HW3
CS559

Xin Li

1.

$$P_0(u) = (u^3 \; u^2 \; u \; 1)\, M \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{pmatrix}$$

$$P_1(u) = (u^3 \; u^2 \; u \; 1)\, M \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{pmatrix}$$

$$\Rightarrow \; P_0'(u) = (3u^2 \; 2u \; 1 \; 0)\, M \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{pmatrix}$$

$$P_1'(u) = (3u^2 \; 2u \; 1 \; 0)\, M \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{pmatrix}$$

$$\Rightarrow \; P_0''(u) = (6u \; 2 \; 0 \; 0)\, M \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{pmatrix}$$

$$P_1''(u) = (6u \; 2 \; 0 \; 0)\, M \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{pmatrix}$$

$$P_0'(1) = (3 \; 2 \; 1 \; 0)\, M \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{pmatrix} = -3P_1 + 3P_3$$

$$P_1'(0) = (0 \; 0 \; 1 \; 0)\, M \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{pmatrix} = -3P_1 + 3P_3$$

$$P_0''(1) = (6 \; 2 \; 0 \; 0)\, M \begin{pmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \end{pmatrix} = 6P_1 - 12P_2 + 6P_3$$

$$P_1''(0) = (0 \; 2 \; 0 \; 0)\, M \begin{pmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{pmatrix} = 6P_1 - 12P_2 + 6P_3$$

So $P_0'(1) = P_1'(0)$ and $P_0'(1) = P_1''(0)$

The curve is C^2 continuous.

2. $p(u) = (u^3\ u^2\ u\ 1)\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

$p(0) = d$

$p(1) = a + b + c + d$

$p(\frac{1}{2}) = \frac{1}{8}a + \frac{1}{4}b + \frac{1}{2}c + d$

$p'(\frac{1}{2}) = \frac{3}{4}a + b + c$

so $\begin{pmatrix} p(0) \\ p(1) \\ p(\frac{1}{2}) \\ p'(\frac{1}{2}) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ \frac{1}{8} & \frac{1}{4} & \frac{1}{2} & 1 \\ \frac{3}{4} & 1 & 1 & 0 \end{pmatrix}\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

$\Rightarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} -4 & 4 & 0 & -4 \\ 8 & -4 & -4 & 6 \\ -5 & 1 & 4 & -2 \\ 1 & 0 & 0 & 0 \end{pmatrix}\begin{pmatrix} p(0) \\ p(1) \\ p(\frac{1}{2}) \\ p'(\frac{1}{2}) \end{pmatrix}$

3. $p(u) = (u^3\ u^2\ u\ 1)\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$

$p(0) = d$

$p(\frac{1}{3}) = \frac{1}{27}a + \frac{1}{9}b + \frac{1}{3}c + d$

$p(\frac{2}{3}) = \frac{8}{27}a + \frac{4}{9}b + \frac{2}{3}c + d$

$p(1) = a + b + c + d$

So
$$\begin{pmatrix} p(0) \\ p(\frac{1}{3}) \\ p(\frac{2}{3}) \\ p(1) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 1 \\ \frac{1}{27} & \frac{1}{9} & \frac{1}{3} & 1 \\ \frac{8}{27} & \frac{4}{9} & \frac{2}{3} & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$$

$$\Rightarrow \begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix} = \begin{pmatrix} -\frac{9}{2} & \frac{27}{2} & -\frac{27}{2} & \frac{9}{2} \\ 9 & -\frac{45}{2} & 18 & -\frac{9}{2} \\ -\frac{11}{2} & 9 & -\frac{9}{2} & 1 \\ 1 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p(0) \\ p(\frac{1}{3}) \\ p(\frac{2}{3}) \\ p(1) \end{pmatrix}$$

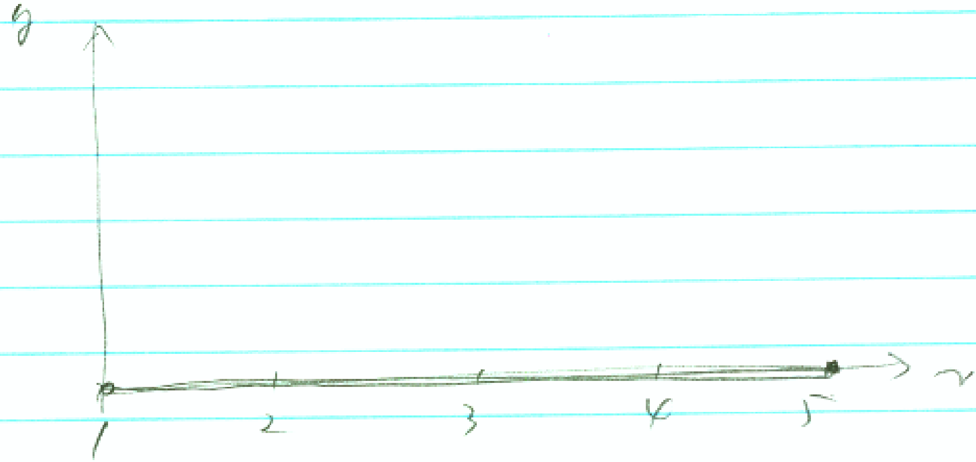
4.
$$p(u) = (u^4 \quad u^3 \quad u^2 \quad u \quad 1) \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix}$$

