (\mathbf{p} - \mathbf{x}) = 0$ or $ax + by + cz + d = 0$, where $\mathbf{p}$ is any point on the plane and $\mathbf{n}$ is the normal vector. What are the values of a, b, c and d in terms of $\mathbf{n} = (n_x, n_y, n_z)$ and $\mathbf{p} = (p_x, p_y, p_z)$? (Hint: Expand out the dot product form of the equation using $\mathbf{x} = (x, y, z)$.)

f. What is the geometric meaning of the quantity d in the implicit equation?

Question 5: What is the result of the following matrix multiplication of a vector?

$$\begin{bmatrix} 0 & 2 & 1 \\ -2 & 0 & 1 \\ 0 & 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$