

Question 1: Curve Derivations (12 points)

Consider a quadratic curve segment, whose canonical form is:

$$f(u) = au^2 + bu + c.$$

We would like to specify this curve segment by providing the position of the beginning, middle and end. That is: $p_0 = f(0)$, $p_1 = f(.5)$, and $p_2 = f(1)$

Find M^{-1} , the *inverse* of the matrix that maps from the three control points to the canonical parameters for this curve.

Hint: go through the process of finding M as we did with cubics, just skip the step that's hard to do by hand.

$$\begin{aligned} f(u) &= au^2 + bu + c \\ p_0 = f(0) &= c \\ p_1 = f(.5) &= \frac{1}{4}a + \frac{1}{2}b + c \\ p_2 = f(1) &= a + b + c \end{aligned}$$

$$p = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & \frac{1}{2} & 1 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

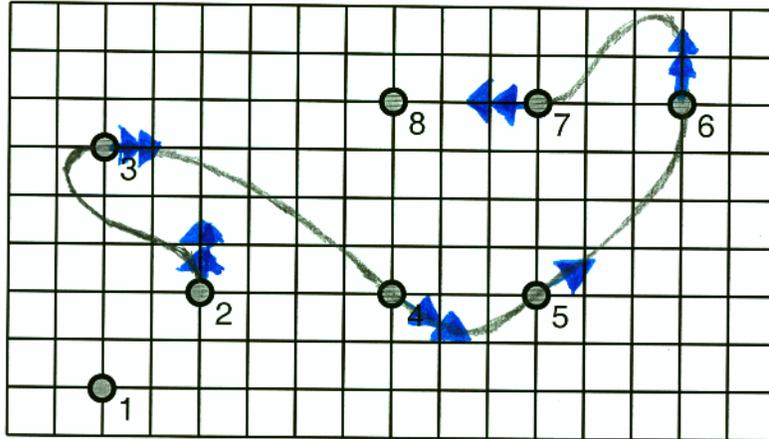
$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \left(\begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & \frac{1}{2} & 1 \\ 1 & 1 & 1 \end{bmatrix} \right)^{-1} p$$

$$M^{-1} = \begin{bmatrix} 0 & 0 & 1 \\ \frac{1}{4} & \frac{1}{2} & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

Question 2: Catmull-Rom Splines (10 pts)

Sketch the Catmull-Rom (a.k.a Cardinal cubic spline with tension 0) through the following control points. Draw arrows for the tangent to the curve at its beginning and end.

The control points are numbered 1 through 8 and do not cycle.



NOTE: You ONLY HAD TO DRAW ARROWS AT POINTS 2 AND 7

points for:

Curve starts @ 2

Curve ends @ 7

arrow at begin and end pointing vertically and horizontally

right general shape

overshooting of 3 and 6

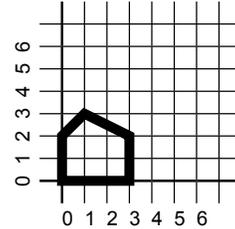
Question 3: 2D Transformations (12 pts)

Write the program (consisting of translate, rotate, scale, and “draw house” commands) that creates each of the following pictures.

The commands are:

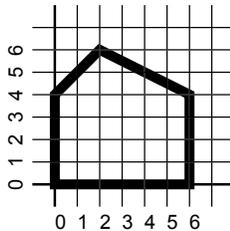
- trans(x,y) - translate by x,y
- rot(theta) - rotate clockwise by theta degrees
- scale(sx,sy) - non-uniform scale
- drawHouse - draws the house

drawHouse() draws this picture:



notice that it is NOT symmetrical

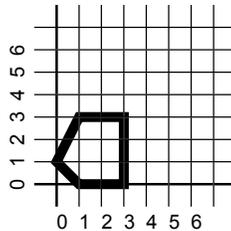
Example: scale(2,2)
draw house



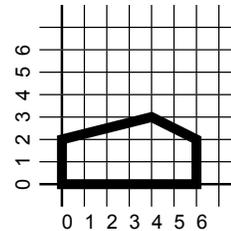
Note:
 The commands affect the current transformation (just like OpenGL).
 The drawHouse command should be your last line.
 Shorter programs are preferable.

WARNING: There are many possible solutions!
 These are just representative!

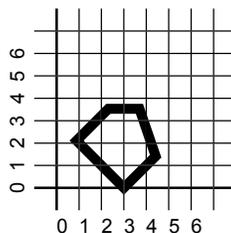
A) trans 3,0
rot -90
draw house



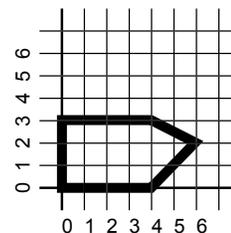
B) trans 6,0
scale -2,1
draw house



C) trans 3,0
rot 45
trans -3,0
draw house



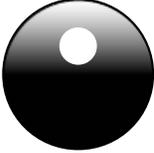
D) rot 90
scale 1,2
trans -3,0
draw house



Note: some students thought the "origin" corner went to (1,2) rather than (3-2*sqrt(2), 2*sqrt(2)). These solutions were accepted.

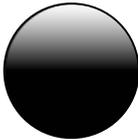
Question 4: Lighting (8 points)

Lit from above using the Phong lighting model, a shiny sphere looks (approximately) like: The light source is straight above the sphere, and the camera is viewing the sphere horizontally.

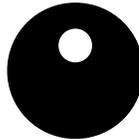


Sketch the way the sphere would look: _____

A) with no specular lighting



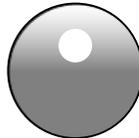
B) with no diffuse lighting



C) with the shininess value raised by a large amount



D) with much more ambient lighting

**Question 5: Viewing (4 pts)**

For each of these properties, say whether they occur for Perspective projection, Orthographic projection, Both, or Neither (mark each P,O,B or N).

- A) Far away objects are smaller **P**
- B) The far clipping plane's position influences how much z-fighting will occur **B**
- C) Can be implemented using homogeneous transformations **B**
- D) Can sight down any axis **B**