$$\begin{pmatrix} p(0) \\ p(\frac{1}{4}) \\ p(\frac{1}{2}) \\ p(\frac{3}{4}) \\ p(1) \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & 1 \\ \frac{1}{256} & \frac{1}{64} & \frac{1}{16} & \frac{1}{4} & 1 \\ \frac{1}{16} & \frac{1}{8} & \frac{1}{4} & \frac{1}{2} & 1 \\ \frac{81}{256} & \frac{27}{64} & \frac{9}{16} & \frac{3}{4} & 1 \\ 1 & 1 & 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix}$$

So
$$\begin{pmatrix} a \\ b \\ c \\ d \\ e \end{pmatrix} = \begin{pmatrix} \frac{32}{3} & -\frac{128}{3} & 64 & -\frac{128}{3} & \frac{32}{3} \\ -\frac{80}{3} & 96 & -128 & \frac{224}{3} & -16 \\ \frac{70}{3} & -\frac{208}{3} & 76 & -\frac{112}{3} & \frac{22}{3} \\ -\frac{25}{3} & 16 & -12 & \frac{16}{3} & -1 \\ 1 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} p(0) \\ p(\frac{1}{4}) \\ p(\frac{1}{2}) \\ p(\frac{3}{4}) \\ p(1) \end{pmatrix}$$

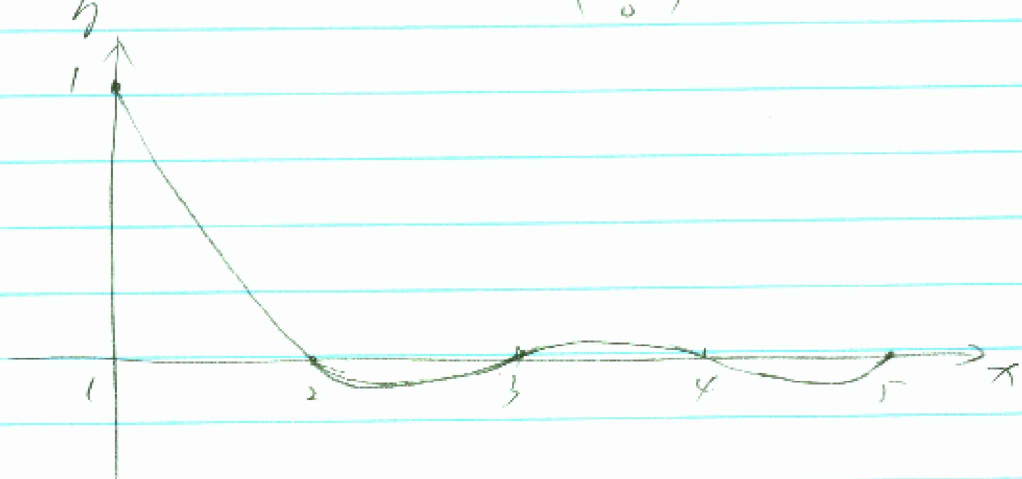
5. For control points $(1,0), (2,0), (3,0), (4,0) \,\&\, (5,0)$

$$\begin{cases} x(u) = (u^4\ u^3\ u^2\ u\ 1)\, M \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} = 4u+1 \\ y(u) = (u^4\ u^3\ u^2\ u\ 1)\, M \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = 0 \end{cases}$$



For control points $(1,1), (2,0), (3,0), (4,0) \,\&\, (5,0)$

$$\begin{cases} x(u) = (u^4\ u^3\ u^2\ u\ 1)\, M \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \end{pmatrix} = 4u+1 \\ y(u) = (u^4\ u^3\ u^2\ u\ 1)\, M \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} = \frac{32}{3}u^4 - \frac{128}{3}u^3 + 64u^2 - \frac{128}{3}u + \frac{32}{3} \end{cases}$$



① The computation of higher order curve is much more expensive.

② The higher order curve has less locality in control. A small change to one control point can cause big change to the whole curve.